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Dual Hop Ostbc Transmission Over Rayleigh Fading Channel For Regenerative System

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Abstract – **This paper presents performance analysis of cooperative multiple-input multiple-output (MIMO) relay network system with single-relay. The MIMO scheme is based on Alamouti space time block coding (STBC) over Rayleigh flat fading channels. The source node, equipped with two transmit antennas, simply broadcasts each STB code to the relay and the destination nodes. The relay node, equipped with multiple antennas amplifies-and-forwards (AF) the received STB codes. The destination node applies a maximum ratio combining (MRC) and exploits the diversity gain obtained by the direct and the indirect link simultaneously. The moment generating functions (MGF) of the signal-to-noise ration (SNR) for the direct and the indirect link are given in a closed forms. These statistical results are then applied to derive a lower bound of the symbol error probability (SEP) for a particular signal of M-ary-quadrature-amplitude modulation (M-QAM) and to obtain the outage probability through numerical evaluations. Subsequently, simulation results of the SEP and the outage probabilities are presented to illustrate the performance improvement given by the MIMO cooperative diversity systems based on STBC schemes.**

Keywords – **MIMO relay channel, space-time block coding, MRC scheme, moment generating function (MGF), probability outage.**

I. Introduction

Cooperative communication has recently attracted a lot of interest due to its ability to realize the performance gains and coverage extension [1] [2] [3] . Typically, it concerns a system where users share and coordinate their resources to enhance the transmission quality and to optimize the power allocation. The combination of relaying system with MIMO processing is a straightforward extension of both concepts. Moreover, this combination gives additional degrees of freedom to improve the capacity of the overall cooperative system [4] [5] .

Recently, it has been demonstrated that cooperation based on space time block codes (STBC) represents an effective way to introduce spatial diversity in wireless scenarios where we can not take the full benefit of the uncorrelated channels from the multi-antenna systems. Cooperative diversity gains can be achieved through creating distributed virtual antennas across different terminals in the network. Actually, there exist two ways to apply STBC technologies in cooperative system. In the first way, cooperation using distributed STBC is applyed in order to create a virtual transmit array in a distributed multiple relay network [6] [7] [8] . For example, in [8] , the Alamouti space time code [9] is employed with a distributed manner in a cooperative relay network over Rayleigh fading environment. In the second way, the STBC matrix is completely broadcasted to the relay and the destination. Several time slots are employed during the transmission of each STBC matrix. For an Alamouti STBC, cooperative relay network with single relay system and two receive antennas can be looked as a virtual MIMO system with four receive antennas, in consequence, the performane of the cooperative diversity system can be improved without increasing the number of receive antennas. In [10] , the authors neglect the source destination link and consider that one source transmits an STBC matrix via one relay node using AF protocol. Specifically, they derive the exact SEP for maximum likelihood (ML) decoding of orthogonal space time block codes in dual-hop relay channels and they do not consider the direct link.

Another work related to the use of STBC as in the second way is given in [11] . In their paper, the authors have investigated the performances of MIMO relaying systems with decode-and-forward (DF) protocol where the source, the relay and the destination are multiple-antenna nodes. Specifically, the authors derive a closed form expression for the outage probability of the SNR.

Two recent papers related to our contribution give performance analysis of cooperative diversity system based on STBC scheme over some particular scenarios [13] [14] . In [13] , Safari and Uysal derive an upper-bound on the pairwise error probability (PEP) for cooperative diversity schemes over log-normal fading channels and the distributed SIMO, MISO and MIMO systems. In their contributions, the authors derive a Chernoff bound on PEP and a union bound on bit error rate (BER) performance where each node is equipped with single antenna. In [14] , Muhaidat and Uysal give a derivation of the PEP by including an extension to multiple antennas nodes. In their paper, the authors derive a closed form of the PEP for dual-hop relaying scheme and channel state information (CSI) assisted AF, blind AF and DF. In their work, the source node transmits a general STBC matrix and the destination applies a maximum likelihood decoding (ML) considering only the indirect link (the direct link is neglected). However, this derivation seems to be incomplete since in most realistic cases the diversity is increased by considering both the direct and indirect links.

For MIMO relay channel, there has been no work to derive the symbol error probability at the destination when the source broadcasts Alamouti STBC matrix codes and amplify-and-forward strategy are used in MIMO systems. We complete the contributions of [14] as follows:

1) Deriving the SEP for more general cooperative diversity by including the direct and indirect links.

2) Avoiding the complexity of the ML decoder by using an MRC based decoder determining the SEP at the demapper front end for general M-QAM constellation.

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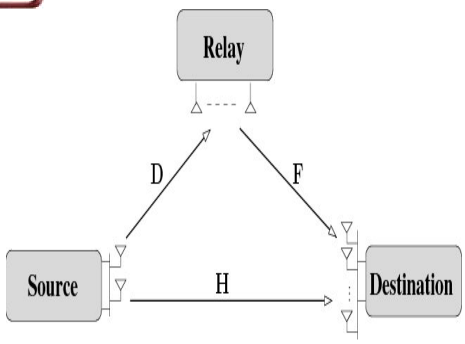

Fig.1. MIMO cooperation scheme with source node broadcasting Alamouti STBC matrix.

In this paper, we present performance analysis of a cooperative MIMO relay system based on Alamouti scheme [9] (Fig. 1). The moment generating functions (MGF) of the signal-to-noise ratio for the direct and the indirect link are given in a closed forms. These MGF are then applied to derive a closed form of the lower bound of the SEP and to obtain the outage probability.

The paper is organized as follows. In Section II, we introduce system model. Section III presents the SNR analysis at the output of the destination for the direct link. In Section IV, we develop an upper bound of the SNR at the output of the destination for the indirect link. In Section V, using the MGF of the SNR for the direct and indirect links, we give a lower bound on the SEP of the cooperative link taking into account the diversity given by these two links. Performances related to the outage probabilities and the diversity gains are detailed in section VI. We discuss and analyze numerical results in Section VII. Finally, conclusions are drawn in Section VIII.

II. SYSTEM MODEL

In this section, we describe a MIMO cooperative system consisting of a source, relay and destination nodes equipped with multiple antennas. We consider the amplify-and-forward cooperative MIMO relay channel as shown in Fig. 1. The source, relay and destination nodes have Nt, Nr and Nr antennas, respectively. In order to provide an efficient coding rate, we use 2×2 Alamouti matrix code, we consider $Nt = 2$ transmits antennas. With a slight modification in the MIMO STBC model, we have transformed the channel matrix $\mathbf{H}$ into a modified channel matrix $\mathbf{H}[\mathrm{X}_2]$ with orthogonal columns and $2Nr\times 2$ entries. According to the same modification in MIMO STBC channel model, the source-destination, the source-relay and the relay-destination channel matrices are respectively $\mathbf{H}[\mathrm{X}_2]\in \mathbb{C}^{2Nr\times 2}$, $\mathbf{D}[\mathrm{X}_2]\in \mathbb{C}^{2Nr\times 2}$ and $\mathbf{F}[\mathrm{X}_2]\in \mathbb{C}^{2Nr\times 2Nr}$.

We assume a half duplex relaying protocol so, the transmission from the source to the destination is made in two separate time-slots as in time division multiple access (TDMA) systems [16] . In the first time-slot, the source sends its Alamouti encoded signal $\mathbf{x}$ to the relay and the destination, where $\mathbf{x} = [s_1, s_2]^T$ is the vector symbols that composes the Alamouti matrix. The relay simply amplifies the received signal before forwarding it to the destination during the second time-slot. Finally, the destination combines the signals of two time-slots coming from the relay and the source nodes using maximum ratio combining.

The vector signal transmitted by the source and received by the destination is given as follows:

$$\mathbf{y}_0 = \sqrt{\alpha_{sd}\mathbf{E_s}}.\mathbf{H}[\mathrm{X}_2]\mathbf{x} + \mathbf{n}_0$$

where $\mathbf{n}_0$ is the modified $2Nr\times 1$ noise vector measured at the destination which is composed by zero mean complex Gaussian random variables with variance $\mathbf{N}_0$. $\mathbf{E_s}$ and α_{sd} are the symbol energy and the pathloss factor applied at the direct link, respectively.

At the destination node, according to the MIMO STBC modelization given above, the received signal is a vector $\mathbf{y}_0$ given by

$$\begin{bmatrix} y_{1,1} \\ y_{1,2}^* \\ \vdots \\ y_{Nr,1} \\ y_{Nr,2}^* \end{bmatrix} = \sqrt{\alpha_{sd}\mathbf{E_s}} \begin{bmatrix} h_{1,1} & h_{1,2} \\ h_{1,2}^* & -h_{1,1}^* \\ \vdots & \vdots \\ h_{Nr,1} & h_{Nr,2} \\ h_{Nr,2}^* & -h_{Nr,1}^* \end{bmatrix} \mathbf{x} + \begin{bmatrix} n_{1,1} \\ n_{2,1}^* \\ \vdots \\ n_{Nr,1} \\ n_{Nr,2}^* \end{bmatrix} \quad (2)$$

if we consider the relayed link, the vector signals observed at the relay is

$$\mathbf{y}_r = \sqrt{\alpha_{sr}\mathbf{E_s}}.\mathbf{D}[\mathrm{X}_2]\mathbf{x} + \mathbf{n}_r \quad (3)$$

Where α_{sr} is the path loss factor related to the source relay link (first hop link), and $\mathbf{n_r}$ is the $2Nr\times 1$ modified vector noise applied at the relay, which is composed by a zero mean complex random variables with variance $\mathbf{N}_0$. At the relay, the system applies AF protocol with a matrix gain $\mathbf{G}$ defined as

$$\mathbf{G} = \sqrt{\frac{\alpha_{rd}\mathbf{E_r}}{\alpha_{sr}\mathbf{E_s}\|\mathbf{D}\|_F^2 + \mathbf{N}_0}}.\mathbf{I}_{2Nr} \quad (4)$$

Where $\|\mathbf{D}\|_F$ is the Frobenius norm of the matrix $\mathbf{D}$.

In order to simplify the analysis, we assume a fixed gain, $\mathbf{G} = \sqrt{\frac{\alpha_{rd}\mathbf{E_r}}{\alpha_{sr}\mathbf{E_s}+\mathbf{N}_0}}.\mathbf{I}_{2Nr} = g\mathbf{I}_{2Nr}$, where $\mathbf{I}_{2Nr}$ is the $2Nr\times 2Nr$ identity matrix. We notice here, that fixed gain $\mathbf{G}$ used for AF relaying is considered in many papers in the literature, (see for example [17]). At the destination, the relayed signal is given by

$$\mathbf{y}_d = \sqrt{\alpha_{sr}\mathbf{E_s}}\mathbf{FGD}[\mathrm{X}_2]\mathbf{x} + \mathbf{FG}[\mathrm{X}_2]\mathbf{n_r} + \mathbf{n_d} \quad (5)$$

the equivalent virtual MIMO, $4Nr\times 2$, representation of the proposed MIMO relay system is given as follows

$$\begin{bmatrix} \mathbf{y}_0 \\ \mathbf{y}_d \end{bmatrix} = \begin{bmatrix} \sqrt{\alpha_{sd}\mathbf{E_s}}\mathbf{H}[\mathrm{X}_2] \\ \sqrt{\alpha_{sr}\mathbf{E_s}}\mathbf{FGD}[\mathrm{X}_2] \end{bmatrix}\mathbf{x} + \begin{bmatrix} \mathbf{I}_{2Nr} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{FG}[\mathrm{X}_2] & \mathbf{I}_{2Nr} \end{bmatrix} \begin{bmatrix} \mathbf{n}_0 \\ \mathbf{n_r} \\ \mathbf{n_d} \end{bmatrix} \quad (6)$$

The equivalent source-relay-destination channel matrix is

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given by $\mathbf{U}[X_2]' = \mathbf{FGD}[X_2]$ let $\mathbf{U} = \mathbf{FD}[X_2]$.

$$\mathbf{U}[X_2] = \begin{bmatrix} u_{1,1} & u_{1,2} \\ u^*_{1,2} & -u^*_{1,1} \\ \vdots & \vdots \\ u_{Nr,1} & u_{Nr,2} \\ u^*_{Nr,2} & -u^*_{Nr,1} \end{bmatrix}. \quad (7)$$

with the **FD** matrix product, $u_{i,j}$ are defined as

$$u_{i,j} = \sum_{k=1}^{Nr} f_{i,k} d_{k,j} \quad (1 \le i \le Nr),\ (1 \le j \le 2) \quad (8)$$

The equivalent noise measured at the output of the relayed link is

$$\mathbf{n}_e = \mathbf{FG}[X_2]\mathbf{n}_r + \mathbf{n}_d \quad (9)$$

In order to evaluate the SEP at the output of the MRC at the destination, we decompose the problem into two sub-problems. In the first one, we derive the SNR at the MRC output considering the direct link only. In the second sub-problem, we calculate the SNR at the MRC output for the relayed link and finally we use the MGF of each SNR to give the symbol error probability and to evaluate de outage probability of the cooperative and non cooperative links.

III. SNR OF THE DIRECT LINK

The modified matrix $\mathbf{H}[X_2]$ describing the equivalent (*source-destination*) channel has orthogonal columns [15, p. 285]. Applying the MRC to the received vector signal $\mathbf{y}_0$, we have as output

$$\tilde{y}_0 = \mathbf{H}[X_2]^H.\mathbf{y}_0 = \sqrt{\alpha_{sd}\mathbf{E_s}}\left(\sum_{i=1}^{Nr}|h_{i,1}|^2 + |h_{i,2}|^2\right)\mathbf{x} + \tilde{\mathbf{n}}_0 \quad (10)$$

where $\tilde{\mathbf{n}}_0$ is the equivalent noise measured at the output of the MRC given as

$$\tilde{\mathbf{n}}_0 = \mathbf{H}^H[X_2].\mathbf{n}_0 = \sum_{i=1}^{Nr}\begin{pmatrix} h_{i,1}n_{i,1} + h_{i,2}n^*_{i,2} \\ h^*_{i,2}n_{i,1} - h_{i,1}n^*_{i,2} \end{pmatrix} \quad (11)$$

The covariance matrix of the equivalent noise $\tilde{\mathbf{n}}_0$ is

$$\varepsilon\{\tilde{\mathbf{n}}_0\tilde{\mathbf{n}}_F^H\} = N_0 \|\mathbf{H}\|_F^2 \mathbf{I}_2 \quad (12)$$
$$\varepsilon\{\tilde{\mathbf{n}}_0\} = \mathbf{0}_{2,1}$$

where $\varepsilon\{.\}$ is the expectation of $\{.\}$. 1 The effective channel for the data symbols $s_i,\ _{i\in\{1,\ldots,M\}}$ is

$$z_i = \sqrt{\alpha_{sd}\mathbf{E_s}}\|\mathbf{H}\|_F^2 s_i + \tilde{n}_{0_i} \quad s_i,\ _{i\in\{1,\ldots,M\}} \quad (13)$$

Hence, according to (12) and (13), the SNR of the signal transmitted through the source-destination link and measured at the output of the MRC is given by

$$\gamma_{sd} = \frac{\alpha_{sd}\mathbf{E_s}.\|\mathbf{H}\|_F^2}{N_0} \quad (14)$$

Since the channel of the direct link with matrix $\mathbf{H}$ is flat Rayleigh fading then all the entries $h_{i,j}$ of $\mathbf{H}$ are complex Gaussian random variables each with $N(0,1)$ distribution. Hence, γ_{sd} is a random variable equal to the sum of $4Nr$ Gaussian random variables each of which with $N(0,1)$ distribution. Then, γ_{sd} is a Chi-squared random variable with degrees of freedom $4Nr$. Thus, the probabiliy density function (pdf) of γ_{sd} is

$$f_{\gamma_{sd}}(\gamma) = \frac{1}{2^{2Nr}\Gamma(2Nr)}\frac{\gamma^{2Nr-1}}{\overline{\gamma}^{2Nr}}\exp\left(-\frac{\gamma}{2\overline{\gamma}_{sd}}\right) \quad (15)$$

where $\overline{\gamma}_{sd} = \alpha_{sd}.\frac{\mathbf{E_s}}{N_0}(2Nr\sigma_h^2)$ is the average SNR of the source-destination link and $\Gamma(.)$ is the Gamma function [18, eq. (8.310.1)] defined as $\Gamma(n) = (n-1)!$ where n is an integer, $n > 0$. The moment generating function, $(M_{\gamma_{sd}}(s))$, of γ_{sd} is then given by

$$M_{\gamma_{sd}}(s) = \int_{-\infty}^{\infty} f_{\gamma_{sd}}(\gamma)\exp(-s\gamma)d\gamma \quad (16)$$

Since γ_{sd} is a Chi-squared random variable with mean $\overline{\gamma}_{sd}$,thenusing $\int_0^{\infty}\gamma^n\exp(\beta\gamma)d\gamma = (n!)\beta^{-(n+1)}$, the MGF of γ_{sd} can be easily found as

$$M_{\gamma_{sd}}(s) = (1 + 2\overline{\gamma}_{sd}s)^{-m_0} \quad (17)$$

where $m_0 = 2Nr$

IV. SNR OF THE INDIRECT LINK

In this section, we evaluate the SNR of the relayed link (γ_{srd}) at the output of the MRC. The modified matrix $\mathbf{U}[X_2]$ describing the equivalent relayed link **FD** has orthogonal columns. Applying the MRC to the vector signal $\mathbf{y}_d$, according to [15, p.285], the output of the MRC is

$$\tilde{\mathbf{y}}_d = \mathbf{U}'[X_2]^H.\mathbf{y}_d$$
$$= \sqrt{\alpha_{sr}\mathbf{E_s}}\left(\sum_{i=1}^{Nr}|u_{i,1}|^2 + |u_{i,2}|^2\right)\mathbf{x} + \tilde{\mathbf{n}} \quad (18)$$

Where $\tilde{\mathbf{n}}$ is the equivalent noise channel measured at the output of the MRC

$$\tilde{\mathbf{n}} = \mathbf{U}'^H[X_2].\mathbf{n}_e \quad (19)$$

Substituting (9) in (19), the covariance matrix of the equivalent noise $\tilde{\mathbf{n}}$ is given by

$$\varepsilon\{\tilde{\mathbf{n}}\tilde{\mathbf{n}}^H\} = N_0\sum_{i=1}^{Nr}\left(|u_{i1}|^2 + |u_{i2}|^2\right)\left(1 + g^2\sum_{k=1}^{Nr}|f_{ik}|^2\right) \quad (20)$$

Then, the effective channel for the data symbols $s_i,\ _{i\in\{1,\ldots,M\}}$ is

$$z_i^r = \sqrt{\alpha_{sr}.\mathbf{E_s}}g\|\mathbf{U}\|_F^2 s_i + \tilde{n}_i \quad _{i\in\{1,\ldots,M\}} \quad (21)$$

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by letting $\bar{\gamma}_{sr} = \frac{\alpha_{sr}\mathbf{E_s}}{N_0}$ as the mean SNR of the source-relay link, the instantaneous SNR γ_{srd} of the source-relay-destination link will be given by

$$\gamma_{srd} = \frac{\bar{\gamma}_{sr} g^2 \left[\sum_{i=1}^{Nr} |u_{i1}|^2 + |u_{i2}|^2\right]^2}{\sum_{i=1}^{Nr}\left(\left[|u_{i1}|^2 + |u_{i2}|^2\right]\left[1 + g^2 \sum_{k=1}^{Nr} |f_{ik}|^2\right]\right)} \tag{22}$$

Deriving a closed-form expression of the PDF of the instantaneous SNR γ_{srd} is too hard to accomplish. Hence, it is more easy to use an upper bound result of the SNR and make a comparison with the exact analysis obtained by simulation. The upper bound of the SNR in (22) is obtained by neglecting the term $g^2 \sum_{k=1}^{Nr} |f_{ik}|^2$ in (22)

$$\gamma_{srd} \le \bar{\gamma}_{sr} . g^2 \left[\sum_{i=1}^{Nr} |u_{i1}|^2 + |u_{i2}|^2\right] \tag{23}$$

We consider the case where $Nr \ge 4$. In order to derive a closed-form of the upper bound of γ_{srd}, we assume that $\mathbf{F}$ and $\mathbf{D}$ are random matrices with complex Gaussian entries $f_{i,l}$ and $d_{i,l}$, respectively. Precisely, each real and imaginary part is $\mathrm{N}(0,1)$ random variable

$$f_{il} = a_{il} + jb_{il} \text{ and } d_{il} = a'_{il} + jb'_{il} \tag{24}$$

Substituting (24) in (8) and evaluating the expression in more compact form we have

$$u_{i,l} = \lambda_{i,l} + j\mu_{i,l} = \sum_{k=1}^{Nr}\left(a_{ik}d'_{kl} - b_{ik}b'_{kl}\right) + j\left(a_{ik}b'_{kl} + b_{ik}d'_{kl}\right)$$
$$(1 \le i \le Nr),(1 \le l \le 2) \tag{25}$$

Without loss of generality, omitting the indices i,k and l, the random variables a, a', b and b' are $\mathrm{N}(0,\sigma^2)$. Then, according to the Gaussian random variable properties, the product $x = aa'$, is a random variable equal to the product of two independent Gaussian random variables with zero mean and variance σ_1 and σ_2 respectively. According to [19, Ch. 6], this product is a zero mean random variable and its pdf is given by

$$p_X(x) = \frac{1}{\pi\sigma_1\sigma_2} K_0\left(\frac{|x|}{\sigma_1\sigma_2}\right) \tag{26}$$

Where $K_0(.)$ is the Bessel function of the second kind and order zero. Fig. 2 illustrates the pdf of X obtained analytically and by simulation.

The real part $\lambda_{i,l}$ and the imagine part $\mu_{i,l}$ of the matrix entries $u_{i,l}$ in (25) are equal to the sum of $2\,Nr$ zero mean random variables with modified Bessel function distribution of order 0. According to the central-limit theorem, $\lambda_{i,l}$ and $\mu_{i,l}$ may be approximated by Gaussian random variable with zero mean and variance $2Nr(\sigma_f^2\sigma_d^2)$.

As shown in Fig. 3 the pdf of $\lambda_{i,l}$ is Gaussian function centered at zero.

Hence, $|u_{il}|^2 = \lambda_{il}^2 + \mu_{il}^2$ are exponential random variables with probability density function, $p_U(u) = \exp(-u/\sigma)/\sigma$ for $u > 0$. Thus, $\|\mathbf{U}\|_F^2$ equal to the sum of four exponential random variables with parameter $2\sigma^2$, is a Chi-squared random variable with degree of freedom $2 \times 2\,Nr$. The pdf of γ_{srd} is upper bounded by

$$f_{\gamma_{srd}}(\gamma) = \frac{1}{2^{(2Nr)}\Gamma(2Nr)} \frac{\gamma^{(2Nr-1)}}{\bar{\gamma}_{srd}^{-(2Nr)}} \exp\left(-\frac{\gamma}{2\bar{\gamma}_{srd}}\right) \tag{27}$$

Where $\bar{\gamma}_{srd} = (\frac{\alpha_{sr}\mathbf{E_s}}{N_0} g^2)(2Nr\sigma_f^2\sigma_d^2)$.Since, the SNR at the output of the relayed link γ_{srd} is a Chi-squared random variable with degrees of freedom $4Nr$ and using the same derivation as for equation (17), the MGF of γ_{srd} is given by denoting $m_1 = 2Nr$

$$M_{\gamma_{srd}}(s) = (1 + 2\bar{\gamma}_{srd}s)^{-m_1} \tag{28}$$

V. *SEP* OF THE COOPERATIVE SCHEME

The cooperation is based on the use of two independent branches: the direct and indirect links. The SEP must average the two branches conditional over the pdf of γ_{sd} and γ_{srd}. For M-QAM constellation, the average SEP expression, obtained by the MGF method, can be written as the sum of two terms, denoted by I_1 and I_2 [20],

$$SEP(\gamma) = \overbrace{\frac{4q}{\pi} . \int_0^{\frac{\pi}{2}} M_{\gamma_{sd}}\left(\frac{\mathrm{G_{QAM}}}{\sin^2\theta}\right) M_{\gamma_{srd}}\left(\frac{\mathrm{G_{QAM}}}{\sin^2\theta}\right) d\theta}^{\mathrm{I}_1} - \underbrace{\frac{4q^2}{\pi} \int_0^{\frac{\pi}{4}} M_{\gamma_{sd}}\left(\frac{\mathrm{G_{QAM}}}{\sin^2\theta}\right) M_{\gamma_{srd}}\left(\frac{\mathrm{G_{QAM}}}{\sin^2\theta}\right) d\theta}_{\mathrm{I}_2} \tag{29}$$

Where $\mathrm{G_{QAM}} = 3/[2(M-1)]$, $q = 1 - 4/\sqrt{M}$.

For the first term in (29), if we substitute (17) and (28) in (29) and we make the change of variable $t = \cos^2\theta$, after some manipulations, we obtain

$$\mathrm{I}_1 = \frac{2q}{\pi} M_{\gamma_{sd}}(\mathrm{G_{QAM}}) M_{\gamma_{srd}}(\mathrm{G_{QAM}}) \int_0^1 t^{-\frac{1}{2}} (1-t)^{-\frac{1}{2}+m_0+m_1} \left(1 - \frac{1}{1 + 2.\mathrm{G_{QAM}}\bar{\gamma}_{sd}}.t\right)^{-m_0} \left(1 - \frac{1}{1 + 2.\mathrm{G_{QAM}}\bar{\gamma}_{srd}}.t\right)^{-m_1} dt \tag{30}$$

For the second term, upon making the change of variable $t = 1 - \tan^2\theta$, we obtain

$$\mathrm{I}_2 = \frac{4q^2}{\pi} M_{\gamma_{sd}}(2\mathrm{G_{QAM}}) M_{\gamma_{srd}}(2\mathrm{G_{QAM}}) \int_0^1 t^{-\frac{1}{2}} (1-t)^{-\frac{1}{2}+m_0+m_1}$$

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$$\left(1-\frac{1+2G_{QAM}\bar{\gamma}_{sd}}{1+4G_{QAM}\bar{\gamma}_{sd}}t\right)^{-m_0}\left(1-\frac{1+2G_{QAM}\bar{\gamma}_{srd}}{1+4G_{QAM}\bar{\gamma}_{srd}}t\right)^{-m_1}\left(1-\frac{1}{2}t\right)^{-1}dt \quad (31)$$

In order to continue the derivation of the SEP, we define the Lauricella multivariate Hypergemetric function $F_D^{(n)}$ [21] as

$$F_D^{(n)}=(a,b_1,\ldots,b_n;c;x_1,\ldots,x_n)$$
$$\sum_{i_1,\ldots,i_n=0}^{\infty}(a)_{i_1+\ldots+i_n}\frac{(b_1)_{i_1}\ldots(b_n)_{i_n}}{(c)_{i_1+\ldots+i_n}}\frac{x_1^{i_1}}{i_1!}\ldots\frac{x_n^{i_n}}{i_n!}$$
$$\max\{|x_1|\ldots|x_n|\}<1$$
$$=\frac{\Gamma(c)}{\Gamma(a)\Gamma(c-a)}\cdot\int_0^1 t^{\alpha}-1(1-t)^{c-a-1}\prod_{i=1}^{L}(1-x_it)^{-b_i}\,dt. \quad (32)$$
$$\mathrm{Re}(c)>\mathrm{Re}(a)>0$$

where $(a)_n=\Gamma(a+n)/\Gamma(a)$ is the Pochhammer symbol, with $(a)_0=1$. Therefore, with the help of (32), the average *SEP* of square M-QAM constellation can be deduced as

$$SEP=\frac{2q\Gamma\left(\frac{1}{2}+m_0+m_1\right)}{\sqrt{\pi}\Gamma(1+m_0+m_1)}M_{\gamma_{sd}}(G_{QAM})M_{\gamma_{srd}}(G_{QAM})F_D^2$$
$$\left(\frac{1}{2},m_0,m_1;1+m_0+m_1;\frac{1}{1+2G_{QAM}\bar{\gamma}_{sd}},\frac{1}{1+2G_{QAM}\bar{\gamma}_{srd}}\right)-$$
$$\frac{q^2}{\pi\left(\frac{1}{2}+m_0+m_1\right)}M_{\gamma_{sd}}(2G_{QAM})M_{\gamma_{srd}}(2G_{QAM})$$
$$F_D^3\left(1,m_0,m_1,1;\frac{3}{2}+m_0+m_1;\frac{1+2G_{QAM}\bar{\gamma}_{sd}}{1+4G_{QAM}\bar{\gamma}_{sd}},\frac{1+2G_{QAM}\bar{\gamma}_{srd}}{1+4G_{QAM}\bar{\gamma}_{srd}},\frac{1}{2}\right) \quad (33)$$

If we consider only the indirect link, the SEP is simply given by

$$SEP=\frac{2q\Gamma\left(\frac{1}{2}+m_1\right)}{\sqrt{\pi}\Gamma(1+m_1)}M_{\gamma_{srd}}(G_{QAM})$$
$$F_D^1\left(\frac{1}{2},m_1;1+m_1;\frac{1}{1+2G_{QAM}\bar{\gamma}_{srd}}\right)$$
$$-\frac{q^2}{\pi\left(\frac{1}{2}+m_1\right)}M_{\gamma_{srd}}(2G_{QAM})F_D^2 \quad (34)$$
$$\left(1,m_1,1;\frac{3}{2}+m_1;,\frac{1+2G_{QAM}\bar{\gamma}_{srd}}{1+4G_{QAM}\bar{\gamma}_{srd}},\frac{1}{2}\right)$$

It is easily to generalize the derivation of the SEP for a MIMO cooperative diversity system based on Alamouti STBC sheme. The SNR of each indirect link will be upbounded as in equation () and then the lowerbound of the SEP is as

$$SEP=\frac{2q\Gamma\left(\frac{1}{2}+m_0+\ldots+m_k\right)}{\sqrt{\pi}\Gamma(1+m_0+\ldots+m_k)}M_{\gamma_{s0d}}(G_{QAM})\ldots M_{\gamma_{skd}}(G_{QAM})$$
$$F_D^{(k+1)}\left(\frac{1}{2},m_0,\ldots,m_k;m_0+\ldots+m_k;\frac{1}{1+2G_{QAM}\bar{\gamma}_{s0d}},\ldots,\frac{1}{1+2G_{QAM}\bar{\gamma}_{skd}}\right)$$
$$-\frac{q^2}{\pi\left(\frac{1}{2}+m_0+\ldots+m_k\right)}M_{\gamma_{s0d}}(2G_{QAM})\ldots M_{\gamma_{skd}}(2G_{QAM})$$
$$F_D^{(k+2)}\left(1,m_0,\ldots,m_k,1;\frac{3}{2}+m_0+\ldots+m_k;\frac{1+2G_{QAM}\bar{\gamma}_{s0d}}{1+4G_{QAM}\bar{\gamma}_{s0d}},\ldots,\frac{1+2G_{QAM}\bar{\gamma}_{skd}}{1+4G_{QAM}\bar{\gamma}_{skd}},\frac{1}{2}\right) \quad (35)$$

where the index k term in m_k and in γ_{skd} refers to the source destination link for $k=0$ and to the source-k th-relay-destination link for $k>=1$.

VI. Outage Prbability erformances

In addition to the average SEP, outage probability, denoted by $P_{out}(\gamma_{th})$, is another standard performance criterion of cooperative diversity systems. It is defined as the probability that the instantaneous error rate exceeds a specified value or equivalently that the (instantaneous) combined SNR γ_{cop} falls below a certain specified threshold (γ_{th}), i.e.,

$$P_{out}=P[0\le\gamma_{cop}\le\gamma_{th}]=\int_0^{\gamma_{th}}p_{\gamma_{cop}}(\gamma_{cop})d\gamma_{cop} \quad (36)$$

where $p_{\gamma_{cop}}(\gamma_{cop})$ is the probability density function of γ_{cop}. Mathematically speaking, the outage probability coincides with the cumulative distribution function (CDF) of γ_{cop} evaluated at γ_{th} which is equal to the inverse Laplace transform $\mathcal{L}^{-1}(\bullet)$ of the ratio $\mathcal{M}_{\gamma_{cop}}(-s)/s$ evaluated at γ_{th} [22]

$$P_{out}=\mathcal{L}^{-1}\left(\frac{M_{\gamma_{cop}}(s)}{s}\right)_{|\gamma_{th}} \quad (37)$$

According to the assumption of mutually independent channels, the MGF, $\mathcal{M}_{\gamma_{cop}}$ of the cooperative link can be expressed as

$$\mathcal{M}_{\gamma_{cop}}(s)=\mathcal{M}_{\gamma_{sd}}(s)\,\mathcal{M}_{\gamma_{srd}}(s). \quad (38)$$

where $\mathcal{M}_{\gamma_{sd}}(s)$ and $\mathcal{M}_{\gamma_{srd}}(s)$ are the MGF of the direction and the indirect link given by the results in (17) and (28). We notice here, that the inverse Laplace transform can be derived analytically or using simple numerical techniques. Using the results in [23, equ. 9.186] the equation (37) can be developed as

$$P_{out} = P_{\gamma_{cop}}(\gamma_{th}) = \frac{2^{-K} e^{A/2}}{\gamma_{th}} \sum_{k=0}^{K} \binom{K}{k}$$

$$\times \sum_{n=0}^{N+k} \frac{(-1)^n}{\alpha_n} \Re\left\{ \frac{M_{\gamma_{cop}}\left(-\frac{A+2\pi jn}{2\gamma_{th}}\right)}{\frac{A+2\pi jn}{2\gamma_{th}}} \right\} + E(N,K) \qquad (39)$$

$$= 2^{-K+1} e^{A/2} \sum_{k=0}^{K} \binom{K}{k} \sum_{n=0}^{N+k} \frac{(-1)^n}{\alpha_n} \Re\{ \left(1 - \frac{\bar{\gamma}_{sd}}{\gamma_{th}}(A+2\pi jn)\right)^{-m_0}$$

$$\times \left(1 - \frac{\bar{\gamma}_{srd}}{\gamma_{th}}(A+2\pi jn)\right)^{-m_1} \frac{1}{(A+2\pi jn)} \} + E(N,K) \qquad (40)$$

where $\Re$ denotes the real part and the overall error term $E(N,K)$ is approximated by

$$E(N,K) \cong \frac{e^{A/2}}{\gamma_{th}} \sum_{k=0}^{K} 2^{-K} (-1)^{N+1+k} \binom{K}{k} \Re\left\{ \frac{M_{\gamma_{cop}}\left(-\frac{A+2\pi j(N+k+1)}{2\gamma_{th}}\right)}{\frac{A+2\pi j(N+k+1)}{2\gamma_{th}}} \right\} \qquad (41)$$

$$\cong e^{A/2} \sum_{k=0}^{K} 2^{-K+1} (-1)^{N+1+k} \binom{K}{k} \Re\{ \left(1 - \frac{\bar{\gamma}_{sd}}{\gamma_{th}}(A+2\pi j(N+k+1))\right)^{-m_0}$$

$$\times \left(1 - \frac{\bar{\gamma}_{srd}}{\gamma_{th}}(A+2\pi j(N+k+1))\right)^{-m_1} \frac{1}{(A+2\pi j(N+k+1))} \} \qquad (42)$$

The simple expressions of the outage probability and the corresponding numerical error can be easily computed using commonly used mathematical packages such as Mathematica.

VII. Numerical Results

In this section, numerical results and Monte-Carlo Simulations are provided to discuss the impact of the relay, the number of antennas Nr and the constellation size M on the performance of MIMO STBC cooperative diversity reception with M-QAM scheme. Monte-Carlo simulation are obtained by running the simulators for five runs (each with 10^7 samples). It is assumed that the channel matrices $\mathbf{H}$, $\mathbf{F}$ and $\mathbf{D}$, of the direct, first hop and second hop links, respectively are constant during one frame of 512 symbols, which is essentially the case of quasi-static fading.

In Figs. 4 and 5, the SEP (equation (33)) is shown as a function of the average-to-noise ratio of the relayed link for several values of M and number of receive antennas Nr. The SEP curves, plotted in Fig. 4, shows the analytical lower bound on the SEP and the simulation results obtained by Monte-Carlo simulation for 4-QAM modulation and $Nr = 2{,}3$ and 4 receive antennas. It is clear that the analytical results provide a tight lower bound on the SEP for the overall SNR range. We can notice also the great increase on the SEP performance with the number of receive antennas and this can be justified by the diversity gain obtained by the use of the relay with multiple antennas and the diversity obtained by the receiver equiped by Nr receive antennas.

From Fig. 5, one can see that an increase in the constellation size M affects the system's error performance. It can be seen that a transition from $M = 16$ to $M = 4$ leads to a performance improvement greater than 9 dB for $\mathrm{SEP} = 10^{-3}$. This is related to the intersymbol interference due to the important number of symbols in 16-QAM compared to 4-QAM.

It is clear that the proposed lower bound and the simulation results on the SEP are in excellent agreement. We can notice also the tightness of the derived lower bound of the SEP improves as SNR increases; however this bound loses its tightness at low SNR.

In the two last Figs. 6 and 7, the outage probability of the cooperative diversity system with STBC Alamouti scheme over Rayleigh fading channels is given versus the average signal-to-noise ratio of the relayed link and for Nr=2,3 and 4 receive antennas considering two scenarios for the pathlosses. The curves give numerical representation of (40) and (42) using commonly used mathematical packages such as Mathematica with A set to equal $10\log 10;23.026$ to guarantee a discretization error of less than 10^{-10} and with $K = 15$ and $N = 30$. From these figures, it is clear that the outage probability significantly increases with Nr. It is obvious also, that the proposed lower bound and the simulation results are in excellent agreement. We can notice the tightness of the derived lower bound of the P_{out} improves as SNR increases; however this bound loses its tightness at low SNR.

VIII. Conclusion

In this paper, we have used an upper bound for the signal-to-noise ratio to derive the error probability and the outage probability of the MIMO Relay channel based on Alamouti STBC transmission. The analysis is achieved using the MGF of the SNR of the cooperative links measured at the destination. A lower bound expression of the SEP is given for MRC M-QAM in Rayleigh flat fading channels. The validity of our analytical results is confirmed by Monte-Carlo simulations.

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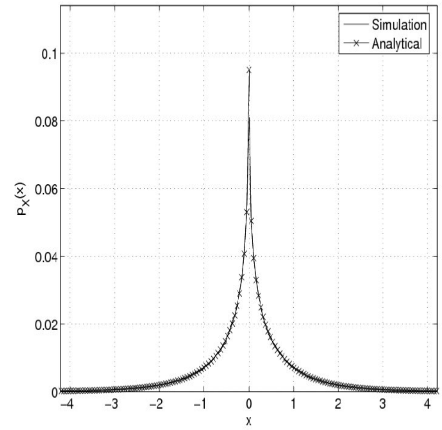

Fig. 2. Pdf of the the random variable $X = ab$, given by

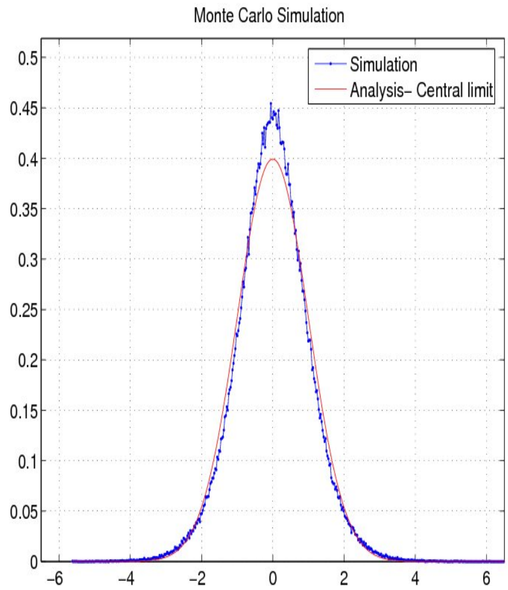

Fig.3. pdf of $\lambda_{i,l}$.

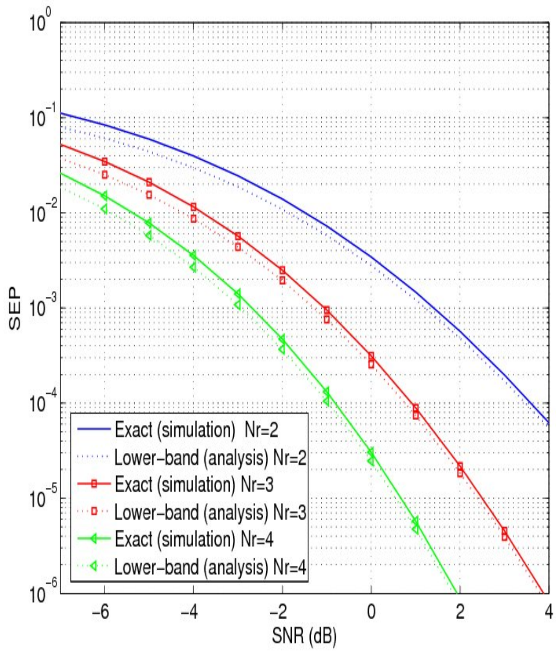

Fig.4. SEP versus SNR for the cooperative link for $Nr = 2,3,$ and 4 antennes, with 4-QAM modulation.

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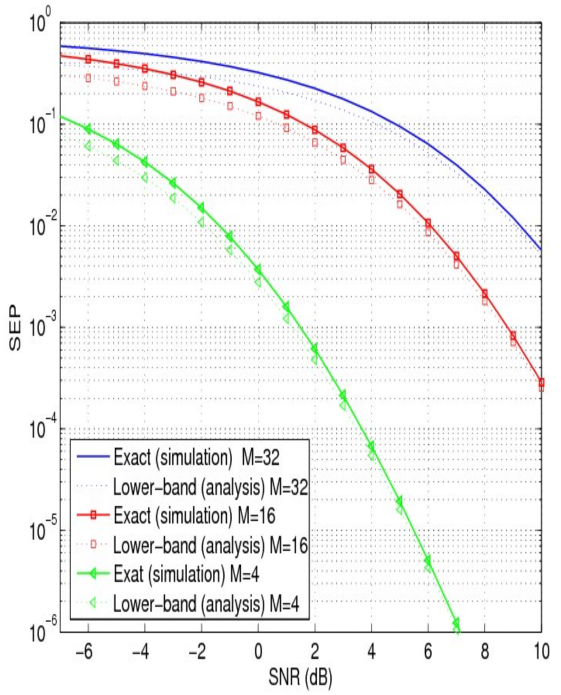

Fig.5. SEP versus SNR for the cooperative for 4-QAM, 16-QAM, and 32-QAM for $Nr = 2$.

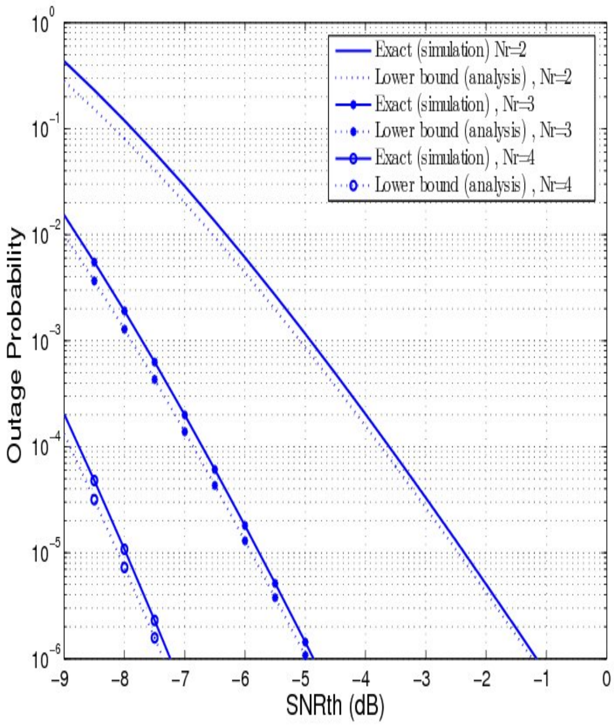

Fig.6. Outage probability of the cooperative MIMO system ($Nr = 2$, 3 and 4 receive antennas) γ_{th}, pathloss of the direct and indirect links are $\alpha_{sd} = 1$ and $\alpha_{srd} = 1$

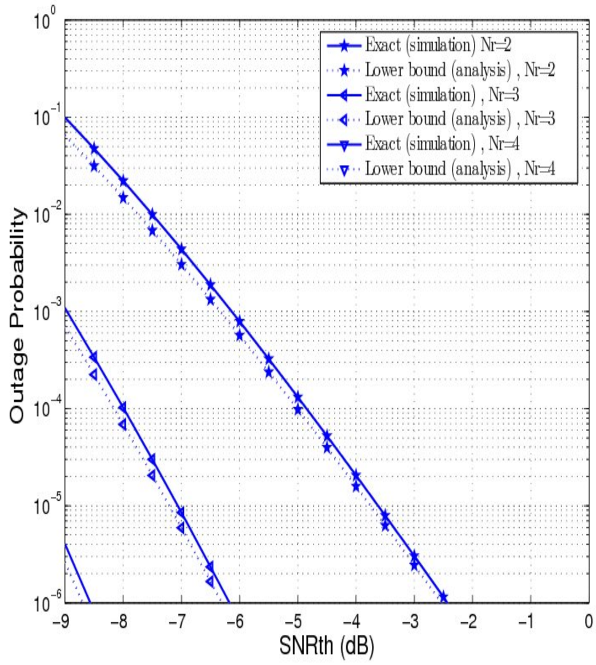

Fig.7. Outage probability of the cooperative MIMO STBC system ($Nr = 2$, 3 and 4 receive antennas) γ_{th}, pathloss of the direct and indirect links are $\alpha_{sd} = 0.5$ and $\alpha_{srd} = 0.5$.

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