

# Diversity Analysis of MIMO-STBC using various Coding and Receiving Techniques

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**Abstract** — In this paper, we present a low-complexity algorithm for detection in high rate, space-time block coded (STBC) large MIMO systems that achieve high spectral efficiencies. STBC is a powerful technique to achieve full spatial diversity in Rayleigh fading channels with low decoding complexities. This complexity gain becomes greater when the number of transmit antennas or the constellation size becomes larger. STBC is a MIMO transmit strategy which exploits transmit diversity and high reliability. In most of the existing space-time code designs, achieving full diversity is based on maximum-likelihood (ML) decoding at the receiver that is usually computationally expensive. Sphere detection has emerged as a powerful means of finding the maximum likelihood solutions to the detection problem of multiple antenna (MIMO) systems. In this paper, we analyze the performance and computational complexity of sphere detector and compare it with that of already known MIMO receiver techniques.

**Keywords** — Alamouti STBC, full diversity, maximum likelihood (ML) detector, multiple-input multiple-output (MIMO), sphere decoder.

## I. INTRODUCTION

The wireless communication system aims to integrate features such as high data rate access, high quality of service, spectral efficiency and reliability. This increase demand has led to the requirement of higher network capacity and improved performance. A MIMO system has emerged as an attractive technique for achieving high bit rate data transmission with high bandwidth efficiency. Multiple-input multiple-output wireless technology seems to meet these demands by offering increased spectral efficiency through spatial multiplexing gain and improved link reliability due to antenna diversity gain. In order to make them more reliable, several transmitter and receiver diversity techniques utilizing space-time or space-frequency codes can be used. MIMO has the capacity of producing independent parallel channels and transmitting multipath data streams and thus meets the demand for high data rate wireless transmission. Alamouti based space-time coding technique is one of the most effective transmitter diversity methods [1],[2].

## II. MIMO SYSTEM MODEL

In MIMO systems, a transmitter sends multiple data streams through multiple transmit antennas. The transmit streams go through a channel matrix which consists of all  $N_t N_r$  paths between the  $N_t$  transmit antennas at the transmitter and  $N_r$  receive antennas at the receiver. Then, the receiver gets the received signal vectors by the

multiple receive antennas and decodes received signals vectors into the original information. MIMO is mostly used in wireless local area networks (WLANs), such as IEEE 802.16 and IEEE 802.11n to acquire throughputs as high as 600 Mbps. Multiple antennas allow MIMO systems to perform precoding (multilayer beam forming), diversity coding (space-time coding) & spatial multiplexing. Fig. 1 shows a simplified block diagram of such a MIMO-STBC system.

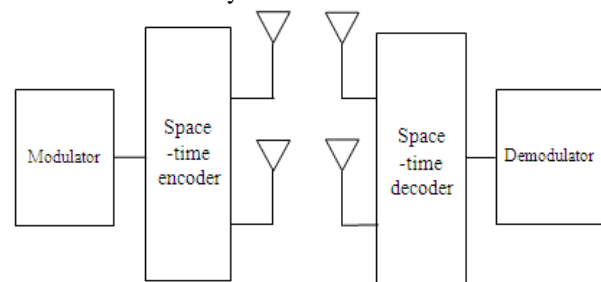


Fig.1. Block diagram of MIMO-STBC system

MIMO transmission can be characterized by the time-variant channel matrix

$$H = \begin{bmatrix} h_{1,1} & \dots & h_{1,N_t} \\ \vdots & \ddots & \vdots \\ h_{N_r,1} & \dots & h_{N_r,N_t} \end{bmatrix} \quad (1)$$

Where the general element  $h_{n_r,n_t}$  represents the complex time-variant channel transfer function at the path between the  $n_t$ -th transmitter antenna and the  $n_r$ -th receiver antenna.  $N_t$  and  $N_r$  represent the number of transmitter and receiver antennas respectively.

Multi antenna systems can be classified into three main groups. Multiple antennas at the transmitter side are usually applicable for beam forming purposes. Transmitter or receiver side multiple antennas suit also for realizing frequency or space diversity schemes. The third class includes systems with multiple transmitter and receiver antennas realizing spatial multiplexing [3]. In the paper, we confine our attention to systems exploiting the benefits of spatial multiplexing. These multifunction of MIMO helps in attaining improved performance in terms of both their bit error rate as well as throughput. Each antenna element on a MIMO system operates on the same frequency and therefore does not require extra bandwidth. The total power through all antenna elements is less than or equal to that of a single antenna system, i.e.

$$\sum_{k=1}^N p_k \leq P \quad (2)$$

Where  $N$  is the total number of antenna elements,  $p_k$  is the power allocated through the  $k^{th}$  antenna element, and  $P$  is the power if the system had a single antenna element [4].

### Capacity of MIMO Channel

Derived from the Shannon's law, the channel capacity for MIMO system can be approximated to  $C = m \cdot \log(1 + SNR)$  (3)

Where  $m$  is the number of antennas in the transmitter and receiver sides.

### III. CHANNEL MODEL

The wireless communication channel is mainly characterized as a doubly selective channel, i.e. it is both frequency and time selective. While multi path propagation gives rise to frequency selectivity, mobility and/or carrier frequency offset gives rise to time selectivity. The frequency-selectivity implies inter-symbol interference (ISI) that is the transmitted signal is spread in time. On the other hand, time-selectivity results in spreading of the transmitted signal in the frequency domain, called as Doppler spread. Therefore, in order to provide reliable communication, advance and efficient channel equalization techniques are necessary.

A typical model use in research to model non-line-of-sight (NLOS) scenarios is the Rayleigh model. The Rayleigh model assumes NLOS, and is used for environments with a large number of scatterers. This model has independent identically distributed (i.i.d.) complex, zero mean, unit variance channel elements.

### IV. SPACE TIME BLOCK CODE

One of the methodologies for exploiting the capacity in the MIMO system consist of using the additional diversity of MIMO systems, namely spatial diversity, to combat channel fading. This can be achieved by transmitting several replicas of the same information through each antenna. By doing this, the probability of losing the information decreases exponentially [5]. The diversity order or diversity gain of a MIMO system is defined as the number of independent receptions of the same signal. A MIMO system with  $N_t$  transmit antennas and  $N_r$  receive antennas has potentially full diversity (i.e. maximum diversity) gain equal to  $N_t N_r$ .

The different replicas sent for exploiting diversity are generated by a space-time encoder which encodes a single stream through space using all the transmit antennas and through time by sending each symbol at different times. This form of coding is called space-time coding (STC). Due to their decoding simplicity, the most dominant form of STCs is space-time block codes (STBC).

#### A. Alamouti STBC

Alamouti's scheme was the first STBC that provides full diversity at full data rate for two transmit antennas. Alamouti proposed a simple transmit diversity scheme which improves the signal quality at the receiver by simple processing across two antennas at the transmitter side. The proposed block coding technique has been theoretically extended by Tarokh [6] in the framework of orthogonal STBCs, so that codes with

comparable properties for more than two transmit antennas are available. The Alamouti STBC is specified as

$$S = \begin{pmatrix} s_1 & -s_2^* \\ s_2 & s_1^* \end{pmatrix} \quad (4)$$

During the first period, signals  $s_1$ ,  $s_2$  are transmitted at the same time by antenna 1 and antenna 2 respectively while during the second period, signal  $-s_1^*$  is transmitted by antenna 2 and  $s_2^*$  by antenna 1, where  $s_1^*$  is the complex conjugate number of  $s_1$ .

The Alamouti STBC offers two important advantages for a practical implementation, namely: (1) It effectively orthogonalizes the flat fading channel, thus decreasing the effect of fading (diversity) and (2) as for all orthogonal STBCs; the optimum maximum likelihood receiver reduces to a symbol-by-symbol decision after a zero forcing receiver.

#### B. Orthogonal Space-Time Block Codes

For complex orthogonal space-time block codes, due to the orthogonality of their codes, their maximum likelihood (ML) decoding is linear and hence they achieve full diversity with linear receivers [7]. OSTBCs can be expanded to any number of transmit antennas. The real orthogonal designs exist only for  $N = 2, 4$ , and  $8$ . STBCs based on real designs have transmission rate of 1; a number codes based on generalized real designs are constructed explicitly for  $N = 8$ .

Consider MIMO-STBC systems with  $N$  transmit and  $M$  receive antennas. The received signal vector is given by

$$y = Hs + n \quad (5)$$

Where  $y$  is a  $M$ -dimensional received complex vector,  $s$  is a  $N$ -dimensional transmitted complex vector whose entries have real and imaginary parts that are integers,  $H$  is the channel matrix and  $n$  is the i.i.d. complex additive white Gaussian noise (AWGN) vector with zero-mean and co-variance matrix  $\sigma^2 I$ . The receiver has perfect knowledge of the random channel matrix  $H$ , which remains constant over one or more symbol periods (slow fading).

### V. SPHERE DECODER

Maximum likelihood decoding (MLD) is the optimum decoding algorithm that is used for MIMO system [8]. MLD complexity increases exponentially with the number of antennas and the constellation order being used for modulation [9]. Therefore sphere decoder (SD) was proposed as an alternative to provide MLD performance with less complexity.

Sphere decoder algorithms [10, 11, 12] achieved maximum likelihood performance by providing an efficient way for generating all candidate solutions that lie inside a hyper-sphere defined by the channel matrix and the received signal vector. In the high signal-to-noise ratio (SNR) region, the radius of the sphere can be chosen small enough so that only few candidates are found inside this sphere. This search space is therefore drastically smaller than the ML search which consists of  $q^N$  points for a  $q$ -ary signal constellation. Thus, the number of operations (complexity) is roughly polynomial in  $N$  in the high SNR region.

Assuming  $H$  is known at the receiver, the ML detection is given by

$$\hat{s} = \underset{s \in \omega}{\operatorname{arg\,min}} \|y - Hs\|^2 \quad (6)$$

Solving (6) becomes impractical and exhaustive for high transmission rates, and the complexity grows exponentially [13]. Therefore, sphere detector solves this problem by searching for the closest point among all lattice points that lie inside a sphere centered around the received vector  $y$  and of radius  $d$ . The algorithm runs recursively until all lattice point inside the sphere are found. If no points were found inside the sphere, then we increase the radius and start over again. Now, introducing this radius constraint on (6) changes the problem to

$$\hat{s} = \underset{s \in \omega}{\operatorname{arg\,min}} \|y - Hs\|^2 < d^2 \quad (7)$$

## VI. EQUALIZATION TECHNIQUES

In general, equalizers are classified according to the structure, namely linear or non-linear equalizers. Equalizers can also be classified according to the optimization criterion such as zero-forcing (ZF), when a zero forcing solution is sought, or minimum mean square error (MMSE), when the equalizer optimizes the mean-squared error (MSE) of the symbol estimate, or maximum likelihood (ML) when the maximum likelihood sequence estimation (MLSE) criterion is utilized.

### 1. Zero-Forcing Equalizer:

The ZF equalizer applies the inverse of the channel frequency response to the received signal, to restore the signal after the channel [14]. The name zero forcing corresponds to bringing down the inter symbol interference (ISI) to zero in a noise free case. For a channel with frequency response  $F(f)$ , the zero-forcing equalizer  $C(f)$  is constructed by

$$C(f) = \frac{1}{F(f)} \quad (10)$$

Thus combination of channel and equalizer gives a flat frequency response and linear phase  $F(f)C(f) = 1$ .

### 2. Minimum Mean Square Error (MMSE):

Although the ZF equalizer removes ISI, may not give the best error performance for the communication system because it does not take into account noises in the system. An equalizer that takes noise into account is the MMSE equalizer. It is based on the mean square error (MSE) criterion. A MMSE estimator describes the approach which minimizes the mean square error, which is a common measure of estimator quality. The MMSE estimator is then defined as the estimator achieving minimal MSE. Basically, MMSE is the expected value of the squared difference between desired data signal and the estimated data signal [15]. Its output can be expressed in the form

$$\hat{f} = \underset{f}{\operatorname{arg\,min}} E|X(f) - \hat{X}(f)|^2 \quad (11)$$

where  $X$  are data samples.

### 3. Maximum Likelihood Decoding Equalizer:

Maximum likelihood estimation is a totally analytic maximization procedure (MAP). The techniques discussed previously were not optimal in terms of minimizing the

average symbol error probability. Since the effect of a symbol is spread to other symbols, it is intuitive that the optimal receiver should observe not only the segment of received signal concerning the desired symbol, but the whole received signal instead [16]. Using the whole received signal, we can employ the MAP principle to develop the optimal symbol-by-symbol detector, which decides one transmitted symbol at a time, to minimize the average symbol error probability.

We know that the system equation is expressed as

$$r = Hs + v \quad (12)$$

where  $r$  is the received signal,  $Hs$  is the noise free signal constellation and  $v$  is the receiver noise.

The optimum detector compares the received signal vector  $r$  to every possible noise free constellation point  $H(s)$  [17].

## VII. SIMULATION RESULTS

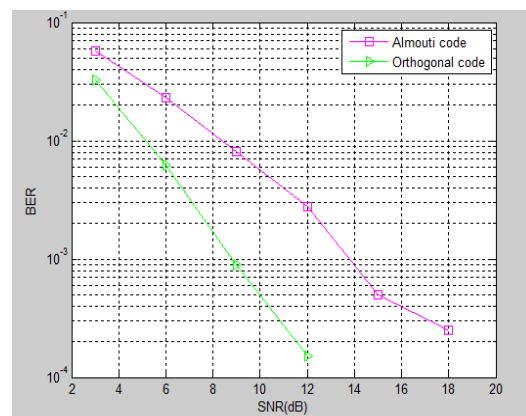


Fig.2. BER plot for Alamouti code vs Orthogonal code

Figure 2 shows the bit error rate performance of Alamouti code and the orthogonal code. It is seen in the graph that orthogonal code performs better than the Alamouti code. The analysis is done on the basis of SNR vs BER graph for a Rayleigh fading channel. Using PSK modulation and orthogonal codes at the transmitting end, in order to reduce the effective bit error rate in MIMO system from  $10^{-1}$  to  $10^{-4}$  may require only 3-12 dB SNR. Achieving the same in the Alamouti code may require up to 18 dB.

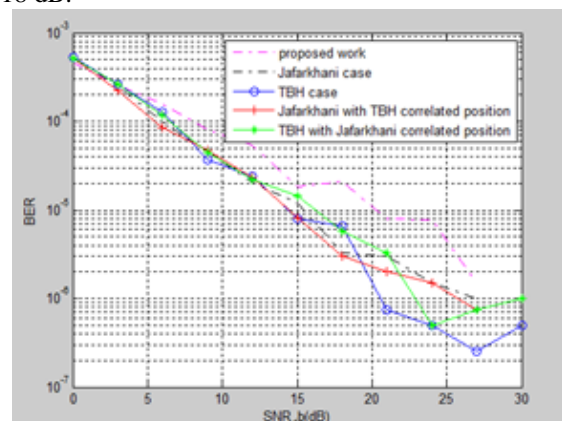


Fig.3. BER performance of various coding techniques

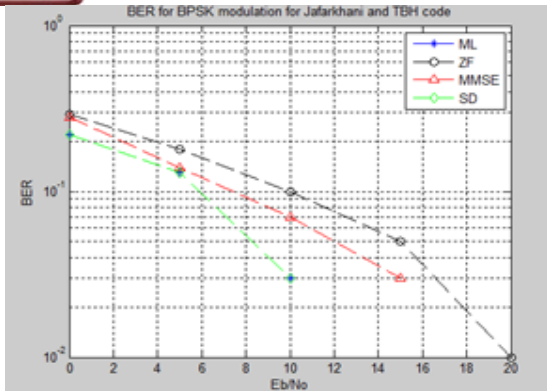


Fig.4. BER performance of various receiving techniques

Figure 3 and 4 shows the graphs for the performance of various coding and receiving techniques. In a fading channel, using typical modulation and sphere decoding schemes at the receiver side, reducing the effective bit error rate in MIMO systems from  $10^{-1}$  to  $10^{-2}$  may require only 5-10 dB SNR. Achieving the same in zero-forcing receiver may require greater than 14 dB.

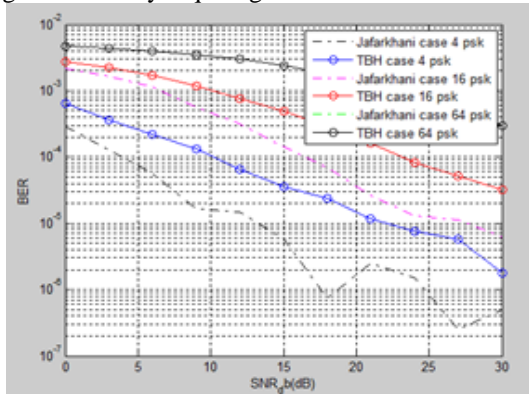


Fig.5. Comparison between different modulation techniques

It is observed that sphere decoder performs very similar to the maximum likelihood detector. The bit error rate of sphere decoder approaches to that of the maximum likelihood detector, which is lower as compared to the zero-forcing equalizer and the minimum mean square equalizer, as illustrated in table 1. Also, the implementation and decoding complexity is reduced as compared to the other techniques. Therefore, sphere decoder can be used as an alternative solution to the maximum likelihood equalizer.

Energy Per bit ( $E_b/N_o$ )	Bit Error Rate			
	Maximum Likelihood Detector	Zero-Forcing Detector	Minimum Mean-Square Error	Sphere Decoder
0	0.200	0.240	0.230	0.200
5	0.100	0.170	0.160	0.100
10	0	0.090	0.040	0.00050
15	0	0.020	0.010	0
20	0	0	0	0

Table 1: Comparison of BER performance of various receiving techniques

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