

An Enhancement in Transient Response of LMS Adaptive Filtering

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Abstract — This paper studies the statistical behavior of an Affine combination of the outputs of two variable step size normalized least mean-square (LMS) adaptive filters that simultaneously adapt using the same white Gaussian inputs. The purpose to combine two filters is to obtain a new LMS adaptive filter with fast convergence and small steady-state mean-square deviation (MSD). The combination factor $\lambda(n)$ is optimized which minimizes the mean-square error (MSE) and gives good steady state and transient response. By using this affine combination, we can cancel the noise signal with high convergence speed.

Keywords – Index terms- affine combination.

I. INTRODUCTION

The common problem in designing adaptive filters is to overcome the trade-off between the convergence speed and final misadjustment, i.e., faster converging filter gives large steady state deviation and slowly converging filter gives small deviation from steady-state value. This trade off can be controlled by this affine combination.

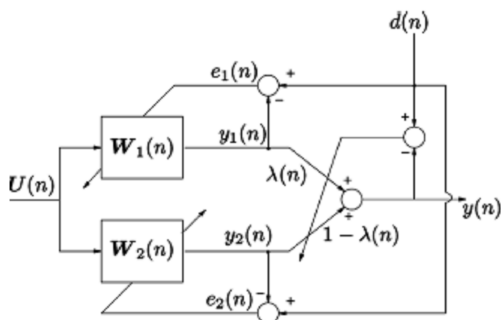


Fig. 1. Adaptive combining of two transversal adaptive filters.

In this scheme where adaptive filter $W_1(n)$ uses a larger step size than adaptive filter $W_2(n)$. The main goal is the selection of the scalar mixing parameter $\lambda(n)$ for combining the two filter outputs. The mixing parameter is defined as a sigmoid function whose free parameter is adaptively optimized using a stochastic gradient search which minimizes the quadratic error of the overall filter, which leads to an optimal affine sequence $\lambda_0(n)$.

Finally, two realizable schemes for updating $\lambda(n)$ are proposed. The first scheme is based on a stochastic gradient approximation. The second scheme is based on the relative values of averaged estimates of the individual error powers. Both schemes are briefly studied, and their support the theoretical findings and show that the analysis performances are compared to that of the optimal affine

combiner. Finally we apply this scheme to the noise cancellation

II. OPTIMAL AFFINE COMBINER

The affine combination is shown in Fig. 1. Each filter uses the LMS adaptation rule but with different step sizes μ_1 and μ_2

$$W_i(n+1) = W_i(n) + \mu_i e_i(n) U(n) \dots (1)$$

Where

$$e_i(n) = d(n) - W_i^T(n) U(n) \dots (2)$$

$$d(n) = e_0(n) + w_0^T U(n) \dots (3)$$

Where $W_i(n), i=1, 2$ are the N-dimensional adaptive coefficient vectors, is assumed zero-mean, and statistically independent of any other signal in the system. $U_i(n)$ is the input vector. It will be assumed, without loss, that $\mu_1 > \mu_2$, so that will, in general, $W_1(n)$ converges faster than $W_2(n)$. Also, $W_2(n)$ will converge to the lowest individual steady-state weight misadjustment. The weight vectors $W_1(n)$ and $W_2(n)$ are coupled both deterministically and statistically through $U(n)$ and $e_0(n)$.

The outputs of the two filters are combined as

$$Y(n) = \lambda(n) y_1(n) + [1 - \lambda(n)] y_2(n) \dots (4)$$

Where $Y_i(n) = W_i^T(n) U(n), i=1,2$ and over all system error is

$$e(n) = d(n) - y(n) \dots (5)$$

Equation (4) can be re-written as

$$Y(n) = \lambda(n) W_1^T(n) U(n) + [1 - \lambda(n)] W_2^T(n) U(n) = \{ \lambda(n) [w_1(n) - w_2(n)] + w_2(n) \}^T U(n)$$

$$(i) \quad = \{ \lambda(n) w_{12}(n) + w_2(n) \}^T U(n) \dots (6)$$

Where $W_{12}(n) = W_1(n) - W_2(n)$

Equation 6 shows that $y(n)$ can be interpreted as a combination of $W_2(n)$ and a weighted version of the difference filter $W_{12}(n)$. It also shows that the combined adaptive filter has an equivalent weight vector given by $w_{eq} = \lambda(n) w_{12}(n) + w_2(n) \dots (7)$

Subtracting (1) for $i=2$ from (1) for $i=1$ yields a recursion for $w_{12}(n+1) =$

$$[I - \mu_1 U(n) U^T(n)] w_{12} + (\mu_1 - \mu_2) e_2(n) U(n) \dots (8)$$

A rule to find λ , which minimizes MSE

$$e(n) = e_0(n) + [w_{02}(n) - \lambda(n) w_{12}(n)]^T U(n) \dots (9)$$

$$= -2E [e(n) w_{12}^T(n) U(n) / w_2(n), w_{12}(n)] = 0 \dots (10)$$

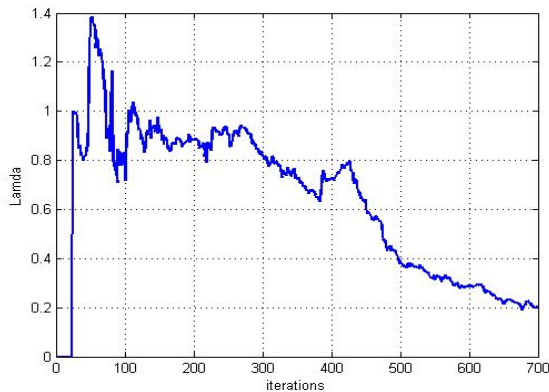
$$[w_{02}(n) - \lambda(n) w_{12}(n)]^T R_u w_{12}(n) = 0 \dots (11)$$

$$\lambda_0(n) = w_{02}^T(n) R_u w_{12}(n) / w_{12}^T(n) R_u w_{12}(n) \dots (12)$$

Which is the expression for the optimum affine combiner, as a function of unknown system response.

3. Iterative Algorithms to Adjust Affine Combiner:

The previous derivation of the optimal linear combiner was based upon prior knowledge of the unknown system response.. Clearly, this is not the case in reality. However, the theoretical model and its derived properties can be used to upper bound the performance of practical algorithms for adjusting without such knowledge. Algorithms that yield close-to-optimal performance for typical unknown responses can be considered as good candidates for practical applications.



Performance close to the optimal suggests that further analytical study of a new algorithm could be worth the effort. This is especially important for the adaptive combiner structure. There are two algorithms for the adjustment of optimal affine combiner. The first algorithm is based upon a stochastic gradient search for the optimal. The second is based on the ratio of the average error powers from each individual adaptive filter. The performances of these algorithms are then compared to the optimal performance.

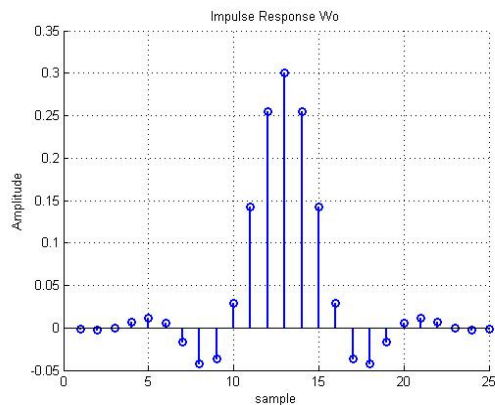


Fig.2. Un known System Response for the behavior of the optimum adaptive filter affine combination are presented. The unknown system response is shown in Fig.2 .

A. Stochastic Gradient Approach:

Consider a stochastic gradient search to estimate the optimum instantaneous value of λ . The stochastic gradient algorithm to update λ is

$$\lambda(n+1) = \lambda(n) + \mu [d(n) - w_{12}^T(n)U(n)]w_{12}^T(n)U(n) \dots (13)$$

$$w_{12}(n) = \lambda(n)w_1(n) = [1 - \lambda(n)]w_2(n) \dots (14)$$

Equation (13) is a linear first order stochastic time-varying recursion in the scalar parameter $\lambda(n)$. The stochastic

behavior of this recursion has been analyzed. The accuracy of the theoretical analysis and the performance of the proposed algorithm for adjusting $\lambda(n)$ are evaluated here. Appropriate values of μ were chosen so that the algorithm was able to track the adaptation of $w_1(n)$ and $w_2(n)$. Some difficulties were encountered regarding the tradeoff between stability of the recursion and the algorithm's tracking capabilities in the initial phase of adaptation. Sufficiently small values of μ were found so that (13) was stable. However, these values were not large enough to track the adaptation of $w_1(n)$ and $w_2(n)$. Larger values of μ improved the tracking but led to instability during the early phase of the adaptation in (13). Considering the optimum desired behavior for λ , solution was obtained by constraining λ to be less than or equal to 1 for all n . Larger (smaller) signal-to-noise ratios require larger (smaller) μ , which in turn requires the application of the constraint to $\lambda(n)$ for longer (shorter) periods.

The stochastic gradient algorithm requires a good estimate of the noise power to reasonably select and mildly constrain λ in recursion (13). The accuracy of this estimate could limit the usefulness of the stochastic gradient algorithm for some applications. A different scheme for choosing λ , based on the average error powers of the two filters is proposed.

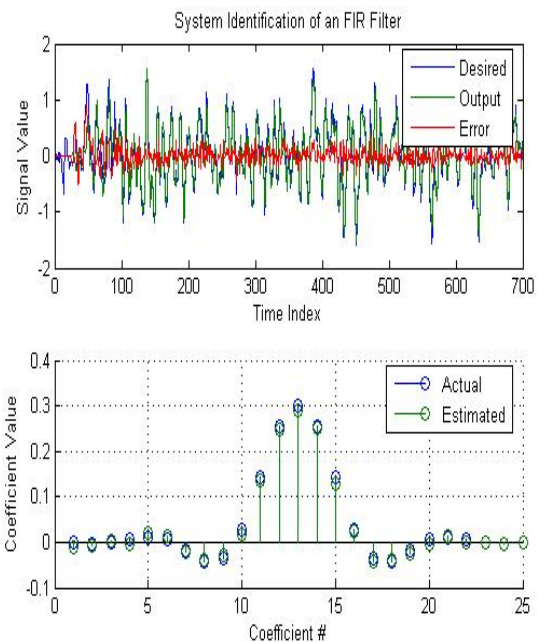


Fig3. System Identification using Affine VSS-NLMS of Stochastic Gradient Approach

MSE of affine combination is always less than either LMS1 and LMS2. This behavior is expected from an optimal combiner and verified. These curves represent the best performance that could be obtained using two LMS

B. Error Power Based Scheme:

A function of time averaged error powers could be a good candidate for an estimator of the optimum $\lambda(n)$ for each n . The individual adaptive error powers are good

indicators of the contribution of each adaptive output to the quality of the present estimation of $d(n)$. These errors are readily available and do not need an estimate of the additive noise power.

Consider a uniform sliding time average of the instantaneous Errors

$$e_1^2(n) = 1/k \sum_{m=n-k+1}^n e_1^2(m) \dots \dots \dots (15)$$

$$e_2^2(n) = 1/k \sum_{m=n-k+1}^n e_2^2(m) \dots \dots \dots (16)$$

where k is the averaging window. Then, consider the instantaneous value of $\mu(n)$ determined as

$$\mu(n) = 1 - \text{kerf} (e_1^2(n)/e_2^2(n)) \dots \dots \dots (17)$$

$$\text{erf}(x) = 2/\sqrt{\pi} \int_0^x e^{-t^2} dt \dots \dots \dots (18)$$

Fig shows the behaviors of using error power based scheme to update affine combiner and MSE of individual and combined. These results clearly show that the proposed algorithm leads to a very good practical implementation of the linear combiner.

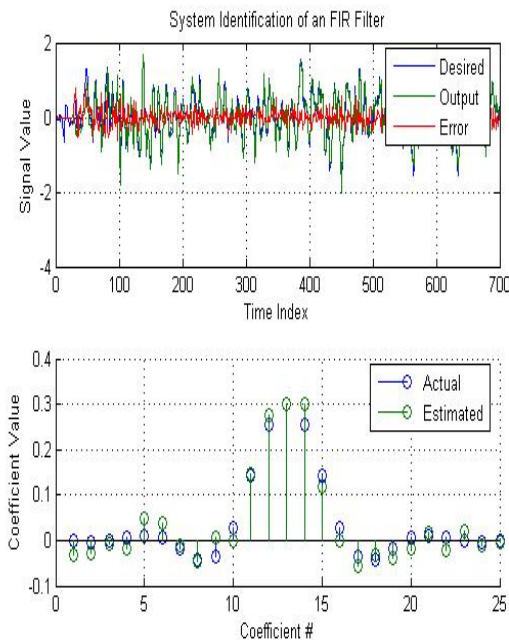


Fig.4. System identification using error power based scheme

III. TIME VARYING NLMS ALGORITHM

The conventional LMS algorithm is then used to find an optimal value of μ at that frequency. This optimal value μ_0 is used to update the time varying convergence parameter μ_n according to the following formula:

$$\mu_n = \alpha_n \mu_0$$

Where α_n is a decaying factor. We will consider the following decaying law:

$$\alpha_n = C / (1 + a n^b)$$

where C, a, b are positive constants that will determine the magnitude and the rate of decrease for α_n . According to the above law, C has to be a positive number larger than 1. When

$C = 1$, α_n will be equal to 1 and the new algorithm will be the same as the conventional LMS algorithm.

IV. NOISE CANCELLATION SIMULATION & RESULTS

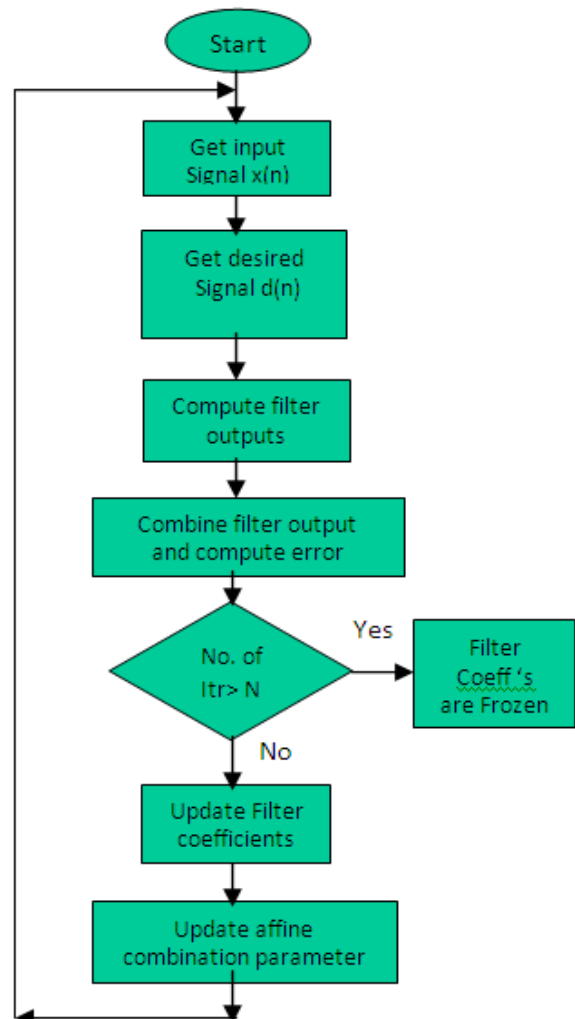


Fig.5. Flow chart for noise cancellation procedure

A. Description of the Simulation Setup

The ECG signal used for simulation is characterized by the following parameters.

Parameters of ECG Input:

Length: 15

Noise Parameters:

Amplitude: 0.15

Type: Gaussian

Mean: 0

Variance: 1

Initial Seed: 10

Filter Parameters

Filter Type: FIR

Order: 32

Structure: Direct form-I

Window: Rectangular

No. Of Iterations:

Convergence Factor: time varying

Desired signal parameters:
 Original ECG signal.

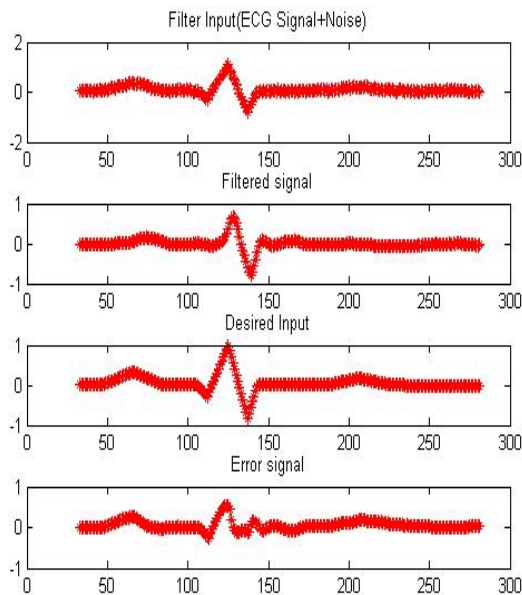
B. Results:

As first 32 samples are used to fill input array, no output is present till 32nd iteration. This can be observed in the simulated result shown in figure 6.2. But the updating of the coefficients has to be done for 250 times so the output is present till 282 iterations. Mean square error at the end of 250 iterations is 0.0197. Mean square error is taken as the performance criteria for comparison of adaptive algorithms.

Simulation procedure is repeated for different number of iterations ranging from 100 to 500 and the obtained MSE values are tabulated in the table given below.

Iterations	LMS1	LMS2	AFFINE	AFFINE TVNLMS
50	0.0484	0.0586	0.0323	0.030
100	0.0349	0.0412	0.0289	0.0252
150	0.0276	0.0323	0.0231	0.0212
200	0.0229	0.0287	0.0202	0.0180
250	0.0197	0.0232	0.0153	0.0132

the corresponding graph showing the convergence of the algorithm



V. CONCLUSIONS AND FUTURE SCOPE

This paper has studied the statistical behavior of an affine combination of the outputs of two VSS-NLMS adaptive filters that simultaneously adapt using the same inputs. The purpose of the affine combination is to obtain an LMS adaptive filter with fast convergence and small steady-state MSD. The affine combination studied is a generalization of the convex combination where the combination factor is restricted to the interval (0,1). Here the viewpoint was taken that the two filters each produce dependent estimates of the unknown channel. Thus, there exists a sequence of optimal affine combining coefficients which minimizes MSE.

REFERENCES

- [1]. Sadaoki Furui and M. Mohan Sondhi, "Advances in Speech Signal Processing", Marcel Dekker, Inc, 1992.
- [2]. Lester S.H Ngia, "System Modeling using Basis Functions and Application to Echo Cancellation", Ph. D. Dissertation, Chalmers University of Technology.
- [3]. "Adaptive combination of proportionate filters for sparse echo cancellation" jerónimo arenas-garcía, member, ieee, and aníbal r. figueiras-vidal, senior member, ieee IEEE Transactions on Audio, Speech, and Language Processing, vol. 17, no. 6, August 2009.
- [4]. "Affine combination of two LMS adaptive filters-transient mean square analysis-2008" jerónimo arenas-garcía, member, ieee, and aníbal r. figueiras-vidal, senior member, ieee IEEE Transactions on Audio, Speech, and Language Processing, vol. 17, no. 6, August 2008.
- [5]. "Affine combination of two LMS adaptive filters-transient mean and steady-state mean square analysis-2010" jerónimo arenas-garcía, member, ieee, and aníbal r. figueiras-vidal, senior member, ieee IEEE Transactions on Audio, Speech, and Language Processing, vol. 17, no. 6, August 2010.

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