

# Comparison of Different Design Techniques of Linear 2-DFIR Filtering for Detection of Edges in Image

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**Abstract** – In this paper Five different design techniques of linear phase two dimensional Finite impulse response (FIR) filter has been developed and The mathematical basis of these techniques is reviewed and differences in the techniques are noted. The comparison among these techniques has been done on the basis of frequency response and their effect on detecting edges of the image.

**Keywords** – DFT, FIR, IIR

## I. INTRODUCTION TO 2-D FILTERS

There are two most common classes of digital filters one is Finite Impulse Response (FIR) non recursive and other is Infinite Impulse Response(IIR) recursive.IIR filter efficiently realized by factoring their z- transforms into cascade of real or complex pole-zeros network.0 In two dimensions, however, the two-dimensional z-transforms cannot generally be factored into lower order systems [1], [2] .This leads to many design problems, such as the difficulty in determining the stability of the filter, as well as in finding an efficient realization where the coefficients may be truncated to a reasonable number of bits For FIR digital filters, the problems of stability do not exist in one dimension as the z-transform is a finite polynomial. Thus design techniques in one dimension are often directly extendable to two or more dimension. 2-D digital filter[4] are discrete system and widely used in image and video processing system[3].

Traditional approaches to the design of such filters include the window method[4], in which a smoothing window is applied to the Fourier series coefficients of the ideal frequency response; The frequency transformation method[5], in which a one-dimensional(1-D) prototype filter response is mapped into a function of two frequency variables using an appropriate transformation function; The frequency-sampling method[6], in which desired frequency response values are specified at certain sample points in the frequency domain.

### A. Theory of 2-d filters

Stable filter impulse response must satisfy the following condition:

$$\sum_{n_1=-\infty}^{\infty} \sum_{n_2=-\infty}^{\infty} |h(n_1, n_2)| < \infty \dots \dots \dots (1)$$

Let  $h(n_1, n_2)$  finite sequence defined from  $0 < n_1 < N_1 - 1$  and  $0 < n_2 < N_2 - 1$

$$H(Z_1, Z_2) = \sum_{n_1=0}^{N_1-1} \sum_{n_2=0}^{N_2-1} h(n_1, n_2) Z_1^{-n_1} Z_2^{-n_2} \dots \dots \dots (2)$$

And output of FIR filter is:

$$y(n_1, n_2) = \sum_{m_1=0}^{N_1-1} \sum_{m_2=0}^{N_2-1} h(m_1, m_2) x(n_1 - m_1, n_2 - m_2) \dots \dots \dots (3)$$

Where  $x(n_1, n_2)$  input to the filter

In recent years, several techniques for the design of two-dimensional (2-D) FIR digital filters have been investigated. Much effort has been spent on designing filters which are optimal in the minimax (Chebyshev) sense. The minimax design yields an equiripple filter and avoids the overshoot problem at the edge of the passband produced by least squares error criterion. However, it requires the use of sophisticated optimization tools such as Remez exchange algorithm [10], frequency transformation algorithm [11], extended the Remez exchange algorithm for one-dimensional (1-D) to the design of 2-D FIR filters. The optimal solution is not necessarily unique and may fail to converge, since the alternation theorem does not hold in two dimensions [12], and Frequency sampling method.

The paper is organized as follows In section II Different techniques of designing 2-D FIR filter has been described. Simulation and results of different design techniques discussed in section II is shown in Section III. Section IV offers some conclusion on the basis of results.

## II. DESCRIPTION OF DESIGN METHOD

### A. Frequency transformation method

The frequency transformation method transforms a one-dimensional FIR filter into a two-dimensional FIR filter. The frequency transformation method preserves most of the characteristics of the one-dimensional filter, particularly the transition bandwidth and ripple characteristics. This method uses a *transformation matrix*, a set of elements that defines the frequency transformation.

#### 1. Remez Exchange Technique

The Remez algorithm or Mc-Clellan transform [8] starts with the function  $f$  to be approximated and a set  $x$  of  $n + 2$  sample points  $x_1, x_2, \dots, x_{n+2}$  in the approximation interval, usually the chebyshev [9] nodes linearly mapped to the interval. The steps are:

1. Solve the linear system of equations

$$b_0 + b_1 x_i + \dots + b_n x_i^n + -1^i E = f(x_i) \dots \dots \dots (4)$$

(Where  $i = 1, 2, \dots, n + 2$ ), for the unknowns

- $h_0, h_1, \dots, h_n$  and  $E$ .
- Use the  $b_i$  as coefficients to form a polynomial  $P_n$ .
  - Find the set  $M$  of points of local maximum error  $|P_n(x) - f(x)|$ .
  - If the errors at every  $m \in M$  are of equal magnitude and alternate in sign, then  $P_n$  is the minimax approximation polynomial. If not, replace  $x$  with  $M$  and repeat the steps above. The result is called the polynomial of best approximation, the Chebyshev approximation, or the minimax approximation. Results of Mc-Clellan Transform are shown in fig. 1

### 2. Frequency sampling based FIR filter design using transformation

The output filter coefficients,  $b$ , are ordered in descending powers of  $z$ .

$$b(z) = b(1) + b(2)z^{-1} + \dots + b(n+1)z^{-n} \dots \dots \dots (5)$$

This method always uses an even filter order for configurations with a pass band at the Nyquist frequency. This is because for odd orders, the frequency response at the Nyquist frequency is necessarily 0. If we specify an odd-valued  $n$ , and it will increments it by 1. Results of using frequency transform techniques are shown in fig 2

### B. Frequency sampling method

The frequency sampling method creates a filter based on a desired frequency response. Given a matrix of points that define the shape of the frequency response, this method creates a filter whose frequency response passes through those points. Frequency sampling places no constraints on the behavior of the frequency response between the given points; usually, the response ripples in these areas.

Computes the filter by taking the inverse discrete Fourier transform of the desired frequency response. If the desired frequency response is real and symmetric (zero phase), the resulting filter is also zero phase.

The discrete Fourier transforms (DFT) relations for the filter at discrete set of values is given by

$$H(k_1, k_2) = \sum_{n_1=0}^{N_1-1} \sum_{n_2=0}^{N_2-1} h(n_1, n_2) \cdot \exp \left[ -j2\pi \left( \frac{k_1 n_1}{N_1} + \frac{k_2 n_2}{N_2} \right) \right] \dots \dots (6)$$

Frequency interpolation formula from DFT is given by:

$$H(e^{jw_1}, e^{jw_2}) = \sum_{n_1=0}^{N_1-1} \sum_{n_2=0}^{N_2-1} H(k_1, k_2) A(k_1, k_2, w_1, w_2) \dots (7)$$

where  $A(k_1, k_2, w_1, w_2) = \frac{1}{N_1 N_2} \left[ \frac{1 - \exp(-jN_1 w_1)}{1 - \exp[j(2\pi \frac{k_1}{N_1} - w_1)]} \right]$

$\cdot \left[ \frac{1 - \exp(-jN_2 w_2)}{1 - \exp[j(2\pi \frac{k_2}{N_2} - w_2)]} \right] \dots \dots (8)$

Results of frequency sampling techniques are shown in fig. 3

### C. Window method

In [7], a 2-D windowing technique has been presented in which a 1-D window is extended to 2-D windows. The technique is simple and has a short design time. The choice of the window function in the design is governed by the requirement that  $H(e^{j2\pi f_1}, e^{j2\pi f_2})$  approximate  $I(e^{j2\pi f_1}, e^{j2\pi f_2})$ . This implies that  $w(e^{j2\pi f_1}, e^{j2\pi f_2})$

should approximate a two-dimensional impulse function. Two related types of window functions which satisfy this requirement have been used. The first is the circularly symmetric window function described by Huang [7], results of 1-D window are shown in fig. 4

$$w(n_1, n_2) = w_1 \sqrt{n_1^2 + n_2^2} \dots (9)$$

Where  $w_1(x)$  is a good 1-D window function. This window function forms two separable 2-D window

$$w(n_1, n_2) = w_1(n_1)w_2(n_2) \dots \dots (10)$$

2-D window is used to design two dimensional FIR digital filters using inverse Fourier transform and multiply by 2-D window. Results of simulation are shown in fig. 5

$$h_d(n_1, n_2) = \frac{1}{2\pi^2} \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} H_d(w_1, w_2) e^{jw_1 n_1} e^{jw_2 n_2} dw_1 dw_2 \dots (11)$$

$$h(n_1, n_2) = h_d(n_1, n_2)w(n_1, n_2) \dots \dots (12)$$

## III. SIMULATION AND RESULTS

### A. Remez Exchange Technique

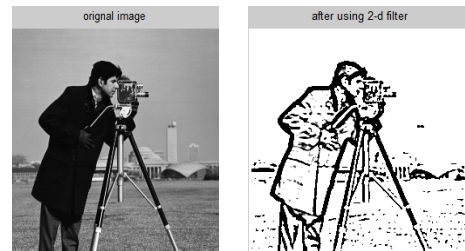
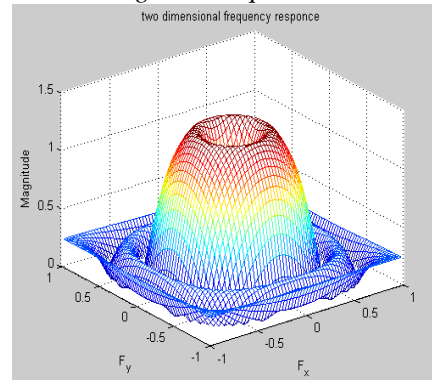
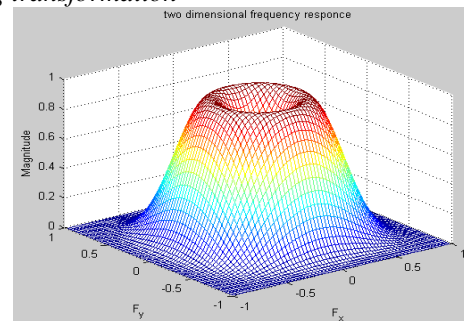


Fig.1. Frequency response of filter order 10\*10 by using MC-Clellan Transform

### A. Frequency sampling based FIR filter design using transformation



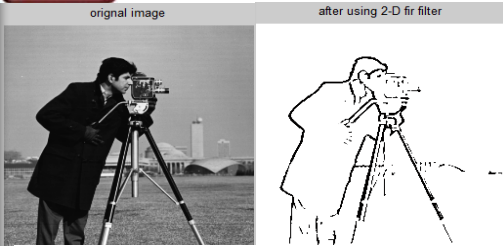


Fig.2. Frequency response of filter order 10\*10 by using frequency sampling based transform.

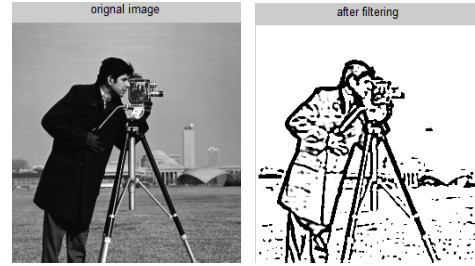


Fig.4. Frequency response of filter order by using 1-D window method using kaiser window

**B. Frequency sampling method**

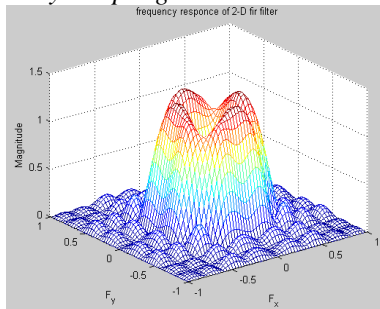


Fig.3. Frequency response of filter order 10\*10 using frequency sampling method.

**C. 2-D Window method**

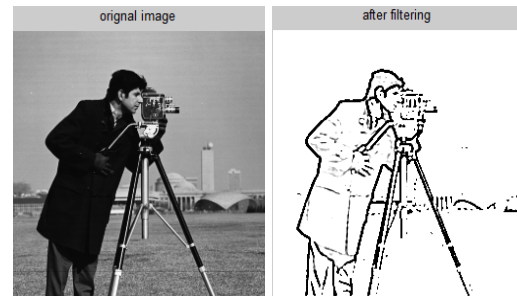
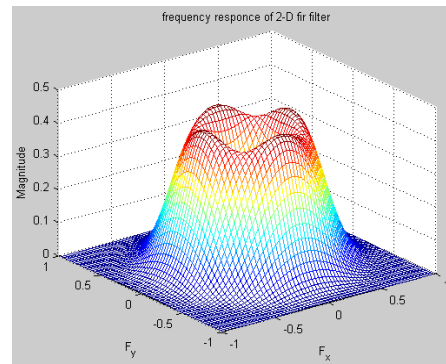
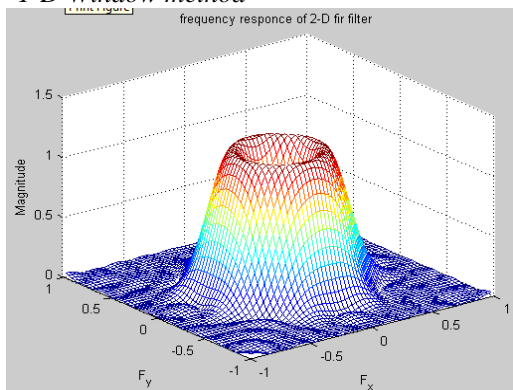


Fig.5. Frequency response of filter order 10\*10 by using 2-D window method.

**B. 1-D Window method**



**Comparison of Techniques:**

Five different techniques of FIR filter designing were compared by designing with order of 10\*10 filters on the basis of magnitude response, phase response and impulse response with order of filter

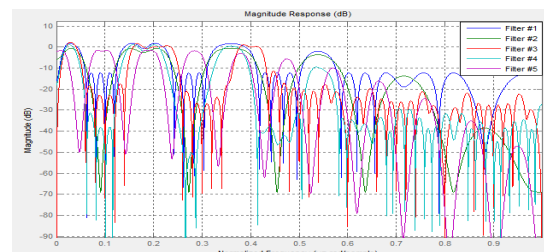


Fig.6. Magnitude Response

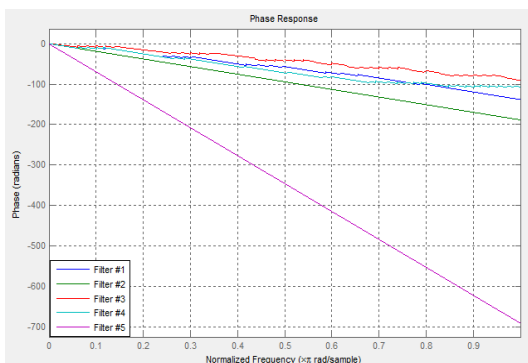


Fig.7. Phase Response

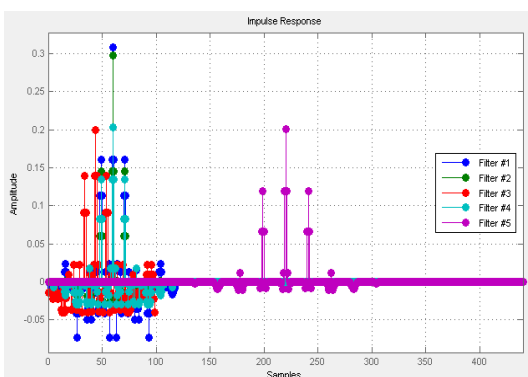


Fig.8. Impulse Response

#### IV. CONCLUSION

The various filter design techniques discussed in this paper have linear phase response except frequency sampling technique and also all exhibit stable filtering. All the techniques are direct form II transposed type filter. The disadvantage of the frequency sampling technique was that the frequency response gave errors at the points where it was not sampled. By using 2-D window technique implementation cost is high but it is lowest in case of frequency sampling method and 1-D window. From analysis it is concluded that window techniques is best technique for designing 2-D FIR filter and detection of edge in image is better in case of window technique. Window method has short design time and simplest technique.

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