Algebraic-Geometrical Energy Axioms and Logic

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Abstract – In an axiomatic mathematical model some old proved and new results show that the energy mathematics in the quantum range has to be revised. The hedgehog model belongs to a fiber bundle from the SU(3) geometry and describes energy exchanges of a nucleon with its environment. The fusion model shows how from two protons a weak rotor (the wheel) for deuteron arises. It sets Eucliden space coordinates. Gravity is described for length contraction/expansion by a spiralic projection and later with central projections for metrical rescalings. The necessary coordinate extension from spacetime (x, y, z, t) to octonians uses a mathematical proof (figure 9) for the Hilbert space subspace lattice OML, - central astroids have the doubled up octonians with (e0, m, f, e7) added. The new first and last coordinates form a G-compass for color charges, m is mass, f frequency [12]. The neutral color charge of nucleons is taken as rgb-graviton whirl (figure 9) necessary for the OML structure. The measuring octonian Gleason operators act like additive probabilistic states (figure 8). The Minkowski rscaling of measure units is added and several models for the new theory are shown in the last figure.

Keywords – Axiomatic Mathematical Model, Algebraic-Geometrical, Energy Axioms, Logic.

I. INTRODUCTION

In 1936 G. Birkhoff and J. von Neumann suggested for quantum structures that projective geometrical subspace lattices can serve for solving quantum problems in physics. They called this a quantum logic. It exists today as research under the name quantum structures. Qubit research may use it for constructing another generation of quantum computers. As projective logistics, for a finite dimensional Hilbert space Hn model, the projection operators of a Hn split Hn into a (closed) subspace U and its orthogonal subspace in Hn = U + U┴. The algebraic model OML has these subspaces as elements provided with the orthogonality U┴┴ = U. A dimension function exists on the orthomodular lattice OML. Its atoms directly above the smallest elements with dimension 0, drawn as point in a lattice diagram, present lines of the projective geometry. The dimension 1 OML for this is drawn as an example for spin in figure 7 as MON for n spin up/down oriented vectors, Instead of drawing parallel or anti-parallel spin vectors when they arise in composed systems like atomic kernels AK, the orthogonality for spin is listed in a Boolean sublattice MO1 with atoms xj for spin up and xj' for spin down. A selection function chooses from every block either xj or xj' . The tuple of n members from MON is then listed by replacing the atoms by its two possible Boolean values where u = 1 is for spin up, u = 0 for spin down and u' is Boolean computed 0' = 1, 1' = 0. Changing spin states of AK means then that some u are replaced by u' in this list. The algebraic modulo 2 computation can be extended to higher dimensional OML. In the figures 9 are drawn two atomic block diagrams in dimension 4. The blocks of this OML are Boolean algebras 24 whose subspace elements present a set of commuting operators and Boolean laws can be used as a logic. They fail for non-commuting operators in different blocks. But the weaker algebraic lattice models OML allow to predict for finite dimensional projective geometries results: the MINT-Wigris research of the author can demonstrate with technical running macroscopic models that such a new orthomodular not Boolean logic is mathematical proved necessary for the quantum range. The model in figure 2 for the fusion of two protons to deuteron as Cooper pairing of a proton with a neutron shows that these two nucleon states are in a weak interaction isospin exchange.
and that the three observed red-green-blue color charged quarks are not only confined in a nucleon by the strong interactions gluon exchange between Cooper paired u-, d-quarks, but that in a crystalic (chemistry model) a tetrahedron with an rgb-graviton at the center of the figure exists and is added to the quark triangle as base. As mathematical proof for the rgb-graviton belonging to a nucleon, in [11] the existence of its block (curved drawn in figure 9 left) for the red-green-blue vertices of the triangle in figure 9 is mentioned. This theorem also contains the proof for an octonians astroid (in figure 9 at right) belonging to a xyzt-spacetime 4-cycle which has these coordinates as vertex atoms. The other 4 cycle atoms belong to other blocks of a projective 4-dimensional orthomodular lattice.

Concerning the model of figure 2, the symmetry of the tetrahedron is S₄, the permutation of 4 elements, and factorizes through the CPT Klein group symmetry to the quark triangle symmetry D₃. In a projective 4-dimensional geometry as a finite dimensional Hilbert space, the orthomodular Boolean block structure arises through the direct decompositions of such a spacetime presentation into a subspace and its orthogonal subspace. The associated projection operators are in general not commuting, but satisfy the orthomodular law, also for their presentation in a modular ortho subspace lattice. The modular law is stronger than the orthomodular law and arises through the projective geometrical axiom that any two lines have a point in common. A finite lattice model of this kind is MO_n in figure 7 which is a logic for the spin orientations when they arrange in composed systems parallel or antiparallel. This is observed for atomic kernels, composed by Cooper paired deuterons. The common center for the deuteron in figure 2 can be replaced by an exciton which allows the two nucleons to be stored in a distance. If the distance gets too large, the observed atomic kernel decays of quarks occur since the isospin energy exchange reaches not the partner nucleon. The periodic system of chemistry has only a finite number of atoms.

For the 4-dimensional projective geometry the subspace lattice OML is the set theoretical union of its blocks which are Boolean and arise from commuting sets of projection operators. In lattice diagrams the lines above a smallest element 0 are projective points as lines in one dimension higher [x, y, z, ict, w] through the origin. The four projective points in a block directly above 0 are called atoms. A way to draw simplified subspace diagrams is setting for a block in n-dimensional projective geometries n point/atoms on an interval as in figures 8,9 for the cases n = 3, 4. The orthomodular proved result is that for a 3-cycle at left in figure 4 a block has to exist which contains the three atoms for the r-, g-, b-space coordinates of Euclidean space. This is then interpreted as a quasi particle rgb-graviton whirl as neutral superposition of the three quarks color charges, observed for all nucleons. As a twisting action it induces the 6-cycle of the strong interactions SI rotor which is a representation of the D₃ symmetry. It allows integrations of forces to potentials or speeds like \[ \int b/r^2 \, dr = -b/r, \] \[ b = \int (du²/d²t) \, dt = b \cdot du/dt \] for kinetic or rotational energies and also area integration for magnetic induction and force, volume integration for heat bubbles in spacetime with entropy inside the bubble. In addition, the six energies for this have as projective closures in a complex projective 2-dimensional CP² presentation an atmosphere of bounding S² spheres. Three for the three xyz-space coordinates are Heegard decomposed into two color charged hemispheres as Heisenberg Cooper pairs. At their center they carry a spin-like whirl vector for each of the six color charge whirls. It can turn like spin its normal direction for an input or output energy exchange of the nucleon, deuteron or atomic kernel [2], [3]. A threshold opens a valve for this exchange. Energies associated with red/ length as radius or the x-line or electrical force as potential can be exchanged, green/ angle φ, the y-line, heat as phonon bubbles are exchanged as well as for the other four hemispheres blue/ a new octonian
energy coordinate for kinetic energy, setting momentum vectors \( p = mv \) and \( Hz \) measured frequencies, turquoise (as anticolor to red)/angular momentum measured as \( L = rzp \), \( r \) radius for a matter system, rotating about a barycenter in a plane with an orthogonal normal rotation axis added to the plane in space or the \( z \)-coordinate of space. In this case a new octonian vector dimension \( e_0 \) sets in spherical coordinates an \( \theta \) angle towards the \( z \)-axis for its vector, observed for electrons \( n \)th roots of unity spin directions distribution when located on atoms shells.

In another action, \( e_0 \) attributes a measuring unit vector to every octonian energy or space line, numbered by indices of octonian vectors 1, 2, ..., 6 as in figure 8. In combination of three, the Fano interval 123 is for space coordinates, 145 is an octonian subspace for the electrical charges cross product of magnetic momentum orthogonal to the plane in which an electrical charge on a loop is rotating which defines induction as an angular momentum for the loops rotation in the 145 subspace. The other Fano triples as measuring Gleason frames for Gleason operators are explained in the article or [1]. They are spin-like base triples in the 3-dimensional subspaces and carry on an unit sphere \( S^2 \) real, complex or quaternionic scalars for measuring purposes. Spin has meter as measure for length, an \( rgb \)-graviton in a 126 subspace measures its named three quark color charges and sets the tetrahedron crystal in nucleons. 126 is a strong interactions \( SU(3) \) subspace, generated by the first three GellMann matrices. As projection operators they project the geometrical \( S^3 \) space of \( SU(3) \) down to the weak interactions Hopf sphere \( S^3 \) as unit sphere in spacetime 1234. The second geometrical factor \( S^5 \) of the \( SU(3) \) twisted \( S^2 \times S^5 \) unit sphere product space has a similar fiber bundle presentation as the Hopf fiber bundle for spacetime and \( S^3 \). The \( S^5 \) is the unit sphere in the octonian subspace 123456 where an octonian Einstein energy plane 56 for a mass \( m \) and a frequency \( f \) coordinate is added to space-time. It contains the Einstein line \( mc^2 = hf \) and has a projective closure \( P^2 \) with coordinates \( [m, f, w] \). The coordinate \( w \) at projective infinity serves for the observed Kepler conic sections orbits of planets when gravity lets them rotate or comets are hitting a star or escaping when reaching the second cosmic speed. The Einstein correction through general relativity is explained differently in another projective interpretation.

Concerning the \( S^5 \) fiber bundle: it has like the Hopf \( SU(2) \) bundle a circular loop \( S^1 \) observed as the electromagnetic interactions symmetry \( U(1) \) in the standard models symmetry \( U(1) \times SU(2) \times SU(3) \).

The base of the bundle is the complex \( CP^2 \), drawn in figure 1 as hedgehog having a color charge atmosphere about the atomic kernel or nucleon. Its states are presented by the SI rotor. The factorizing of the \( S_3 \) to \( D_3 \) allows the tables listing mentioned in [1] where a row contains six color charges. In the columns to a color charges factored class is added a coordinate 1, ..., 6, a symmetry which can come from \( D_3 \) or the \( SU(2) \) Pauli matrix quaternionic generators and one of the six energies under consideration, named in an abbreviation \( \text{EM(pot)} \) (electrical potential), \( E(q) \), \( q = \text{heat, kin, magn, pot, rot} \). Beside this factorization, the \( S_4 \) symmetry has many geometrical unused applications for physics in the quantum range.

The 8-dimensional space extensions of 1234 is for \( SU(3) \) or octonians, where in the last case quaternions are doubled up and Cooper associate in (Heisenberg) pairings the coordinates which have either as two rays a location on a coordinate line or generate as nonlinear vectors a projective extended complex plane 15 (position-momentum), 27 (polar rolled coordinates for the map \( 2 \rightarrow 7 \), complex \( \phi \rightarrow \exp(i\phi) \)), 30 (angle \( \theta \) as \( n \)th roots needle vector \( e_0 \)), 46 (time-energy). The measures for 7 and 0 are \( cd \) (candela) and color charge. The octonians extension give for their GF’s another multiplication table as the \( SU(3) \) GellMann matrices. At least 14 GF’s (7
are in figure 7) exist and combinations with the $e_0$ coordinate can add more GF’s. Figure 5 shows the model where a turn from the diagonals in the left part of figure 5 for 1234 spacetime quaternionic coordinates are complex orthogonal bases split into the former plane pairings of octonian coordinates (right part in figure 5). The mathematical construction is different and due to Cayley-Dickson. For further (complex or functional) computations, better used in physics, the complex $C^4$ space, real 8 dimensional $R^8$, are suggested. Hilbert space decompositions into a subspace and its orthogonal subspace are also used. An example for projections is given in form of a unification: E. Schmutzer [7] uses a projective interpreted subspace $R^5$, having a 5-dimensional projector; the EM (pot), $E(pot)$ are described as a common 5-dimensional field and the projector splits this into the two 4-dimensional electrical and gravity potential fields and a neutral (scalar or spactime) field [7].

The projective Birkhoff-von Neumann view for gravity and quantum structures have been accepted and proved in many publications [11]. It has a non Boolean orthomodular subspace logic which is applied as a logistics for the former facts, observed for nucleons, quasiparticles and the modern findings for the particle range. It includes also the recent findings of Higgs bosons for a mass setting Higgs field and the findings of graviton waves in the sense of the particle-wave duality. As particle the author name the $rgb$-graviton whirls and adds to the physics duality a third whirl (octonian coordinate 0) energy character to particles (octonian coordinate 5) and waves (octonian coordinate 7). For the octonian spacetime coordinates 1234 the Hopf and $S^5$ fiber bundles allow changing dimensions and projective spaces allow by the use of correlations the projective duality which associates with projective points (as lines through an origin in a real or complex space of one dimension higher) a hyperplane with the constants of the point and extends this for k dimensional projective subspaces by taking the intersecting hyperplanes for their k points generation. The logistics includes beside geometrical projectivity or projection operators like the central (stereographic) or (spiralic) orthogonal projections, the GF’s as measuring apparatus for spins and other 3-dimensional base triples in octonian coordinates. They are available for most quasiparticles.

All this new research has been done since 1936, but physics still hides most of it and makes high energy technology instead. The MINT-Wigris technology has to be payed in order to construct at least 20 more models than the ones in the patented overview as last figure in this article, the patented videos or figures which partly show proposed new MINT-Wigris tools to be invented.

2. The Hedgehog, Deuteron and Gleason

In my recent papers I suggested different geometries, finite symmetries, Moebius transformations [4], new measuring apparatus as Gleason frames GF which generate measuring Gleason operators T. If a vector space V has a distance measure, for instance as Euclidean measure $<w,w> = \Sigma w^2$ then T acts on V and changes the measure to $<wT,w>$. In addition it can change for an observable the state of a physical system P.

Example:

Spin is an observable of fermions P and mostly measured through its attached magnetic momentum in the gyromagnetic relation. P has a Euclidean measure for its spin presentation in space xyz-coordinates as vector $s = (sx, sy, sz)$ where the coordinates form an orthogonal base triple with the above Euclidean measure for the three space coordinates. In the Stern-Gerlach experiment the s vector is prepared, for instance showing in the positive x-line direction. When P (composed from several systems, a wave package for instance) enters the measuring
apparatus, the magnetic influence forces the spin direction to change either in +z or -z space direction, called spin up or down. It does this nearly in a probability distribution to 50 percent. The state of P has changed through the experiment.

In composing three quarks in a nucleon P, their three spins can have parallel projections which mostly are drawn by listing the three vectors parallel or anti-parallel in ↑↓↓, ↑↑↑, ↑↓↑,… The logical treatment requires beside choosing a Euclidean space geometry for the quarks spin energy to provide the lines $u = x, y, z$ with an orientation $+u$ or $-u$. In the 3-dimensional case as an additional number $+1$ or $-1$ the coordinates are extended projective to $[x, y, z, w]$ where $w$ takes only the values $+1$ or $-1$ and the normings are $[wx, wy, wz, 1]$. Diometrical opposite points on a 3-dimensional unit sphere with $\Sigma u^2 = 1$ present the same projective point in $P^3$ as factor space of $R^3$. In the weak interactions $SU(2)$ symmetry coordinate presentation to $x, y, z$ are attached the non-commuting Pauli matrices $\sigma_j, j = 1, 2, 3$ in quaternionic form $i, j, k$. Listing for a 4-dimensional Euclidean space $(x, y, z, w)$ its lines through an arbitrarily chosen origin $O$ as projective points of $P^3$ for the nucleons spin computation above means that there is a Gleason measure $S$ for $P$ which can change a chosen orientation for a spin vector. Mathematically, this is computed by choosing for a spin rotation an oriented triple as base and taking their determinant, normed to its sign $w = +1$ or $w = -1$. Take two base vectors $e_1 = (1, 0, 0)$ and $e_2 = (0, 1, 0)$ where spin points as normal to this plane in up or down direction. Then the determinant with first row $(1, i, j, k)$ and one of the other vectors as second row, followed by the second vector. The combination with second row $e_1$ gives the vector $(0, 0, 1)$ while the second row as $e_2$ gives the vector $(0, 0, -1)$. Adding the spin orientation means that it presents two states of $P$ while in the projective version they are identical systems. In a difference equation approach, the characteristic polynomial $z^2 - 1$ with solutions $+1$ and $-1$ is taken, as initial conditions for the spin $S$ solution are taken $a_0 = 0$, $a_1 = n$ and as Fibonacci like sequence for solutions are obtained as $S_n = (-1)^n$, $n$ a natural number, which are for the two states of $P$ listed as $w = +1$ or $w = 1$. In a dihedral $D_1$ notation the Pauli spin matrices are for the $w$-coordinates replaced by its two transformations, the identity id and a reflection. In the $P^3$ presentation of $P$ this action is disregarded.

How can the spin change in the Stern-Gerlach experiment be explained geometrically? $P^3$ has as subspace $P^2 = R^2 \cup R \cup \{ \infty \}$. It contains a Moebius strip MB. If the spin vector is set as normal to the MB and is rotated by 360 degrees along a central circle of MB, ist normal direction exchanges to the oppositely oriented vector in the environment: MB is not orientable. For a nucleon is therefore assumed that it has a spherical $S^2$ bounding sphere for $R^4$, taken as a complex projective plane $CP^2 = C^2 \cup C \cup \{ \infty \} = C^2 \cup S^2$. The boundary $S^2$ splits like a Heegard decomposition of the $S^3$ unit sphere of $SU(2)$ into two hemispheres at an equator and diametrically opposite points on an equator are identified for its presentation as $P^2$. For $P$ as a 3-dimensional system it means that along the normal $S_n$ vectors in direction of one of the three space axes $x, y$ or $z$ it can change its state from pointing inside a volume for $P$ in $R^3$ or outside for an energy exchange of the nucleon with its environment. The vector $S_n$ is then not a spin vector but a vector presenting one of six energies, listed as nucleons electrical $EM, EM, EM, EM, EM, EM$ or mass $E$ potentials ($+x$ or $-x$-axis), as its kinetic rotational $E(\text{rot})$ ($-y$-axis) or kinetic $E(\text{kin})$ ($-z$-axis) energies or as $E(\text{heat})$ ($+y$-axis) for its entropy inside the volume or $E(\text{magn})$ ($+z$-axis) for its magnetic energy. Exchanged energies use quasiparticles or field quantums like phonons for heat, proposed $rgb$-gravitons for $E(\text{pot})$, transformed $EM(\text{pot})$ frequency in form of photons or helicons, magnons can be used for $E(\text{magn})$, rotons for $E(\text{rot})$ and maybe holons for $E(\text{kin})$. 

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Fig. 1. Hedgehog atmosphere about a nucleon, deuteron or atomic kernel, drawn as color charge carrying polar caps with input/output vectors in the center.

The two hemispheres about an axis can be seen as a Cooper pair which generates the three Heisenberg uncertainties. The pairing is obtained for the deuteron fusion where two protons are joined at a common barycenter B as in figure 2 at left with the quark triangles parallel in space. There are two u-quarks along one of the three vertical intervals connecting the quarks. In a weak decay the upper u-quark decays, emits a positron and a neutrino [10] as energy in the environment and a rotational energy is set which rotates the upper proton against the lower one in space such that in a central projection the triangles form a hexagon as in the right part of figure 2, the two rays of the plane generated by the vectors from B to a quark and its vertical located partner quark is then projected onto two rays of a 1-dimensional space coordinate x, y or z. A weak interactions isospin exchange between partner u-,d-quarks on a ray can occur as a weak rotor. It is mathematically used for differentiating functions of one or several spacetime coordinates. Integrations of such functions are performed by a strong interaction SI rotor which uses a presentation of the quark triangle dihedral symmetry D₃ [1]. As examples, a potential force vector b/r², b a real constant is radial integrated to a potential as -b/r or a kinetic or rotational energy vector as d²u/dt² is integrated to a linear or angular speed vector du/dt. A volume integration is for E(heat) and E(magn) has the electromagnetic EM inductions area integration or differentiation.

The space for this is obtained by taking the complex cross product of the spacetime coordinates, written as complex vectors by the use of the quaternion generators as z₁ = z +ict, c speed of light, and z₂ = x +iy which can have also polar (r, φ) radius/angle coordinates. The energy plane z₃ = (u, w) = z₁ x z₂ carries mass measures in kg on the u-coordinate and Hz frequency measures on the w-coordinate. The plane contains the Einstein line mc² = hf such that frequencies Hz can be transformed into kg of mass and added to a systems mass, set at a barycenter of a matter system with different subsystems like a nucleon. For the SI rotor is observed that it sets three barycentrical coordinates in the quark triangle and that 90 percent of the observed nucleon mass (set at the intersection of the three barycentrical coordinates by a Higgs field or boson) is added by its inner rotations and speeds. Only about 10 percent is due to the quarks added masses.
3. Quaternions, Octonians

The use of quaternions and the Pauli matrices generators is due to presenting the three space coordinates as three rotational Euler angles which rotate as matrix transformation the plane orthogonal to the axis: to x an angle δ, to y an angle φ and to z an angle θ is used. As transformations they generate the non-commutative Pauli spin matrix multiplication of quaternions 1, i, j, k with \( u^2 = 1 \) for \( u = i, j, k \). As geometrical model the wheel is in use: the rotation is generated by two oppositely oriented force vectors in the plane orthogonal to the axis.

Fig. 3. A nucleon hanging above its projection plane in space, at three different heights the gravity generated shadow projection in the plane changes its diameter (at left), at right wheel with generated space coordinate axes.

Fig. 2. Fusion of two protons to deuteron.

Explaining figure 3: a deuteron bubble is hanging in a higher dimensional space above spacetime, the wave or whirl like action of graviton waves w or rgb-graviton whirls generate a central projection of the bubble into it, drawn as a plane as in the Hopf stoch projection. Then a radial distance between two points as physical system can be rescaled through a spiralic turn of a nucleon triangle in form of the three dogs (figure 4) as points on a
circumference circle containing the quark vertices, chasing one another with the same speed or running away from one another on the same spiralic curve. Referring to the Einstein tensors metrical $g_{\mu\nu}$ components (general relativity), the w induced propagation of spacetime ripples has an added tensor $h_{\mu\nu}$ satisfying a linear field equation $\mathbf{V}$ (using $\frac{2}{c^2}\partial^2/\partial t^2$ applied to the $h_{\mu\nu}$) and for w travelling along a space z-direction in physics a tensorial equation using the function $\cos(\omega(t-z))$, t time, $\omega = 2\pi f$, and for $f$ as frequency the same equation as EMI waves $\lambda f = c$, $\lambda$ wave length. The observed particles distance measure changes with this, - in spacetime they are kept at rest. In figure 3 this fact is drawn differently as a shadow projection, showing the changing distances. It uses a central projection.

For the SU(2) Hopf geometry $S^3$ with the Hopf fiber bundle map $h$: $S^3 \rightarrow S^2$ is observed that the three Pauli matrices map the four spacetime coordinators $(x, y, z, i)$ to three space coordinates $x, y, z$ in such a manner that the x-projection gives the complex dot product, measuring a distance in space, the y-projection the complex cross product and the z-projection relates to the $S^1$ fiber of the bundle as $(x^2 + y^2) - (z^2 + ct^2)$. Every point in $S^3$ is blown up to a circle. The south pole is mapped to a central core of expanding concentric torus locations for a rotating charge. The charge is a leaning circle on the torus such that its orthogonal magnetic momentum plus spin and the torus can rotate about a central torus axis of $S^1 \times S^1$.

Fig. 4. Gravitational spiralic curve for a central projection in octonians, showing the change of the distance measure between the dogs as a pendulum like oscillation; if $<x,x>$ is a bilinearform for a Euclidean or Minkowski distance metric, a rescaling identity operator changes the observed oscillation of distance in the proportions $a_j^{1/2} : 1 : 2$ to $a_j \cdot <x,x>$; if wanted the endpoints of the changing distance as interval I measured in a plane (figure 3) between two deuteron quarks in the triangles as 1/2 can be explained as a conic rotation where I is oscillating upwards or downwards and forms two circle-like curves between the endpoints of I with length 1 or 2 (figure at right); in the list of quasiparticles also solitons (middle figure) which change density of matter are travelling like the (amplitude) ripples at right.

The third component of the Hopf base is also available as a 4-dimensional block matrix with first row $(A \ 0)$ and second row $(0 \ B)$ where $A, B$ are 2x2-matrices for rotating one of the $S^1$ copies of the torus. The spin vector sits as normal to $S^2$ at its north pole. The coupling of spin with the magnetic momentum is due to the superposition of two GF, named below as 123 for spin and 145 for EM and 4 as magnetic momentum vector, joined to spin at their endpoint and north pole. The change of direction in the gyromagnetic relation is due to the fact that the direction of the magnetic momentum in 145 is changing with the counterclockwise mpo +1 charge or clockwise cw -1 charge of the lepton. It reverses then ist orientation, also towards spin. A similar helicity property for neutral leptons exists where the magnetic momentum is replaced by momentum as partner of the rotating neutral charge. The charge rotations are on a latitude circle of $S^2$. In a central projection $S^2$ is mapped to an $xy$-plane through the map $\text{st} \circ h (z_1, z_2) = z_2 / z_1$ for $z_1 \neq 0$. The north pole of $S^2$ is in projective extended, normed (complex) coordinates $[0, 1, 0]$ and the projective coordinates $[z_1, z_2, z_3]$ give the normed complex spacetime $[z_1 / z_2, z_2 / z_3, 1]$. The boundary of this plane is $[1, z_2 / z_1, 0]$ with $[0, 1, 0]$ for $z_1 = 0$ added as north pole.
for the $S^2$ projection st. The st projection of $S^2$ adds for the Bohr sphere location of an electron in an atom's shell the possible energy change of radius in form of the main quantum numbers. Its angular frequency is transformed, using the Rydberg constant scaling, into the EMI frequency of its energy, expanding as helix line on a circular $U(1)$, $S^1$ cylinder with the cylinder's axis as its linear world line. One winding in time on the fiber bundle space $U(1)$ of the cylinder quantizes the spin with natural numbers $n = 1, 2, \ldots$. As spin values also such multiples of 1/2 can occur. The initial conditions for a Fibonacci like sequence, changing spins up/down direction is replaced in this case by a degenerate numerical orbit of the $D_3$ symmetry as solutions of their presentation as Moebius transformation maps quotients. For the six members of $D_3$ can be used the normed $1/z$ inversion of the first Pauli matrix (and $z$ as identity), together with the complex cross ratios $z/(z-1)$, $1/(z-1)$ and their inverses. Take $z = 2$ then $1/z = 1/2$ for spin length $1/2$, $z/(z-1)$ is 2 for rgb-graviton spins and $1/(z-1)$ is 1 for boson spins. If instead of a difference equation, two suitable Moebius cross ratios are set equal and solved as an equation in the variable $z$, the third complex roots of unity $p_1$, $p_2$ are obtained for the quark triangles $D_3$ symmetry as rotation $\alpha$ with $\alpha^3 = \text{id}$. The map $\alpha$ is a mpo rotation of the triangle by 120 degrees and $\alpha^2$ as a cw rotation by -120 degrees.

The deuteron bubble as $CP^2$ is the base of another fiber bundle, using the twisted product geometry of the strong interaction as $S^3 \times S^5$ of unit spheres. $S^4$ is here replacing the Hopf map $S^3$ and is projective normed to $CP^2$ by a $U(1)$ scaling of its coordinates with fiber of the bundle $S^1$.

There is another dimensional space extension of the complex space $C^3$ from the abstract. A similar figure is available for presenting this as in figure 2, using a leptonic quadrangle with the magnetic group symmetry added.

![Diagram](image-url)

Fig. 5. The magnetic group has as its four vertices the EM charge vector and the neutral charge leptons, combined with magnetic momentum and momentum vector as the two other vertices; drawn are as diagonals at left the four spacetime coordinates, numbered as intervals between j vertices; at right the quadrangle is similarly rotated by an angle, for instance of 135 degrees; in a central projection an 8-edge replaces the hexagon in figure 2; the 1-dimensional diagonals are drawn as two rays which as vectors generate a complex plane, all together form an octonian coordinate space with the quaternions doubled; the 1 and 4 planes can be pairing as in the Heisenberg uncertainties, but 2 as y or $\phi$ coordinate and 3 as $\theta$ angle bifurcate differently; to 2 is added an octonian coordinate $e_7$ which is rolled up for periodic functions, presented by the map $\phi \rightarrow \exp(i\phi)$ for the $U(1)$ polar coordinates complex circle, having a stereographic map onto $e_7$; to the angle $\theta$ is added its spherical coordinate measure which gives an octonian compass needle $e_0$, leaning in this angle towards the $z$-coordinate axis of space, for setting measuring energies unit vectors on the associated octonian coordinate: length in meter and Ampere for EM(pot) on 1, heat in Kelvin on 2, rotation on 3 and magnetic energy, time in seconds on 4 with the known combined measures, on 5 mass in kg, on 6 frequency in Hz; energies get different qualities.
For the \((e_0, e_7/U(1))\) plane the geometry is a G-compass (figure 6). The turning of the vector is discrete with the 6th roots of unity. For color charges the turns act for setting them as charge on a segment between two adjacent positions of the needle. When cut out, the sides of the segment are identified and the color charge has as surface a conic whirl like magnetic field quantaums. They can have super positions like gluons or the rgb-graviton as neutral color charge of the three red-green-blue quarks in a nucleon. For the EM(pot) charges the sixth roots of unity as multiples of \(2\pi/6\) set six possible electrical signed charges with \(1/3, 2/3, 1\), for masses of fermionic series it sets six possible masses and the conjugation operator generates from the six the other 6 of the fermionic 12 series. Also the six energies with different qualities and measures are generated. Beside the G-compass, acting through the use of the \(D_6\) 6th roots of unity, other roots of unity can guide such a dihedral compass, \(D_7\) and \(D_{3, 4}\) are explained earlier. The use of \(D_2\) can be for quarks. It is presented in a Hopf \(S^3\) Heegard decomposition by a surface of genus 2 while leptons have a torus-like surface of genus 1. The \(D_2\) circle contains two poles for its electrical and mass potentials as foci of a central 1-dimensional lemniscate, replacing the core of a torus. The \(D_2\) symmetry of order 4 contains two reflections and one rotation of \(D_2\) by 180 degrees. For the \(D_3\) case and nucleons was postulated a SI rotor for functional integrations and for generating barycentrical coordinates. Several models and videos are constructed for the strong interaction SI rotor. If such a rotor is constructed for single quarks, it has in general no equilibrium state and single quarks have a short lifetime, missing the nucleons SI gluon exchange. They are decaying with the weak interaction WI, releasing a WI boson which decays into an electrical and its partner neutral lepton.

Fig. 6. G-compass using the 6th roots of unity as solutions of a characteristic polynomial \(z^6 - 1\) for six nucleon equilibrium states of the SI rotor of a difference equation.

4. Research Project

As research project [6], [9] is suggested that the use of projective geometry through subspace lattices is logically investigated; for the 4-dimensional projective version of extended spacetime coordinates \([x, y, z, \text{ict}, w]\) subspace lattices holds that they are not containing in the Hasse diagram version a pentagon where a line in this space presents an atom directly above the smallest lattice element 0. A line is a lattice element directly above an atom and a 2-dimensional subspace is directly above such a lattice element. There is a dimension function for the lattice elements. The modular law holds which means that any two lines have an atom as intersection in common. Parallels are not allowed in projective geometrical axioms. In drawing atoms provided with a lattice orthogonality, some authors draw one line as a unit, largest element of the lattice. The finite
diagram is then the modular ortholattice MO2 of figure 7. If this is taken as a logic for the spin up or down states, the listing of several spin n combinations can be an MOn. Mostly spins are drawn by parallel or anti parallel vectors. The subspace lattice of an infinite dimensional Hilbert space fails to have the modular law and has as replacement the orthomodular law. This means that such a lattice, especially in a finite dimensional subspace case, is the set theoretical union of its Boolean sub lattices where the Boolean distributivity allows classical logic. The orthomodular logic has for non-commuting projection operators only a very weak implication and no deduction theorem. For a set of commuting operators classical Boolean logic applies.

If finite orthomodular lattices with a dimension function are like MOn drawn by atoms block diagrams, an interval is drawn having n atoms as points marked on the line. For octonians such a lattice diagram can be the Fano memo (figure 8).

MO\text{On} is for spins with n = 3 drawn earlier as the vector combinations ↑↑↓, ↑↓↑, ↑↑↑,... where one vector is from one block, either in up u or down u' state of the atom. Block diagrams are marked in figure 8. If logical orthomodular block diagrams have Boolean subspace lattices, the projective subspace lattice has no 3-cycles, no GF's.

Fig. 7. Some Hasse diagrams and MOn (below) with n Boolean blocks; 5 is a Boolean 2\text{^4} lattice, 10 a pentagon, 11 a MO2, 12 a Boolean block 2\text{^3}.
Fig. 8. For seven octonian subspaces as intervals with three points for its atoms, carrying each an energy vector, marked with indices of the coordinates 1,…,7; the needle $e_0$ is set apart at left; the subspaces are in matrix form presented by the three Pauli matrices and listed as GF by the triple of their coordinate indices: 123 is for spin and space coordinates a GF, 145 a GF for EM, 167 a GF for EMI, 246 for heat, 257 for Higgs and barycenters with mass, 347 for rotations in form of a triple for rotational momentum as $L = r \times p$, $r$ a radius, $p$ momentum of an orbiting system about a barycenter, 356 is for the SI rotor.

Figure 8 is only for octonians GF’s and not a subspace logic. In diagrams for orthomodular lattices of dimension 3 also no 4-cycles are allowed. For orthomodular subspace lattices in 4 dimensions both are allowed, but there is the request that additional blocks of commuting operators must exist as in figure 9. A research project is to look what logic can be set for modular ortho or orthomodular subspace lattices with a dimension function where the whole space as maximal element 1 of the lattice has dimension 2, 3 or higher. The block structure has no measure interpretation as the Fano diagram for the octonian Pauli matrices GF’s. The spin setting logic is with the MOn sub lattice interpretation and uses the lattice orthogonality for spin up or down vector states.

Figure 9 blocks contain four atoms; as a Boolean subspace diagram for the orthomodular subspace lattice in form of a triangle (at left for red-green-blue color charge energies), the vertices of the triangle have to present three commuting operators which in 4 dimensions are joined to a Boolean subspace lattice by adding a fourth atom; in the case of a 4-cycle (at right), four atoms in form of the drawn astroid have to be added for four blocks, each containing adjacent vertices of the 4-cycle; an octonian interpretation for this is that spacetime coordinates 1234 as vertices of the 4-cycle are doubled to the pairing of octonian coordinates in figure 5 to 15, 27, 30, 46 subspaces. The atoms of the outer 4-cycle come from other blocks in an orthomodular lattice.
In several models, states of a system like a nucleon are macrospocically constructed in the MINT-Wigris Tool bag. The SI rotor presents six (equilibrium) states of the nucleon as a $D_3$ symmetry presentation. The tetrahedron figure of a nucleon in figure 2 adds a central point for a rgb-graviton as superposition of three color charge whirls where a time coordinate as atom for its block presentation can be added. The tetrahedron symmetry is $S_4$ of order 24, the permutations of 4 elements which can be factors by the commutative normal Klein subgroup $Z_2 \times Z_2$ to $D_3$. The Klein group occurs as CPT symmetry in the operator multiplication form $CP = T = PC$, $CT = P = TC$ and $PT = C = TP$. The factorization of $S_4$ was used in older publications of the author in a table having six rows for the six color charges and four lines where the color charges line is extended by a coordinate line for C³ octonian coordinates 123456, by a list of associated symmetry matrices which can be the $D_3$ elements and alternatively for spacetime the SU(2) generating Pauli matrices. In vectorial form, one fourth line contains an energy with its measure set by the $e_0$ octonian vector. Can this be a higher dimensional nucleon logic as orthomodular subspace lattice having blocks of dimension 4? Dimensions 2 are interpreted as the logical spin combinations in figure 7, dimension 3 has some meaning (maybe) for the GF’s. The six states of a deuteron or nucleon in the SI rotor can have 4-dimensional subspace logic where the atoms for the logical lattice have atoms presenting a color charge, a coordinate, a symmetry and an energy. Additional required blocks as in figure 9 of the lattice have another interpretation. For the 3-cycle a quasiparticle as rgb-graviton is additional suggested (having a fourth time coordinate added to a GF 126), for a 4-cycle with 1, 2, 3, 4 vertices the use of four octonian coordinates for subspaces 1257, 2307, 3406 and 1456 are suggested. The 126 GF subspace can be listed by the first three GellMann matrices of the SI symmetry SU(3). SI generates also seven GF’s, similar to the octonian Fano memo GF’s. A Higgs boson has no octonian GF and can be for instance presented by 127, adding 5 as an additional atom which sets a mass scalar at a barycenter, 145 was an octonian GF for EM with the cross product presenting magnetic induction as a rotational vector, the compass needle 0 is added to 346 or 237 as GF. The needle could set a turning angle $\theta$ in 03 or $\beta$ for rescalings according to the Minkowski or Schwarzschild metric in form of $\sin \theta = \nu/c$, $\nu$ special relativistic speed, $\sin \beta = R_s/r$ containing a variable radius $r$ and the Schwarzschild radius $R_s$ of a matter system. The logic is a projection operator in the Minkowski watch figure where two coordinate systems not in gravitational interaction are used for 346 rescalings of energy measures. In an experimental situation where one systems coordinates are in special relativistic speed $\nu$ against the other systems coordinates as symmetries the Lorentz transformations apply. If 346 has a vectorial quasiparticle associated like spin as eigenrotation or the rgb-graviton whirls, in the list of quasiparticles a roton can be used for the setting of the $\theta$ angle with 3, 46 is then for the energy equation in $E = h\nu$, $h$ the Planck constant, with frequency $\nu$ as inverse time interval for a wave package presentation of the measured system where $v$ is obtained by special relativistic rescaled mass differentiation, as the momentums speed of the wave package.

The unit measures on the upper line of figure 10 are measured on the horizontal lower line by multiplying or dividing special relativistic by the $\cos \theta$ factor. The nonlinear additional rescaling of Minkowski to Schwarzschild metrics when the two systems are in a gravitational interaction, is due by setting a common barycenter between them and choosing for the huge mass system its accelerating Schwarzschild radius as the $\cos \beta$ multiplication or division for the differentials $dt$ of time or $dr$ for radius. Used is a central projection 237 (possibly as a quasiparticle orbiton) for this logic with a projective interpretation [1]. In this nonlinear rescaling, 3 sets the angle $\beta$ and 27 presents the complex polar angle $\varphi$ of 2 as exp(i$\varphi$) function of 7 and circle U(1) for
EMI. The action is for an unsymmetrical radius r measure where the huge system Q measures its distance to the possibly rotating system P as |QP| = r and P measures its distance to Q only up to the Schwarzschild radius of Q as |PQ| = r – Rs. The projective nonlinear norming generates \( \cos^2 \beta \) as a complex Moebius transformation with \( z = r \) as the complex variable in \( (z-1)/z \), mentioned shortly after figure 3 for a \( D_3 \) group representation through six complex cross ratios. The G-compass (figure 6) has its coefficient 2x2-matrix of order 6 with first row \((1 \ -1)\) and second row \((0 \ 1)\). For the v Minkowski 2x2-matrix the columns of G are exchanged.

The GellMann 3x3-matrices of SU(3) are extended Pauli matrices where a row and a column of 0 coordinate vectors is inserted. There are only 8 not 9 GellMann matrices since the three extended third diagonal Pauli matrices are linearly dependent and generate as space vectors a plane, not a 3-dimensional space. Their multiplication table is different from the octonian multiplication.

![Diagram](image)

Fig. 10. Minkowski watch as compass.

The research is open for most above listed suggestions. Operators in form of projections onto subspaces are often involved as well as energy exchanges through quasiparticles like spin or rgb-gravitons.

A question is: can be subspace logics in form of finite orthomodular lattices be found which serve with their atoms for the new logical actions through quasiparticles similar to the MOn subspace lattice which combines spin orientations of composed systems. There are only finitely many systems combined which need for state combinations not an infinite Hilbert space and its subspace lattice for projection operators.

5. Shapes and Models for Systems

![Diagram](image)

Fig. 11. Nth roots of unity, spin up/down on a circle, electron spins in atoms.

For the geometrical shapes, the former lists or figures are extended and described for the models and GF’s. Some facts are repeated and figures of the first three sections are not repeated.
The octonion coordinate 0 is used for a compass needle and has the GF 037, including the compass loop/circle U(1) for 7 (figure 6) and for the needle a leaning angle 3 for θ as in figure 11 left.

The spin and space coordinates have the GF 123 (middle and right figure in 11) and an orthogonal base triple (figure 12 left, middle row).

![Diagram](image)

**Fig. 12.** Circle orientations and rotation axis (upper line left), orthogonal (base) triple left/right handed (middle line left), helicity oriented vectors for neutral leptons (last line left), two Lissajous figures (first row right), below three contraction/expansion (Bohr) radii for different shapes and systems (with the three basic spin length added).

The rgb-graviton GF 126 uses for contraction/expansion (figure 4) in a central projection for the CP² bubble the three basic spin length as radii. As a conic whirl (figure 13 left) it is at the tip of the nucleon tetrahedron (figure 2). In figure 9 left, it is 4-dimensional drawn, including a time coordinate, 126 is replaced by its space projection 123 for the other points on the curved line. The orthomodular lattice proof requires it for triangular subspace/block diagrams [11]. Kaluza-Klein loops U(1) (figure 12 upper right) occur independent of a cones bounding circle which arises through a rotating vector, keeping its initial point fixed and having a positive θ angle towards the rotation axis. In the Lissajous figure (12 upper right), two frequencies hit orthogonal in integer proportions. For 1:1 a circle is obtained, for 1:2 a lemniscate (figure 13 lower left) as 1-dimensional core of a quark.

![Diagram](image)

**Fig. 13.** Identify the free red and green intervals for the rgb-graviton, next is a gluon, below a quark lemniscate, at right a weak interaction Heegard decomposition of S³ into two 3-dimensional brezels of genus 2 for quarks and a planar internal energy flow.
For the EM in the figure 14 is drawn for the EM charge the rotating loop of an electrical current. Its inner area is traversed by a magnetic field and induction lets the loop rotating, for an EM charge, the magnetic momentum is orthogonal to the loop area. The GF is 145, 1 is used as coordinate for the EM charge, 4 for the magnetic momentum mu.

![Diagram](image)

Fig. 14. Rotating EM loop at left, rolled coordinates for an EMI cylinder and a lepton torus of genus 1 at right (periodic functions like exp).

The electromagnetic interaction EMI is much later generated in the universes evolution than other forces. Photons could not escape from atoms and neutrinos neither. There is one GF 167 and 27 is for generating the map \( \varphi \rightarrow \exp(i\varphi) \) for the periodic, complex polar exponential function exp. For the 167 GF, 1 is for a wave length, 6 for an EMI frequency and 7 is the cylinders U(1) circle of EMI. Frequency expands on the cylinder in time as a helix line (figure 15 left). As observable, the real component \( \cos \varphi \) serves, presenting the wave (or oscillation) as a function of time and space. Imaginary parts of numbers or functions are not observables, or differentiation has to be applied.

![Diagram](image)

Fig. 15. Circle (or sphere) as boundary for a particles energy, cones for quasiparticles and field quantums, helix for waves at left, double helix at right with a time orientation on the central cylinder axis.

Beside solutions of (systems of) differential equations, difference equations, equations guide the shapes presentation of energies. For 15 and EMI an equation is \( \lambda p = h \), \( \lambda \) wave length, \( p \) momentum, \( h \) the Planck constant. This is also for the first Heisenberg uncertainty position-momentum. A new uncertainty was proposed by the author for speed of light as upper bound \( c \) which involves the coordinate \( \theta \) and \( e_0 \) as octonian coordinates [12]. Heisenberg uncertainties are for two space coordinates (length \( x \) or radius \( r \), angle \( \varphi \)) and time, having a
scaled h lower bound. The dimensional extension of space to spacetime requires by the orthomodular lattice proof that a 4-cycle of (Boolean) blocks, in figure 9 right, has a (central blocks) astroid. The spacetime coordinates are doubled to octonians: 15, 46 (equation $E = hf$) are Heisenberg h pairings, the EMI 27 is added and 30 is for the compass needle (figure 6) where the new c uncertainty (equation $\lambda f = c$ for EMI, f frequency) occurs (figure 16).

For the GF 246 belonging to heat 2, the angular speed $\omega = \frac{d\phi}{dt}$ 6 is used for accoustic whirls (figure 16 right). Phonons for heat are quasiparticles for a volume with energy as entropy inside. The equation is $pV = bT$, $p$ pressure on a volumes surface like $S^2$, $V$ volume in space, $b$ constant, $T$ temperature. The accoustic (Mach cone) whirl front is like an oscillating membran which transfers in time energy and momentum through phonons in matter.

The GF 257 was used for Higgs (field or bosons) setting at barycenters a mass scalar for matter systems. As mentioned earlier, mass 5 can be rescaled when needed, the equation is $mc^2 = hf$ or a Minkowski special relativistic rescaling SR of mass. For EMI 7 sets relativistic mass for its energy frequency. SR uses for 2 an angle $\phi$ for the $\cos \phi$ rescaling with $\sin \phi = v/c$, $v$ relative speed (figure 17 left).

The GF 347 was used for rotations of a system P about a central point Q on a rotation axis 3 which is orthogonal to the systems orbit in a plane. Kepler's second law requests that the radius vector $r$ from Q to P traverses in a time interval 4 the same area $A$, $dA/dt = constant$. The equation for angular momentum is $L = rxp$, $p$ momentum of P. The energy preservation of $L$, $E(rot)$ holds. Since the whole energy $E' = E + E(rot)$ of a
rotating system is preserved, the usual energy preservation $E = E(pot) + E(kin)$ as sum of potential and kinetic energy holds, also for matter waves 7. The GF 356 is used for the SI rotor and is presented by a 6-cycle of the symmetry $D_3$. The model in figure 18 shows that single coordinates have to be set for the conic rotations which allow integrations (listed in the abstract) while some differentiations are due to the wheel rotations (figure 3) about axes as described for 347. For time function $f$ differentiations the coupling 46 adds $df(t)/dt$ as derivatives.

For the compass (figure 6) the GF 037 was used. This is the first model in the following table with 8 figures for existing models.

In this last figure below [5] some technical macroscopic running models are shown for the new axiomatic. For other GF to the in 2020 existing 10 models are needed presentations through a technical model - to be invented. With the template model, hand drawn figures can be made on paper with pencil. The last figure of the table is for dark matter as Horn torus with a point singularity in the middle and for an EMI cylinder where the helix frequency is drawn as a coil and the cylinder is closed at projective infinity by a point for dark energy inside. For whirls the coil is missing and the projective closure of the double cone is a pinched torus, similar to the EMI case. Dark whirls are suggested to exist beside dark mater and dark energy. Observed in astronomy, they can be responsible for the sound coming from the universes big bang.
REFERENCES


AUTHOR’S PROFILE