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Performance of PAPR Reduction Techniques in Terms of BER in LTE-OFDM Signals

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***Abstract* — As a promising technique, OFDM has been widely used in many new and emerging broadband communication systems but the main drawback of Orthogonal Frequency Division Multiplexing (OFDM) is is its high peak-to- average power ratio (PAPR). There are many techniques like Clipping, Companding, Active Constellation Extension, Selective Mapping, Partial Transmit Sequence, Interleaving, etc. All the PAPR reduction techniques have some disadvantages , essentially, BER increases .In this paper two PAPR reduction techniques called Clipping Based Active Constellation Extension (CB-ACE) which is based Repeated Clipping and Filtering (RCF) originally proposed by Jean Armstrong and the other technique called Exponential Companding Transform has been compared on the basis of SNR vs BER performance. The simulation results show that, exponential Companding Transform gives better result for PAPR Reduction and provides low complexity in Algorithm.**

***Keywords* — CB-ACE, Exponential Companding Transform, OFDM, PAPR.**

I. Introduction

OFDM is a powerful modulation technique being used in many new and emerging broadband communication systems, such as digital audio broadcasting (DAB), high-definition television (HDTV), wireless local area network (IEEE 802.11 a and HIPERLAN/2) and Wimax (IEEE802.11). However, because OFDM signals are multicarrier signals that consist of many orthogonal subcarriers with random phase and amplitude, they have large PAPR that requires a linear high-power- amplifier (HPA) with an extremely high dynamic range, which is expensive and inefficient. Furthermore, nonlinearity in amplifier produces inter modulation products between different subcarriers and results in increased BER.

Various a approaches have been proposed to deal with the PAPR problem such as Clipping, Repeated Clipping-and-Filtering (RCF), Coding, Companding Transformation, Inter leaving, Active constellation extension (ACE), Selective Mapping (SLM), Tone Reservation (TR), Partial Transmit Sequence (PTS), and soon [1]. The simplest scheme is RCF proposed by Jean Armstrong which is popularly known as Clipping Based-Active Constellation Extension (CB-ACE), which reduces the PAPR up to a sufficient level and limits the out-of band power to a low level , the computational complexity also increases.[3].This paper gives difference between CB-ACE Algorithm and Exponential Companding Transform.

II. Definition of OFDM Signals and PAPR

OFDM signal can be formed by a block of symbols, $\{X_k, k=0,1,\ldots,N-1\}$, with each symbol modulating one of a set of subcarriers, $\{f_n, n = 0, 1, \ldots, N-1\}$ with equal frequency separation 1/T (required for the orthogonality of the subcarriers), where T is the original symbol period. This can be simply done by inversed is crete Fourier transform (IDFT) for generating the multicarrier symbols. The IDFT of vector $X[k] = [X_0, X_1, \ldots X_{N-1}]$ results in T/N spaced discrete time signal $x[n] = [x_0, x_1, \ldots x_{N-1}]^T$. Thus, the transmitted signal is

$$x_n = \frac{1}{\sqrt{N}} \int_{k=0}^{N-1} \exp j\frac{2\pi kn}{N} \qquad 0 \le k \le N-1 \quad (1)$$

The PAPR of the transmitted signal is the ratio of peak power to the average power of the signal and can be written as [12]

$$PAPR = \frac{\max\limits_{0\le n\le k-1} |x_n|^2}{E\{|x_n|^2\}} \qquad (2)$$

For analysis of PAPR reduction techniques the most widely used performance measure is Complementary Cumulative Distribution function(CCDF), which gives the probability that that the PAPR of a data block exceeds a given threshold z. The CCDF of the PAPR of a data block of N symbols with Nyquistrate sampling is derived as

$$P(PAPR>z)=1-P(PAPR \le z)=1-(1-e^{-z})^N \qquad (3)$$

III. The CB-ACE Algorithm

The basic principle of Clipping-Based Active Constellation Extension (CB-ACE) algorithm involves switching between the time domain and the frequency domain. Filtering and applying the ACE constraint in the frequency domain, after clipping in the time domain, both require iterative processing to suppress the subsequent regrowth of the peak power [3]

The CB-ACE algorithm is first used to clip the peak amplitude of the original Orthogonal Frequency Division Multiplexing (OFDM) signal. The clipping sample obtained after clipping the peak signals, denoted by $C_n^{(i)}$, is given by

$$C_n^{(i)} = \begin{cases} \left(|x_n^{(i)}| - A\right)e^{j\theta_n}, & |x_n^{(i)}| > A \\ 0, & otherwise \end{cases} \qquad (4)$$

where $C_n^{(i)}$ is the Clipping sample of the iteration, $x_n^{(i)}$ is the oversampled OFDM signal, A is predetermined clipping level.

The equation (4) says that the clipping sample is reduced to a value equal to zero when the peak amplitude of the original OFDM signal is less than or equal to the predetermined clipping level, A. If the peak amplitude of the original OFDM signal is greater than the predetermined clipping level, then the clipping sample is given by

$$\left(\left|x_n^{(i)}\right| - A\right)e^{j\theta_n},$$

where the predetermined clipping level is subtracted from the oversampled OFDM signal an is then multiplied by an exponential value [3].

The predetermined clipping level, denoted by A, is related to the target clipping ratio, γ and is given by the equation 5 [3].

$$\gamma = \frac{A^2}{E\{|x_n|^2\}} \quad (5)$$

Where, γ is the target clipping ratio and A is predetermined clipping level.The clipping of the peak signal results to distortion of the original OFDM signal, namely In-Band Distortion and Out-of-Band Distortion . [3], [4].The in-band distortion results in the system performance degradation and cannot be reduced, while, the out-of-band distortion can be minimized by filtering the clipped signals. The signal obtained after filtering the clipped signal is given by [3].

$$x^{(i+1)} = x^i + \mu\tilde{c}^{(i)} \quad (6)$$

where ,μ is positive real number (μ varies from 0.1 to 1) and $\tilde{c}^{(i)}$ is theanti-peak signal at the i^{th} iteration given by

$$\tilde{c}^{(i)} = T^{(i)}c^{(i)} (7)$$

where , $T^{(i)}$ is transfer matrix at the i^{th} iteration which is given by

$$T^{(i)} = \hat{Q}^{*(i)}\hat{Q}^{(i)} \quad (8)$$

where ,$\hat{Q}^{*(i)}$ is conjugate of constellation order and $\hat{Q}^{(i)}$ is the constellation order.

Though, the process of filtering completely eliminates the distortions caused by the clipping process, it introduces peak regrowth at some of the peak signals of the OFDM signal. The peak regrowth can be reduced by repeating the filtering process, which may again introduce some distortions. Therefore, the clipping and filtering processes are to be repeated until the peak signals are completely reduced. Hence, the Clipping-Based Active Constellation Extension (CB-ACE) Algorithm is also named as the Repeated Clipping and Filtering (RCF) process [3]

IV. The Exponential Companding Transform Algorithm

The Exponential Companding Transform is also called as the Nonlinear Companding Transform. The idea of companding comes, from the use of companding in Speech Processing. Since, the Orthogonal Frequency Division Multiplexing (OFDM) signal is similar to that of the speech signal, in the sense that large signals occur very infrequently, the same companding technique can be used to improve the OFDM transmission performance .

The key idea of the Exponential Companding Transform is to effectively reduce the Peak-to-Average Power Ratio (PAPR) of the transmitted or the companded Orthogonal Frequency Division Multiplexing (OFDM) signals by transforming the statistics of the amplitudes of these signals into uniform distribution. The uniform distribution of the signals can be obtained by compressing the peak signals and expanding the small signals. The process of companding enlarges the amplitudes of the small signals, while the peaks remain unchanged. Therefore, the average power is increased and thus the Peak-to-Average Power Ratio (PAPR) can be reduced.

The Exponential Companding Transform can also eliminate the Out-of-Band Interference (OBI), which is a type of distortion caused by clipping the original OFDM signals. The other advantage of the companding transform is that, it can maintain a constant average power level. The proposed scheme can reduce the PAPR for different modulation formats and sub-carrier sizes without increasing the system complexity and signal bandwidth. The Exponential Companding Transform also causes less spectrum side-lobes.

The original Orthogonal Frequency Division Multiplexing (OFDM) signal is converted into the companded signal by using the Exponential Companding Transform. The companded signal obtained by using the Exponential or Nonlinear Companding Transform is given by the equation 9

$$H(x) = sgn\,(x)\sqrt[d]{\alpha\left|1 - exp\left(-\frac{x^2}{\sigma^2}\right)\right|} (12)$$

Where, h(x) – Companded Signal obtained by Exponential Companding Transform, $sgn\,(x)$ -sign Function, α – Average Power of Output Signals, x-original OFDM signal. The average power of the output signals, denoted by α, is required in order to maintain the average amplitude of both the input and output signals at the same level. The average power of the output signals is given by the equation 10.

$$\alpha = \frac{E\left[|S_n|^2\right]}{E\left|\sqrt[d]{\left[1 - exp\left(-\frac{|S_n|^2}{\sigma^2}\right)\right]}\right|^2} (13)$$

Where, α – Average Power of Output Signals d – Power of the amplitude of the Companded Signal

V. Simulation Results

The SNR vs BER plot of the original Orthogonal Frequency Division Multiplexing (OFDM) signal is calculated for the above two algorithms.

From the Figure 1, the Signal-to-Noise Ratio (SNR) is calculated to be equal to 16 dB for a Bit Error Rate (BER) of 10-1 or 0.1 i.e., only 1-bit is in error when a stream of 10-bits are transmitted via a communication channel or medium, when the original Orthogonal Frequency Division Multiplexing (OFDM) signal is transmitted through an Additive White Gaussian Noise (AWGN) channel.

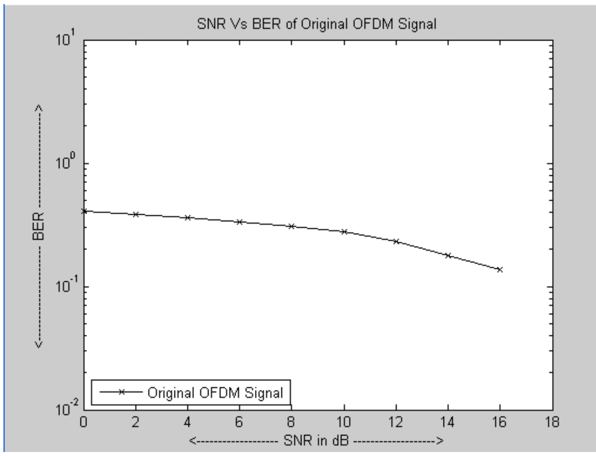

Fig. 1 – SNR Vs BER of Original OFDM Signal

From the Figure 2, the Signal-to-Noise Ratio (SNR) of the Orthogonal Frequency Division Multiplexing (OFDM) signal obtained using the Clipping-Based Active Constellation Extension (CB-ACE) algorithm is equal to 12 dB at a Bit Error Rate (BER) of $10^{-0..4}$ for different constellation orders like 4-Quadrature Amplitude Modulation (4-QAM), 16-Quadrature Amplitude Modulation (16-QAM) and 64-Quadrature Amplitude Modulation (16-QAM).
Here, the Bit Error Rate of $10^{-0.4}$ or 0.398, that means a total of 398-bits are in error when 1000-bits are transmitted via a communication channel or approximately 4-bits are in error when 10-bits are transmitted via a communication channel.

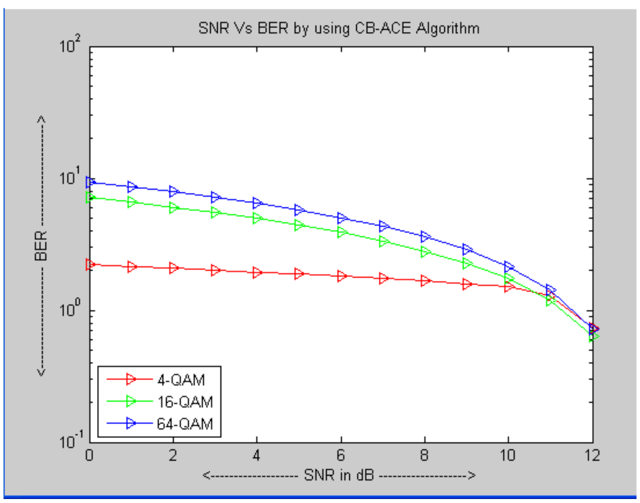

Fig. 2 – SNR Vs BER by using CB-ACE Algorithm
(For Different Target Clipping Ratios)

From the Figure 2, the Signal-to-Noise Ratio (SNR) of the companded Orthogonal Frequency Division Multiplexing (OFDM) signals obtained by using the Exponential Companding Transform is equal to 12 dB at a Bit Error Rate (BER) of $10^{-0.9}$ for 16-Quadrature Amplitude Modulation (16-QAM).

Here, the Bit Error Rate of $10^{-0.9}$ or 0.126, that means a total of 126-bits are in error when 1000-bits are transmitted via a communication channel or approximately 2-bits are in error when 10-bits are transmitted via a communication channel or medium.

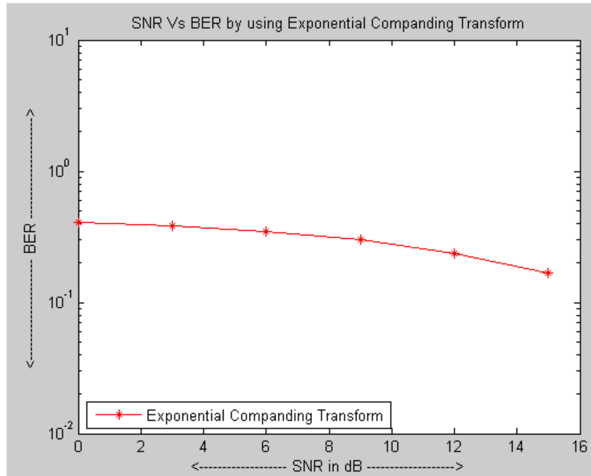

Fig. 4 – SNR Vs BER by using Exponential Companding Transform

From the table 5.1, the Bit-Error Rate (BER) of the OFDM signal is drastically increased by using the PAPR reduction techniques like Clipping-Based Active Constellation Extension (CB-ACE) Algorithm and Adaptive Active Constellation Extension (Adaptive ACE) Algorithm but the BER is slightly increased by using the Exponential Companding Transform, when compared with the original OFM signal.

Table 5.1 – Comparison of SNR vs BER for different techniques

Different Techniques	**SNR (in dB)**	**BER**	**No. of Bits in Error (Out of 1000 Bits)**
Original OFDM Signal	16	10^{-1}	100
Clipping-Based Active Constellation Extension (CB-ACE) Algorithm	12	$10^{-0.4}$	398
Exponential Companding Transform	12	$10^{-0.9}$	126
Adaptive Active Constellation Extension (Adaptive ACE) Algorithm	1.2	$10^{-0.4}$	398

VI. Conclusions

The Clipping-Based Active Constellation Extension (CB-ACE) Algorithm reduces the high Peak-to-Average Power Ratio (PAPR) by clipping and filtering the original OFDM signal. The CB-ACE Algorithm results to peak regrowth, Out-of-Band Interference (OBI), low clipping ratio problem, increase in the Bit Error Rate (BER) and decrease in the Signal-to-Noise Ratio (SNR).

The Exponential Companding Transform improves the Bit Error Rate (BER) and minimizes the Out-of-Band Interference (OBI) in the process of reducing the Peak-to-Average Power Ratio (PAPR) effectively by compressing the peak signals and expanding the small signals. The improved BER transmits the data via a transmission channel with fewer errors, while the minimized OBI reduces the effects caused by clipping.

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