A System Identification Method to Hammerstein Model Based on Recursive Least Squares Method

Jiangtao Zhai
School of Electronics & Information, Jiangsu University of Science and Technology, Zhenjiang Jiangsu, 212003

Chengming Zhu
School of Electronics & Information, Jiangsu University of Science and Technology, Zhenjiang Jiangsu, 212003

Abstract – In this paper, the nonlinear system is taken as the research problem. Hammerstein model as a typical nonlinear model with specific structure, the model is consisting of the static nonlinear module and the linear dynamic module in series form. It can be better reflecting the characteristics of the process, and the Hammerstein model also can be described a large class of nonlinear processes. At present, many scholars have been studied a lot about nonlinear systems and it also proposed some algorithms, thus, a model method based on the recursive least squares algorithm is proposed in this paper, and use the MATLAB simulation software to simulate. Finally, the algorithm was verified by experiments in the accuracy of parameter identification and the advantage of noise resistance.

Keywords – System Identification, Hammerstein Model, Recursive Least Squares, Simulation.

I. INTRODUCTION

The field of system identification uses statistical methods to build mathematical models of dynamical systems from measured data. It’s used to find out a model which equivalent to the given one from the specified category. The purpose of the system identification is to learn the general law of things, and solve practical problems by using the identification results. At present, it has been widely used in the field of transportation, medical treatment, communication and so on. As most of the industrial systems are nonlinear, the research of nonlinear system identification has very important significance. Considering that the expressions of the nonlinear models are more complicated, many nonlinear systems can be modularized through the intraconnection of the memoryless nonlinear gain links and the linear subsystems. As a typical nonlinear system model, Hammerstein model is consist of the static nonlinear module and the linear dynamic module in series form. It has a simple and representative structure, which made it become a popular research object in academia.

At present, the algorithms of the system identification mainly includes least squares algorithm, neural network algorithm and swarm intelligence algorithm. They have been studied a lot, for example, recursive least squares algorithm was used to identify the model parameters by Ding in [1], and neural networks was also used to achieve an effective identification by Linhares in [2]-[4]. Swarm intelligence algorithm specifically includes PSO, artificial colony algorithm, firefly algorithm and fruit flies algorithm et al. In [5], PSO is used for system identification, and in [6]-[7], PSO is modified to identify some specific systems. So in this paper we use recursive least squares (RLS) to identify the Hammerstein model. The algorithm has been used in many field, in [8] we know that recursive least squares has the advantages of tracking unknown parameters ability and it fast convergence’s ability, which attracts the attention of many scholars.

In conclusion, the recursive least squares was used to identify the Hammerstein model, we had been done some experiment and simulation about it, and analyze the result, finally, we prospect for future work.

II. HAMMERSTEIN NONLINEAR MODEL

Hammerstein model is a typical nonlinear system model, and it can be used to describe many industrial processes. The model consists of the memoryless nonlinear module and the linear dynamic module, the structure is shown in Fig. 1 in [9]:

In which the memoryless nonlinear module can be described as (1):

\[ x(k) = f_M(u(k)) \]  

Where u(k) is the input variable. And x(k) is an unknown intermediate variable, which does not necessarily have clear physical meaning. f(.) represents the memoryless nonlinear function of Hammerstein model.

The linear dynamic module can be described as (2):

\[ y(k) = G(z)x(k) + G_0(k) \]  

In Fig.1, the memoryless nonlinear function is written as (3):

\[ x(k) = f_M(u(k)) = \sum_{i=0}^{\rho} r_i u^i(k) \]  

Where \( r_i \) (\( i = 0, 1, \cdots, \rho \)) is the unknown coefficients. The transfer function of the linear dynamic system is as follows:

The transfer function of the linear dynamic system is as follows:

\[ G(z) = \frac{z^{-d}B(z^{-1})}{A(z^{-1})} \]  

The difference equations of \( A(z^{-1}) \) and \( B(z^{-1}) \) is respectively expressed as:

\[ A(z^{-1}) = 1 + a_1 z^{-1} + \cdots + a_{na} z^{-na} \]  

\[ B(z^{-1}) = b_0 + b_1 z^{-1} + \cdots + b_{nb} z^{-nb} \]  

In the above equations, \( z^{-1} \) means the lag operator, \( u(k) \) and \( y(k) \) represents the input and output of the
Besides, when it was off-online under the parameters and is the of original model and parameter model. constraints of output curve keep same between the output suitable nonlinear gain controller function stability theorem in [10], it can be concluded that Hammerstein model can be described as choosing a set of the system is stable.

Linear subsystem parameters matrix; factor as the input part of the linear part. gradually exponential decay then can be "forgotten", at that time can be highlight the role of the new data, and it effective. When the value of can overcome the RLS "data saturation" phenomenon with

A. Recursive Least Squares Algorithm

RLS is the basic parameter estimation method in system identification, in [11], and it has been used in many field. Compared with ordinary least squares algorithm, it has the advantages of needn’t a large matrix inversion, and it’s calculation very small, it also can make the optimization process converge faster and more smoothly to ensure a stable system convergence, and the algorithm can be used real-time online. So RLS algorithm is as follows:

\[
\hat{\theta}(t+1) = \hat{\theta}(t) + K_{e11}(y(t+1) - \Phi^T(t+1)\hat{\theta}(t))
\]

(7)

\[
K_{e11} = \frac{P\Phi(t+1)}{\lambda + \Phi^T(t+1)P\Phi(t+1)}
\]

(8)

\[
P_{e11} = \frac{P - K_{e11}\Phi^T(t+1)P}{\lambda}
\]

(9)

Where \(K_{e11}\) is the gain matrices; \(P_t\) is the covariance matrix; \(P_0 = 10^6 I\) is the initial array, which the is unit matrix, \(\beta\) generally take a large positive integer; \(\theta(0) = [0.001, 0.001, 0.001, 0.001]^T\); \(\lambda\) is the forgetting factor (0 < \(\lambda\) ≤ 1), it purpose was to make the old data gradually exponential decay then can be "forgotten", at that time can be highlight the role of the new data, and it can overcome the RLS “data saturation” phenomenon with effective. When the value of \(\lambda\) is smaller, the “forgotten” become more faster, but if the value is too small will make the precision become lower, and have a large fluctuation. So usually take the \(\lambda \in [0.8, 1]\), when it was off-online identification take \(\lambda = 1\).

B. Parameter Identification of The Hammerstein Model Based on RLS

Given the input \(u(k)\) and output \(y(k)\), the identification of Hammerstein model is uses statistical methods to build mathematical models of dynamical systems from measured data, which make the RLS meet the identification requirements between parameter model output and the actual model output under a specified target function criteria.

Objective function, also known as the fitness function, it can be described as (10):

\[
\max \frac{\hat{\theta}_{a11}(i) - \hat{\theta}_{i11}(i)}{\hat{\theta}_{a11}(i)} < \epsilon
\]

(10)

Where \(\hat{\theta}_{a11}(i)\) is the parameter vector’s element \(\theta\) after \(N + 1\) times recursive calculation results. \(\epsilon\) is the precision requirement of a certain parameter.

The steps of identification are as follows:

Step 1: Collect input and output data like the \(\{u(k), y(k)\}\);

Step 2: Take the initial value \(P(0) = 10^6 I\), \(\hat{\theta}(0) = 0\);

Step 3: By applying Eq.(7), Eq(8), Eq(9), and then calculate every values;

Step 4: Calculate the relative variation of the identification parameter;

Step 5: If the parameter convergence meet the requirements the loop terminates. Otherwise go to step 3.

The concrete flow chart is as follows:

![Flowchart](image)

Fig. 2. The flow chart of algorithm

IV. EXPERIMENTAL RESULTS AND ANALYSIS

In order to verify the effectiveness of the proposed algorithm in, the model is selected as (11):
\( A(q^{-1})y(k) = B(q^{-1})x(k) + \xi(k) \)
\( x(k) = u(k) + 0.5u^2(k) + 0.3u^3(k) \)
\( A(q^{-1})I = 1 + 1.5q^{-1} - 0.7q^{-2} \)
\( B(q^{-1}) = q^{-1} + 0.5q^{-2} \)

Where the input signal \( u(k) \) is a Gaussian White noise sequence and \( \xi(k) \) is a noise sequence. The mean of \( u(k) \) is 0 and the variance is 1, while the mean of \( \xi(k) \) is 0 and the variance is 0.1.

The parameters of RLS are set as follows: the length of the window \( L \) is 50, the initial matrix \( P(0) = 10^6 I, \hat{\theta}(0) = 0 \), which the \( I \) is unit matrix. Optimal parameter estimation of this model is conducted using RLS algorithm on MATLAB, and the algorithms is presented in Fig.3.

In Fig. 3, it can be seen that the output curve of the model proposed in this paper is in accordance with that of the original model. It shows an excellent identification effect, and the algorithm has the advantage of calculation very small, which the optimization process converge faster and more smoothly to ensure a stable system convergence, and it can be used real-time online. Experiments have been done to illustrate the effectiveness of the proposed method more sufficiently, identification of parameter estimation error are presented in Fig.4.

In Fig. 4, it can be seen that identify the model of deviation range based on RLS algorithm between in \([-0.8, 0.8]\), which means high identification precision, and the good effect of identification, Therefore, it is easily concluded that the identification precision of the proposed algorithm is good.

V. CONCLUSIONS AND FUTURE WORKS

In this paper, a Hammerstein model of single input and single output is identified by the introduction of RLS algorithm. From the experiment comparison between the parameter model outputs and the actual system outputs, it can be seen that the algorithm can not only increase the identification accuracy, but also it’s calculate speed very fast, and RLS can be used real-time online, so achieving ideal identification results. However, the algorithm is not strong enough under the circumstance of different signal-to-noise ratio, which needs to be further studied in the future.

REFERENCE


AUTHORS’ PROFILE

First A. Author: Jiangtao Zhai

Jiangtao Zhai received the B.E., M.E. and Ph.D. Degree from Nanjing University of Science and Technology in 2006, 2008 and 2013, respectively. Now, he is a lecturer in Jiangsu University of Science and Technology. He has contributed more than 15 refereed papers covering topics of network steganography.

First B. Author: Chengming Zhu

Chengming Zhu receive the B.E Degree from West Anhui University in 2015. Now he is a master student in Jiangsu University of Science and Technology.

Copyright © 2016 IJECC, All right reserved