

Denoising Gray Scale Images Using Curvelet Transform

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Abstract — Digital images are generally of low contrast and they often have a complex type of noise due to various imperfections in acquisition, transmission storage and display devices and also because of application of different types of quantization, reconstruction and enhancement algorithms. The presence of noise gives an image a mottled, grainy, textured or snowy appearance. Image noise comes from a variety of sources. The goal of image denoising is to estimate the original image from noisy image. A number of methods have been developed for denoising of images but most of the methods causes blurring of images and introduces artifacts. In this paper a multiscale curvelet based denoising technique is presented. Image denoising using curvelet transform produce better results than other standard methods used for denoising images. Experimental results show that the Curvelet based method outperforms the median filter and order filter for reducing the additive noise in grayscale images.

Key Words — curvelet transform, denoising, fast discrete curvelet transform, hard thresholding, unequipped curvelet transform.

I. INTRODUCTION

The need for efficient image restoration methods has grown with the massive production of digital images and movies of all kinds, often taken in poor conditions. No matter how good a camera is, an image improvement is always desirable to extend its range of action. Two main limitations in image accuracy are blur and noise. Blur is intrinsic to image acquisition systems, as digital images have finite number of samples and must satisfy the Shannon Nyquist sampling condition. The second main image perturbation is noise.

Image denoising is an important image processing task, both as a process itself, and as a component in other processes. Many ways to denoise an image or a set of data exist. The main properties of a good image denoising model are that it will remove noise while preserving edges. Hence, in recent years there has been a fair amount of research on non-linear noise removal techniques and prominent among them are the wavelet based denoising techniques. The disadvantage of Wavelet transform is many wavelet coefficients are needed to account for edges i.e. singularities along lines or curves which results into relatively high mean squared error (MSE). A new approach to image denoising is based on a recently introduced family of transforms e.g. curvelet transform which have been proposed as alternatives to wavelet representation of image data. Unlike wavelet, curvelet transforms accurately represent smooth functions using only a few nonzero coefficients. The aim of this paper is to analyze the importance of the newly developed multiscale representation system, namely, the curvelet transform in terms of its digital implementations i.e. transformation based on unequally spaced fast fourier transforms

(USFFT) for denoising a wide variety of gray scale images [1].

The curvelet transform was developed in last few years in an attempt to overcome inherent limitations of traditional multiscale representations such as wavelets. The curvelet transform is a multiscale pyramid with many directions and positions at each length scale, and needle-shaped element at fine scales. Curvelets have useful geometric features that set them apart from wavelets and the likes. For instance, curvelets obey a parabolic scaling relation which says that at scale s , each element has an envelope which is aligned along a ‘ridge’ of length s^2 and width s [2].

II. IMAGE DENOISING

Image denoising can be formally defined as removal of noise present in the image while preserving the important and sharp features of the image. In acquiring, transmitting or processing a digital image for example, the noise induced degradation may be dependent or independent of data which is shown in fig. 1, where noisy image includes the original image and independent identically distributed noise process (n) with variance σ^2 .

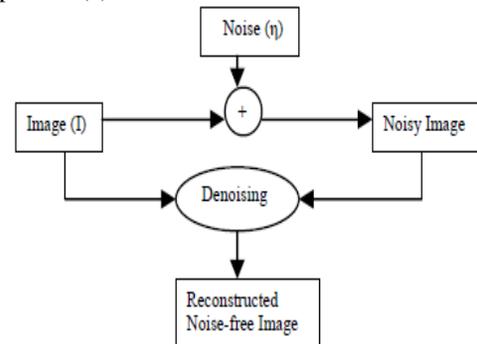


Fig.1 Block diagram of Image Denoising Process

The goal of image denoising is to find an estimate of noise free image based on the knowledge of noise [1]. A more precise explanation of the curvelet based denoising procedure can be given as follows. Curvelet transform is applied to a noisy image. The image I has an image function $u(x, y)$ as a union of modified copies of itself. The net result is that target u is approximated by the attractive fixed point of curvelet transform T that performs the thresholding operation on the image function. Image denoising is a technique of removing unwanted noise from images and retaining as much as possible the important signal features. Image denoising is needed to obtain clear and sharp images because noise leads to blur and unclear image and feature of interest may be misinterpreted in presence of noise signals. Data sets collected by image sensors are generally contaminated by noise. Imperfect instruments, problems with the data acquisition process, and interfering natural phenomena can all degrade the data

of interest. Furthermore, noise can be introduced by transmission errors and compression. Thus, denoising is often a necessary and the first step to be taken before the images data is analyzed. It is necessary to apply an efficient denoising technique to compensate for such data corruption [3].

III. INTRODUCTION TO CURVELETS

Curvelets are a non-adaptive technique for multi-scale object representation. Curvelets are an appropriate basis for representing images which are smooth apart from singularities along smooth curves, where the curves have bounded curvature, i.e. where objects in the image have a minimum length scale. This property holds for cartoons, geometrical diagrams, and text. As one zooms in on such images, the edges they contain appear increasingly straight. Curvelets take advantage of this property, by defining the higher resolution curvelets to be skinnier the lower resolution curvelets. However, natural images do not have this property; they have detail at every scale [4]. Therefore, for natural images, it is preferable to use some sort of directional wavelet transform whose wavelets have the same aspect ratio at every scale. Both forward and inverse curvelet transforms [5] are shown below.

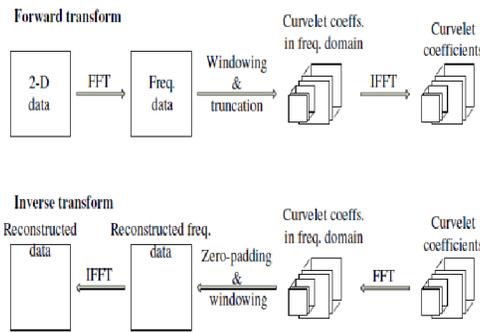


Fig. 2 Data flow structure of the FDCT forward and inverse transforms.

The Fast Discrete Curvelet Transform (FDCT) described in with source code (CurveLab package) is available for academic use at www.curvelet.org. This software package actually contains two discrete implementations of the curvelet transform. The first is called Digital Curvelet Transform via Unequispaced FFT (USFFT), and the second is called Digital Curvelet Transform via wrapping. The FDCT is referred to the wrapping FDCT because this is the version usually used in actual applications. The wrapping FDCT implementation is based on the FFT algorithm. The data flow diagram of the forward and inverse wrapping FDCT are plotted in Fig. 2. The data are first transformed into the frequency domain by forward FFT. The transformed data are then multiplied with a set of window functions. The shape of these windows is defined according to the requirements of the ideal curvelet transform, such as the parabolic scaling rule. The curvelet coefficients are obtained by inverse FFT from windowing data. Since the window functions are

zero except on support regions of elongated wedges, the regions that need to be transformed by the inverse FFT are much smaller than the original data. On the wrapping FDCT, the FFT coefficients on these regions are ‘wrapped’ or folded into rectangular shape before being applied to inverse FFT algorithm. The size of the rectangle is usually not an integer fraction of the size of original data. This process is equivalent to filtering and subsampling the curvelet subband by rational numbers in two dimensions [5].

IV. DENOISING ALGORITHM OF CURVELET TRANSFORM

First of all noise was added to the original image and then denoising algorithm was applied. Following steps are involved in the denoising algorithm of Curvelet Transform [6]:

1. Compute all thresholds for curvelets;
2. Compute norm of curvelets(using monte carlo method);
3. Apply curvelet transform to noisy image;
4. Apply hard thresholding to the curvelet coefficients; and
5. Apply inverse curvelet transform to the result of step 4.

Unequispaced function was used for denoising using curvelet transform. Hard thresholding is applied to the coefficients after decomposition. For the coarse scale elements a value of $3 \cdot \sigma$ is used and in case of fine scale elements a value of $4 \cdot \sigma$ is applied and coefficients which exceed the specified level of thresholding were discarded and the remaining coefficients were used to reconstruct the image using the inverse wrapping function [7]. From mathematical point of view USFFT solves the problem of replacing a singular function by a smooth function so that their Fourier transforms are almost the same within some frequency band [8].

Mean Square Error (MSE) which requires two $m \times n$ grey-scale images I and K where one of the images is considered as a noisy approximation of the other is defined as:

$$\text{MSE} = \frac{1}{m \times n} \sum_{i=1}^m \sum_{j=1}^n (I(i,j) - K(i,j))^2 \quad (1)$$

The PSNR is the most commonly used as a measure of quality of reconstruction in image denoising. The PSNR for both noisy and denoised images were identified using the following formulae:

$$\text{PSNR} = 10 \log_{10} \left(\frac{MAX_I^2}{\text{MSE}} \right) \quad (2)$$

Here, MAX_I is the maximum pixel value of the image. The PSNR of the images denoised is compared using wavelet and curvelet transform for each type of noise mentioned above. Then the mean and standard deviation of each noise was calculated [7].

V. RESULTS AND DISCUSSION

After applying curvelet transform denoising method to noisy images comparison of images is performed. The results were compared qualitatively (visually) and quantitatively (using quality metrics). Results from tables and visual results, it can be proved that values of curvelet methods for all quality metrics are better than the other methods. Table 1 and table 2 shows MSE and PSNR obtained by each method for different gray scale images with standard deviation as 20, 25 and 30. Curvelet method outperforms the median filter and order filter for reducing the noise in images. From table I, we can conclude that larger the MSE value larger is the noise present in the image. The curvelet method has the minimum MSE values so it has the least noise as compared to the other two methods namely the order filter and the median filter. From table II, we conclude that lesser is the noise if larger is the PSNR value, clearly the curvelet method has the maximum PSNR values so it removes the noise much better than the order filter and the median filter. So, Curvelet method outperforms the median filter and order filter for reducing the noise in images.



Fig. 3 Denoising of a Barbara image using curvelet transform, mean filter and order filter.

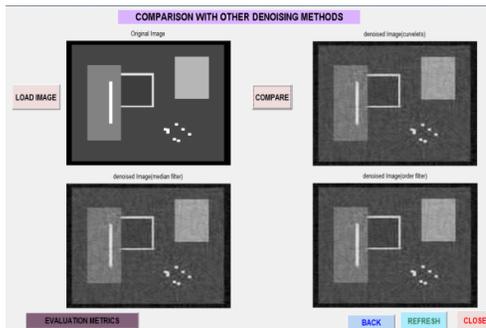


Fig. 4 Denoising of a Synth image using curvelet transform, mean filter and order filter.

Table I Comparison of MSE values with three methods for five different gray scale images and noise added in input image (standard deviation () = 20, 25 and 30).

Image	Standard deviation ()	MSE		
		Curvelet	Median Filter	Order Filter
barbara	20	43.65	56.49	1590.07
	25	52.96	80.13	2156.8
	30	71.40	109.34	2816.85
bld	20	33.02	48.29	1455.11
	25	40.93	72.03	2009.76
	30	56.83	100.11	2699.82
ray	20	40.51	58.03	1757.15
	25	50.09	83.14	2282.7
	30	68.74	108.72	2976.09
a	20	67.44	80.96	3051.62
	25	82.49	106.24	3665.72
	30	102.93	135.12	4384.41
Synth	20	29.35	50.65	1240.34
	25	37.13	73.22	1787.56
	30	54.58	101.08	2444.65

Table II Comparison of PSNR values with three methods for five different gray scale images and noise added in input image (standard deviation = 20, 25 and 30).

Image	Standard deviation ()	PSNR		
		Curvelet	Median filter	Order filter
barbara	20	26.56	25.04	15.47
	25	26.04	24.27	14.25
	30	24.86	23.56	13.25
Bld	20	29.65	28.04	15.72
	25	28.74	26.67	14.47
	30	26.54	25.27	13.33
ray	20	27.74	26.37	14.92
	25	26.98	25.26	13.88
	30	25.29	24.14	12.84
a	20	24.54	24.23	11.99
	25	24.06	23.51	11.39
	30	23.03	22.69	10.75
Synth	20	32.04	28.94	16.95
	25	30.62	27.28	15.30
	30	27.74	25.91	13.97

VI. CONCLUSION

The first objective was to study and implements various techniques for image denoising using curvelets was successfully implemented. It is efficient and simple to implement.

The second objective was to compare the implemented techniques on various metrics. The curvelet transform method was compared visually and using quality metrics PSNR and MSE. The results of the curvelet transform method were better than median filter and order filter and remove the noise to a significant amount.

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