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Approaches to Multiple Attribute Decision Making Based on the I-IVIFHCA Operator with Interval-Valued Intuitionistic Fuzzy Information

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Abstract **– In this paper, we first introduce some operations on the interval-valued intuitionistic fuzzy sets, such as Hamacher sum, Hamacher product, etc., and further develop the induced interval-valued intuitionistic fuzzy Hamacher correlated averaging (I-IVIFHCA) operator. The prominent characteristic of the operators is that they can not only consider the importance of the elements or their ordered positions, but also reflect the correlation among the elements or their ordered positions. We have applied the I-IVIFHCA operators to multiple attribute decision making with interval-valued intuitionistic fuzzy information.**

Keywords **– Interval-Valued Intuitionistic Fuzzy Numbers; Operational Laws; Induced Interval-Valued Intuitionistic Fuzzy Hamacher Correlated Averaging (I-IVIFHCA) Operator.**

INTRODUCTION

Atanassov [1,2] introduced the concept of intuitionistic fuzzy set(IFS) characterized by a membership function and a non-membership function, which is a generalization of the concept of fuzzy set [3] whose basic component is only a membership function. The intuitionistic fuzzy set has received more and more attention since its appearance. Later, Atanassov and Gargov[4-5] further introduced the interval-valued intuitionistic fuzzy set (IVIFS), which is a generalization of the IFS. The fundamental characteristic of the IVIFS is that the values of its membership function and non-membership function are intervals rather than exact numbers. Xu[6] proposed the interval-valued intuitionistic fuzzy weighted averaging (IVIFWA) operato, the interval-valued intuitionistic fuzzy ordered weighted averaging (IVIFOWA) operator and the interval-valued intuitionistic fuzzy hybrid aggregation (IVIFHA) operator and gave an application of the IVIFHA operator to multiple attribute group decision making with interval-valued intuitionistic fuzzy information. Wei and Zhao[7]investigated some multiple attribute group decision making (MAGDM) problems in which both the attribute weights and the expert weights are usually correlative, attribute values take the form of intuitionistic fuzzy values or interval-valued intuitionistic fuzzy values and developed the induced intuitionistic fuzzy correlated averaging (I-IFCA) operator and some desirable properties of the I-IFCA operators are studied, such as commutativity, idempotency and monotonicity. Yu et al.[8] proposed some interval-valued intuitionistic fuzzy aggregation operators such as the interval-valued intuitionistic fuzzy prioritized weighted average (IVIFPWA) operator.

Motivated by the induced Choquet ordered averaging (I-COA) operator[9], in this paper we propose the induced interval-valued intuitionistic fuzzy Hamacher correlated averaging operators based on the Hamacher operations, whose prominent characteristic is that they can not only consider the importance of the elements or their ordered positions, but also reflect the correlations of the elements or their ordered positions. To do so, the remainder of this paper is set out as follows. In the next section, we introduce some basic concepts related to interval-valued intuitionistic fuzzy sets and some operational laws of interval-valued intuitionistic fuzzy numbers. In Section 3 we have developed the induced interval-valued intuitionistic fuzzy Hamacher correlated averaging (I-IVIFHCA) operator. In Section 4, we have developed an approach to multiple attribute decision making based on I-IVIFHCA operator with interval-valued intuitionistic fuzzy information. In Section 5, we conclude the paper and give some remarks.

II. PRELIMINARIES

Atanassov and Gargov[4-5] further introduced the interval-valued intuitionistic fuzzy set (IVIFS), which is a generalization of the IFS. The fundamental characteristic of the IVIFS is that the values of its membership function and non-membership function are intervals rather than exact numbers.

Definition 1. [4-5] Let X be a universe of discourse, An IVIFS $\tilde{A}$ over X is an object having the form:

$$\tilde{A}=\left\{\left\langle x,\tilde{\mu}_A(x),\tilde{\nu}_A(x)\right\rangle \middle| x\in X\right\} \quad (1)$$

where $\tilde{\mu}_A(x)\subset[0,1]$ and $\tilde{\nu}_A(x)\subset[0,1]$ are interval numbers, and $0\le \sup(\tilde{\mu}_A(x))+\sup(\tilde{\nu}_A(x))\le 1$, $\forall x\in X$. For convenience, let $\tilde{\mu}_A(x)=[a,b]$, $\tilde{\nu}_A(x)=[c,d]$, so $\tilde{A}=([a,b],[c,d])$.

Definition 2. Let $\tilde{a}=([a,b],[c,d])$ be an interval-valued intuitionistic fuzzy number, a score function S of an interval-valued intuitionistic fuzzy value can be represented as follows [6]:

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$$S(\tilde{a})=\frac{a-c+b-d}{2},\quad S(\tilde{a})\in[-1,1]. \qquad (2)$$

Definition 3. Let $\tilde{a}=([a,b],[c,d])$ be an interval-valued intuitionistic fuzzy number, a accuracy function H of an interval-valued intuitionistic fuzzy value can be represented as follows [6]:

$$H(\tilde{a})=\frac{a+b+c+d}{2},\quad H(\tilde{a})\in[0,1]. \qquad (3)$$

to evaluate the degree of accuracy of the interval-valued intuitionistic fuzzy value $\tilde{a}=([a,b],[c,d])$, where $H(\tilde{a})\in[0,1]$. The larger the value of $H(\tilde{a})$, the more the degree of accuracy of the interval-valued intuitionistic fuzzy value $\tilde{a}$.

Based on the score function S and the accuracy function H, in the following, Xu[6] give an order relation between two interval-valued intuitionistic fuzzy values, which is defined as follows:

Definition 4. Let $\tilde{a}_1=([a_1,b_1],[c_1,d_1])$ and $\tilde{a}_2=([a_2,b_2],[c_2,d_2])$ be two interval-valued intuitionistic fuzzy values, $s(\tilde{a}_1)=\frac{a_1-c_1+b_1-d_1}{2}$ and $s(\tilde{a}_2)=\frac{a_2-c_2+b_2-d_2}{2}$ be the scores of $\tilde{a}$ and $\tilde{b}$, respectively, and let $H(\tilde{a}_1)=\frac{a_1+c_1+b_1+d_1}{2}$ and $H(\tilde{a}_2)=\frac{a_2+c_2+b_2+d_2}{2}$ be the accuracy degrees of $\tilde{a}$ and $\tilde{b}$, respectively, then if $S(\tilde{a})<S(\tilde{b})$, then $\tilde{a}$ is smaller than $\tilde{b}$, denoted by $\tilde{a}<\tilde{b}$; if $S(\tilde{a})=S(\tilde{b})$, then

(1) if $H(\tilde{a})=H(\tilde{b})$, then $\tilde{a}$ and $\tilde{b}$ represent the same information, denoted by $\tilde{a}=\tilde{b}$;

(2) if $H(\tilde{a})<H(\tilde{b})$, $\tilde{a}$ is smaller than $\tilde{b}$, denoted by $\tilde{a}<\tilde{b}$.

Liu[10] developed some interval-valued intuitionistic fuzzy Hamacher aggregation operators.

Definition 5. Let $\tilde{a}_j=([a_j,b_j],[c_j,d_j])$ $(j=1,2,\cdots,n)$ be a collection of interval-valued intuitionistic fuzzy values, and let IVIFHWA: $Q^n\to Q$, if

$$\mathrm{IVIFHWA}_{\omega}(\tilde{a}_1,\tilde{a}_2,\cdots,\tilde{a}_n)=\bigoplus_{j=1}^{n}(\omega_j\tilde{a}_j)$$
$$=\left(\left[\frac{\prod_{j=1}^{n}(1+(\gamma-1)a_j)^{\omega_j}-\prod_{j=1}^{n}(1-a_j)^{\omega_j}}{\prod_{j=1}^{n}(1+(\gamma-1)a_j)^{\omega_j}+(\gamma-1)\prod_{j=1}^{n}(1-a_j)^{\omega_j}},\ \frac{\prod_{j=1}^{n}(1+(\gamma-1)b_j)^{\omega_j}-\prod_{j=1}^{n}(1-b_j)^{\omega_j}}{\prod_{j=1}^{n}(1+(\gamma-1)b_j)^{\omega_j}+(\gamma-1)\prod_{j=1}^{n}(1-b_j)^{\omega_j}}\right],\right.$$
$$\left.\left[\frac{\gamma\prod_{j=1}^{n}c_j^{\omega_j}}{\prod_{j=1}^{n}(1+(\gamma-1)(1-c_j))^{\omega_j}+(\gamma-1)\prod_{j=1}^{n}c_j^{\omega_j}},\ \frac{\gamma\prod_{j=1}^{n}d_j^{\omega_j}}{\prod_{j=1}^{n}(1+(\gamma-1)(1-d_j))^{\omega_j}+(\gamma-1)\prod_{j=1}^{n}d_j^{\omega_j}}\right]\right) \qquad (4)$$

where $\omega=(\omega_1,\omega_2,\cdots,\omega_n)^T$ be the weight vector of $\tilde{a}_j\ (j=1,2,\cdots,n)$, and $\omega_j>0$, $\sum_{j=1}^{n}\omega_j=1$, then IVIFHWA is called the interval-valued intuitionistic fuzzy Hamacher weighted averaging (IVIFHWA) operator.

Definition 6[10]. Let $\tilde{a}_j=([a_j,b_j],[c_j,d_j])$ $(j=1,2,\cdots,n)$ be a collection of interval-valued intuitionistic fuzzy values. An interval-valued intuitionistic fuzzy Hamacher ordered weighted averaging (IVIFHOWA) operator of dimension n is a mapping IVIFHOWA: $Q^n\to Q$, that has an associated vector $w=(w_1,w_2,\cdots,w_n)^T$ such that $w_j>0$ and $\sum_{j=1}^{n}w_j=1$. Furthermore,

$$\begin{aligned}&\mathrm{IVIFHOWA}_w\left(\tilde{a}_1,\tilde{a}_2,\cdots,\tilde{a}_n\right)\\&=\overset{n}{\underset{j=1}{\oplus}}\left(w_j\tilde{a}_{\sigma(j)}\right)\\&=\left(\left[\frac{\prod_{j=1}^{n}\left(1+(\gamma-1)a_{\sigma(j)}\right)^{w_j}-\prod_{j=1}^{n}\left(1-a_{\sigma(j)}\right)^{w_j}}{\prod_{j=1}^{n}\left(1+(\gamma-1)a_{\sigma(j)}\right)^{w_j}+(\gamma-1)\prod_{j=1}^{n}\left(1-a_{\sigma(j)}\right)^{w_j}},\right.\right.\\&\qquad\left.\frac{\prod_{j=1}^{n}\left(1+(\gamma-1)b_{\sigma(j)}\right)^{w_j}-\prod_{j=1}^{n}\left(1-b_{\sigma(j)}\right)^{w_j}}{\prod_{j=1}^{n}\left(1+(\gamma-1)b_{\sigma(j)}\right)^{w_j}+(\gamma-1)\prod_{j=1}^{n}\left(1-b_{\sigma(j)}\right)^{w_j}}\right],\\&\qquad\left[\frac{\gamma\prod_{j=1}^{n}c_{\sigma(j)}{}^{w_j}}{\prod_{j=1}^{n}\left(1+(\gamma-1)\left(1-c_{\sigma(j)}\right)\right)^{w_j}+(\gamma-1)\prod_{j=1}^{n}c_{\sigma(j)}{}^{w_j}},\right.\\&\qquad\left.\left.\frac{\gamma\prod_{j=1}^{n}d_{\sigma(j)}{}^{w_j}}{\prod_{j=1}^{n}\left(1+(\gamma-1)\left(1-d_{\sigma(j)}\right)\right)^{w_j}+(\gamma-1)\prod_{j=1}^{n}d_{\sigma(j)}{}^{w_j}}\right]\right)\end{aligned}\tag{5}$$

where $\left(\sigma(1),\sigma(2),\cdots,\sigma(n)\right)$ is a permutation of $(1,2,\cdots,n)$, such that $\tilde{\alpha}_{\sigma(j-1)}\geq\tilde{\alpha}_{\sigma(j)}$ for all $j=2,\cdots,n$.

Definition 7[10]. An interval-valued intuitionistic fuzzy Hamacher hybrid averaging (IVIFHHA) operator of dimension n is a mapping IVIFHHA: $Q^n\rightarrow Q$, that has an associated vector $w=\left(w_1,w_2,\cdots,w_n\right)^T$ such that $w_j>0$ and $\sum_{j=1}^{n}w_j=1$. Furthermore,

$$\begin{aligned}&\mathrm{IVIFHHA}_{\omega,w}\left(\tilde{a}_1,\tilde{a}_2,\cdots,\tilde{a}_n\right)\\&=\overset{n}{\underset{j=1}{\oplus}}\left(w_j\dot{\tilde{a}}_{\sigma(j)}\right)\\&=\left(\left[\frac{\prod_{j=1}^{n}\left(1+(\gamma-1)\dot{a}_{\sigma(j)}\right)^{w_j}-\prod_{j=1}^{n}\left(1-\dot{a}_{\sigma(j)}\right)^{w_j}}{\prod_{j=1}^{n}\left(1+(\gamma-1)\dot{a}_{\sigma(j)}\right)^{w_j}+(\gamma-1)\prod_{j=1}^{n}\left(1-\dot{a}_{\sigma(j)}\right)^{w_j}},\right.\right.\\&\qquad\left.\frac{\prod_{j=1}^{n}\left(1+(\gamma-1)\dot{b}_{\sigma(j)}\right)^{w_j}-\prod_{j=1}^{n}\left(1-\dot{b}_{\sigma(j)}\right)^{w_j}}{\prod_{j=1}^{n}\left(1+(\gamma-1)\dot{b}_{\sigma(j)}\right)^{w_j}+(\gamma-1)\prod_{j=1}^{n}\left(1-\dot{b}_{\sigma(j)}\right)^{w_j}}\right],\\&\qquad\left[\frac{\gamma\prod_{j=1}^{n}\dot{c}_{\sigma(j)}{}^{w_j}}{\prod_{j=1}^{n}\left(1+(\gamma-1)\left(1-\dot{c}_{\sigma(j)}\right)\right)^{w_j}+(\gamma-1)\prod_{j=1}^{n}\dot{c}_{\sigma(j)}{}^{w_j}},\right.\\&\qquad\left.\left.\frac{\gamma\prod_{j=1}^{n}\dot{d}_{\sigma(j)}{}^{w_j}}{\prod_{j=1}^{n}\left(1+(\gamma-1)\left(1-\dot{d}_{\sigma(j)}\right)\right)^{w_j}+(\gamma-1)\prod_{j=1}^{n}\dot{d}_{\sigma(j)}{}^{w_j}}\right]\right)\end{aligned}\tag{6}$$

where $\dot{\tilde{a}}_{\sigma(j)}$ is the j th largest of the weighted interval-valued intuitionistic fuzzy values $\dot{\tilde{a}}_j\left(\dot{\tilde{a}}_j=n\omega_j\tilde{a}_j,\ j=1,2,\cdots,n\right)$, $\omega=\left(\omega_1,\omega_2,\cdots,\omega_n\right)^T$ be the weight vector of $\tilde{a}_j\left(j=1,2,\cdots,n\right)$, and $\omega_j>0$, $\sum_{j=1}^{n}\omega_j=1$, and n is the balancing coefficient.

IV. Induced Interval-Valued Intuitionistic Fuzzy Hamacher Correlated Averaging Operator

Definition 8 [9]. Let f be a positive real-valued function on X and m be a fuzzy measure on X. The induced Choquet ordered averaging operator (I-COA) of dimension n is a function I-COA$:\left(R^+\times R^+\right)\rightarrow R^+$, which is defined to aggregate the set of second argument of a list of 2-tuples $\left(\langle u_1,f_1\rangle,\langle u_2,f_2\rangle,\cdots,\langle u_n,f_n\rangle\right)$ according to the following expression:

$$\begin{aligned}&\text{I-COA}_m\left(\langle u_1,f_1\rangle,\langle u_2,f_2\rangle,\cdots,\langle u_n,f_n\rangle\right)\\&=\sum_{j=1}^{n}f_{\sigma(j)}\left[m\left(A_{\sigma(j)}\right)-m\left(A_{\sigma(j-1)}\right)\right]\end{aligned}\tag{7}$$

where $(\sigma(1),\sigma(2),\cdots,\sigma(n))$ is a permutation of $(1,2,\cdots,n)$, such that $u_{\sigma(j-1)}\geq u_{\sigma(j)}$ for all $j=2,\cdots,n$, i.e., $\langle u_{\sigma(j)},f_{\sigma(j)}\rangle$ is the 2-tuple with $u_{\sigma(j)}$ the j^{th} largest value in the set $(u_1,u_2,\cdots,u_n)$, $A_{\sigma(k)}=\left\{x_{\sigma(j)}\,|\,j\leq k\right\}$, for $k\geq1$, and $A_{\sigma(0)}=\phi$.

Thus, I-COA operator has application advantages when target elements has important correlations and ordered positions. Based on the basic aggregation principle for interval-valued intuitionistic fuzzy sets and induced Choquet ordered averaging (I-COA) operator [9], the following part of this paper explores the induced interval-valued intuitionistic fuzzy Hamacher correlated averaging (I-IVIFHCA) operator.

Based on the aggregation principle for IVIFs and induced Choquet ordered averaging (I-COA) operator, in

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the following, we shall develop the induced interval-valued intuitionistic fuzzy Hamacher correlated averaging (I-IVIFHCA) operator with interval-valued intuitionistic fuzzy information.

Definition 9. Let $\tilde{a}_j=\left(\left[a_j,b_j\right],\left[c_j,d_j\right]\right)$ $(j=1,2,\cdots,n)$ be a collection of interval-valued intuitionistic fuzzy values on X, and μ be a fuzzy measure on X, then we call

$$\text{I-IVIFHCA}\left(\langle u_1,\tilde{a}_1\rangle,\langle u_2,\tilde{a}_2\rangle,\cdots,\langle u_n,\tilde{a}_n\rangle\right)=\mathop{\oplus}_{j=1}^{n}\left(\left(\mu\left(A_{\sigma(j)}\right)-\mu\left(A_{\sigma(j-1)}\right)\right)\tilde{a}_{\sigma(j)}\right)\quad(8)$$

the induced interval-valued intuitionistic fuzzy Hamacher correlated averaging (I-IVIFHCA) operator, where $(\sigma(1),\sigma(2),\cdots,\sigma(n))$ is a permutation of $(1,2,\cdots,n)$, such that $\tilde{a}_{\sigma(j-1)}\geq\tilde{a}_{\sigma(j)}$ for all $j=2,\cdots,n$, $A_{\sigma(k)}=\left\{x_{\sigma(j)}\,|\,j\leq k\right\}$, for $k\geq 1$, and $A_{\sigma(0)}=\phi$.

With the operation of interval-valued intuitionistic fuzzy numbers, the I-IVIFHCA operator can be transformed into the following from by induction on n:

$$\begin{aligned}&\text{I-IVIFHCA}\left(\langle u_1,\tilde{a}_1\rangle,\langle u_2,\tilde{a}_2\rangle,\cdots,\langle u_n,\tilde{a}_n\rangle\right)\\&=\mathop{\oplus}_{j=1}^{n}\left(\left(\mu\left(A_{\sigma(j)}\right)-\mu\left(A_{\sigma(j-1)}\right)\right)\tilde{a}_{\sigma(j)}\right)\\&=\left(\left[\frac{\prod_{j=1}^{n}\left(1+(\gamma-1)a_{\sigma(j)}\right)^{\mu(A_{\sigma(j)})-\mu(A_{\sigma(j-1)})}-\prod_{j=1}^{n}\left(1-a_{\sigma(j)}\right)^{\mu(A_{\sigma(j)})-\mu(A_{\sigma(j-1)})}}{\prod_{j=1}^{n}\left(1+(\gamma-1)a_{\sigma(j)}\right)^{\mu(A_{\sigma(j)})-\mu(A_{\sigma(j-1)})}+(\gamma-1)\prod_{j=1}^{n}\left(1-a_{\sigma(j)}\right)^{\mu(A_{\sigma(j)})-\mu(A_{\sigma(j-1)})}},\right.\right.\\&\left.\frac{\prod_{j=1}^{n}\left(1+(\gamma-1)b_{\sigma(j)}\right)^{\mu(A_{\sigma(j)})-\mu(A_{\sigma(j-1)})}-\prod_{j=1}^{n}\left(1-b_{\sigma(j)}\right)^{\mu(A_{\sigma(j)})-\mu(A_{\sigma(j-1)})}}{\prod_{j=1}^{n}\left(1+(\gamma-1)b_{\sigma(j)}\right)^{\mu(A_{\sigma(j)})-\mu(A_{\sigma(j-1)})}+(\gamma-1)\prod_{j=1}^{n}\left(1-b_{\sigma(j)}\right)^{\mu(A_{\sigma(j)})-\mu(A_{\sigma(j-1)})}}\right],\\&\left[\frac{\gamma\prod_{j=1}^{n}c_{\sigma(j)}^{\mu(A_{\sigma(j)})-\mu(A_{\sigma(j-1)})}}{\prod_{j=1}^{n}\left(1+(\gamma-1)\left(1-c_{\sigma(j)}\right)\right)^{\mu(A_{\sigma(j)})-\mu(A_{\sigma(j-1)})}+(\gamma-1)\prod_{j=1}^{n}c_{\sigma(j)}^{\mu(A_{\sigma(j)})-\mu(A_{\sigma(j-1)})}},\right.\\&\left.\left.\frac{\gamma\prod_{j=1}^{n}d_{\sigma(j)}^{\mu(A_{\sigma(j)})-\mu(A_{\sigma(j-1)})}}{\prod_{j=1}^{n}\left(1+(\gamma-1)\left(1-d_{\sigma(j)}\right)\right)^{\mu(A_{\sigma(j)})-\mu(A_{\sigma(j-1)})}+(\gamma-1)\prod_{j=1}^{n}d_{\sigma(j)}^{\mu(A_{\sigma(j)})-\mu(A_{\sigma(j-1)})}}\right]\right)\end{aligned}\quad(9)$$

whose aggregated value is also a interval-valued intuitionistic fuzzy numbers.

It can be easily proved that the I-IVIFHCA operator has the following properties.

Theorem 1. (Idempotency) If all $\tilde{a}_j=\left(\left[a_j,b_j\right],\left[c_j,d_j\right]\right)$ are equal, i.e. $\tilde{a}_j=\tilde{a}$ for all j, then

$$\text{I-IVIFHCA}\left(\langle u_1,\tilde{a}_1\rangle,\langle u_2,\tilde{a}_2\rangle,\cdots,\langle u_n,\tilde{a}_n\rangle\right)=\tilde{a}\quad(10)$$

Theorem 2. (Boundedness) Let $\tilde{a}_j\,(j=1,2,\cdots,n)$ be a collection of IVIFVNs, and let

$\tilde{a}^-=\min_j\tilde{a}_j$, $\tilde{a}^+=\max_j\tilde{a}_j$

Then

$$\tilde{a}^-\leq\text{I-IVIFHCA}\left(\langle u_1,\tilde{a}_1\rangle,\langle u_2,\tilde{a}_2\rangle,\cdots,\langle u_n,\tilde{a}_n\rangle\right)\leq\tilde{a}^+\quad(11)$$

Theorem 3. (Monotonicity) Let $\tilde{a}_j\,(j=1,2,\cdots,n)$ and $\tilde{a}'_j\,(j=1,2,\cdots,n)$ be two set of IVIFVNs, if $\tilde{a}_j\leq\tilde{a}'_j$, for all j, then

$$\text{I-IVIFHCA}\left(\langle u_1,\tilde{a}_1\rangle,\langle u_2,\tilde{a}_2\rangle,\cdots,\langle u_n,\tilde{a}_n\rangle\right) \le \text{I-IVIFHCA}\left(\langle u_1',\tilde{a}_1'\rangle,\langle u_2',\tilde{a}_2'\rangle,\cdots,\langle u_n',\tilde{a}_n'\rangle\right) \tag{12}$$

Theorem 4. (Commutativity) Let $\tilde{a}_j\ (j=1,2,\cdots,n)$ and $\tilde{a}_j'\ (j=1,2,\cdots,n)$ be two set of IVIFVNs, then

$$\text{I-IVIFHCA}\left(\langle u_1,\tilde{a}_1\rangle,\langle u_2,\tilde{a}_2\rangle,\cdots,\langle u_n,\tilde{a}_n\rangle\right) = \text{I-IVIFHCA}\left(\langle u_1',\tilde{a}_1'\rangle,\langle u_2',\tilde{a}_2'\rangle,\cdots,\langle u_n',\tilde{a}_n'\rangle\right) \tag{13}$$

where $\tilde{a}_j'\ (j=1,2,\cdots,n)$ is any permutation of $\tilde{a}_j\ (j=1,2,\cdots,n)$..

IV. An Approach to Interval-Valued Intuitionistic Fuzzy Multiple Attribute Decision Making Problems

In this section, we shall investigate the multiple attribute decision making (MADM) problems based on the IVIFHCA operator in which attribute values take the form of interval-valued intuitionistic fuzzy numbers. Let $A=\{A_1,A_2,\cdots,A_m\}$ be a discrete set of alternatives, and $G=\{G_1,G_2,\cdots,G_n\}$ be the set of attributes. Suppose that $\tilde{R}=(\tilde{r}_{ij})_{m\times n}=\left([a_{ij},b_{ij}],[c_{ij},d_{ij}]\right)_{m\times n}$ is the interval-valued intuitionistic fuzzy decision matrix, where $[a_{ij},b_{ij}]$ indicates the degree that the alternative A_i satisfies the attribute G_j given by the decision maker, $[c_{ij},d_{ij}]$ indicates the degree that the alternative A_i doesn't satisfy the attribute G_j given by the decision maker, $[a_{ij},b_{ij}]\subset[0,1]$, $[c_{ij},d_{ij}]\subset[0,1]$, $b_{ij}+d_{ij}\le 1$, $i=1,2,\cdots,m$, $j=1,2,\cdots,n$.

In the following, we apply the I-IVIFHCA operator to multiple attribute decision making based on interval-valued intuitionistic fuzzy information. The method involves the following steps:

Step 1. Utilize the decision information given in matrix $\tilde{R}$, and the I-IVIFHCA operator

$$\begin{aligned}
\tilde{r}_i &= \left([a_i,b_i],[c_i,d_i]\right) \\
&= \text{I-IVIFHCA}\left(\langle u_{i1},\tilde{r}_{i1}\rangle,\langle u_{i2},\tilde{r}_{i2}\rangle,\cdots,\langle u_{in},\tilde{r}_{in}\rangle\right) \\
&= \bigoplus_{j=1}^{n}\left(\left(\mu\left(A_{\sigma(j)}\right)-\mu\left(A_{\sigma(j-1)}\right)\right)\tilde{r}_{\sigma(ij)}\right) \\
&= \left(\left[\frac{\prod_{j=1}^{n}\left(1+(\gamma-1)a_{\sigma(ij)}\right)^{\mu(A_{\sigma(j)})-\mu(A_{\sigma(j-1)})}-\prod_{j=1}^{n}\left(1-a_{\sigma(ij)}\right)^{\mu(A_{\sigma(j)})-\mu(A_{\sigma(j-1)})}}{\prod_{j=1}^{n}\left(1+(\gamma-1)a_{\sigma(ij)}\right)^{\mu(A_{\sigma(j)})-\mu(A_{\sigma(j-1)})}+(\gamma-1)\prod_{j=1}^{n}\left(1-a_{\sigma(ij)}\right)^{\mu(A_{\sigma(j)})-\mu(A_{\sigma(j-1)})}},\right.\right. \\
&\qquad \left.\frac{\prod_{j=1}^{n}\left(1+(\gamma-1)b_{\sigma(ij)}\right)^{\mu(A_{\sigma(j)})-\mu(A_{\sigma(j-1)})}-\prod_{j=1}^{n}\left(1-b_{\sigma(ij)}\right)^{\mu(A_{\sigma(j)})-\mu(A_{\sigma(j-1)})}}{\prod_{j=1}^{n}\left(1+(\gamma-1)b_{\sigma(ij)}\right)^{\mu(A_{\sigma(j)})-\mu(A_{\sigma(j-1)})}+(\gamma-1)\prod_{j=1}^{n}\left(1-b_{\sigma(ij)}\right)^{\mu(A_{\sigma(j)})-\mu(A_{\sigma(j-1)})}}\right], \\
&\qquad \left[\frac{\gamma\prod_{j=1}^{n}c_{\sigma(ij)}^{\mu(A_{\sigma(j)})-\mu(A_{\sigma(j-1)})}}{\prod_{j=1}^{n}\left(1+(\gamma-1)\left(1-c_{\sigma(ij)}\right)\right)^{\mu(A_{\sigma(j)})-\mu(A_{\sigma(j-1)})}+(\gamma-1)\prod_{j=1}^{n}c_{\sigma(ij)}^{\mu(A_{\sigma(j)})-\mu(A_{\sigma(j-1)})}},\right. \\
&\qquad \left.\left.\frac{\gamma\prod_{j=1}^{n}d_{\sigma(ij)}^{\mu(A_{\sigma(j)})-\mu(A_{\sigma(j-1)})}}{\prod_{j=1}^{n}\left(1+(\gamma-1)\left(1-d_{\sigma(ij)}\right)\right)^{\mu(A_{\sigma(j)})-\mu(A_{\sigma(j-1)})}+(\gamma-1)\prod_{j=1}^{n}d_{\sigma(ij)}^{\mu(A_{\sigma(j)})-\mu(A_{\sigma(j-1)})}}\right]\right), \quad i=1,2,\cdots,m.
\end{aligned} \tag{14}$$

to derive the collective overall preference values $\tilde{r}_i\left(i=1,2,\cdots,m\right)$ of the alternative A_i.

Step 2. Calculate the scores $S\left(\tilde{r}_i\right)\left(i=1,2,\cdots,m\right)$ of the collective overall values $\tilde{r}_i\left(i=1,2,\cdots,m\right)$ to rank all the alternatives $A_i\left(i=1,2,\cdots,m\right)$ and then to select the best one(s) (if there is no difference between two scores $S\left(\tilde{r}_i\right)$ and $S\left(\tilde{r}_j\right)$, then we need to calculate the accuracy degrees $H\left(\tilde{r}_i\right)$ and $H\left(\tilde{r}_j\right)$ of the collective overall preference values $\tilde{r}_i$ and $\tilde{r}_j$, respectively, and then rank the alternatives A_i and A_j in accordance with the accuracy degrees $H\left(\tilde{r}_i\right)$ and $H\left(\tilde{r}_j\right)$.

Step 3. Rank all the alternatives $A_i\left(i=1,2,\cdots,m\right)$ and select the best one(s) in accordance with $S\left(\tilde{r}_i\right)$ and $H\left(\tilde{r}_i\right)\left(i=1,2,\cdots,m\right)$.

Step 4. End.

III. Conclusion

In this paper, we first introduce some operations on the interval-valued intuitionistic fuzzy sets, such as Hamacher sum, Hamacher product, etc., and further develop the induced interval-valued intuitionistic fuzzy Hamacher correlated averaging (I-IVIFHCA) operator. The prominent characteristic of the operators is that they can not only consider the importance of the elements or their ordered positions, but also reflect the correlation among the elements or their ordered positions. We have applied the I-IVIFHCA operators to multiple attribute decision making with interval-valued intuitionistic fuzzy information.

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