

Model for Multiple Attribute Decision Making with Normal Distribution Interval Number Based on Choquet Integral

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Abstract – In this paper, we use the Choquet integral to propose the normal distribution interval number Choquet ordered averaging operator. The operator not only considers the importance of the elements, but also can reflect the correlations among the elements. It is worth pointing out that most of the existing normal distribution interval numbers averaging operators are special cases of our operator. Finally an illustrative example is given to use the operator in the range of uncertain multi-attribute decision-making. The results show that the method proposed in this paper is feasible.

Keywords – Choquet Integral; Normal Distribution Interval Number; The Normal Distribution Interval Number Choquet Ordered Averaging (NDINCOA) Operator.

I. INTRODUCTION

In multi-attribute decision making, the object things are complex, uncertain and Human thinking is ambiguous[1-17]. Therefore, interval numbers are usually more adequate or sufficient to model real-life decision problems than real numbers. Many methods have been proposed for dealing with the problem of multiple attribute decision making which the attribute values are given in terms of interval numbers[1-4], a majority of which are usually treated the attribute values as random variables or subjected to the random distribution. But, in other papers, they investigated the attribute values are subjected to the normal distribution, and they also had got some achievements [5-8]. However, the assumption is too strong to match decision behaviors in the real world. Under the condition that the overall values of alternatives in interval number evaluated are subjected to the normal distribution, Zhang and Fang [5] proposed the concept on the possibility for comparing two interval numbers, and then set up the according pair wise comparison matrix on alternatives. Liu, Chen and Ge [6] introduced that the interval number which obeys normal distribution is used in multi-attribute decision making. Wang and Xiao [7] defined some new aggregation operators, such as the NDINWAA operator, the NDINOWA operator and the NDINHA operator, and then developed an approach for solving multiple attribute group decision making based on normal distribution interval number with incomplete information. Wang and Yang [8] proposed the NDNWAA operator and the DNDNWAA operator and suggested a method for solving dynamic stochastic multiple attribute decision making with incomplete certain information.

In this paper, we propose the normal distribution interval number Choquet ordered averaging (NDINCOA) operator, whose prominent characteristic is that it can not only consider the importance of the elements or their ordered positions, but also reflect the correlations of the elements. Furthermore, a method based on the NDINCOA operator is presented, which can be used to develop a model to solve the multiple attribute decision making problem with correlative weights which the attribute values are given in terms of normal distribution interval numbers. Finally, the method is verified by an example.

II. PRELIMINARIES

Generally, we consider that the overall values in the interval number $[a, b]$ are subjected to the random distribution. This to say, the random variables have the same probability distribution to take a value in the interval number $[a, b]$. But, according to the central limit theorem, we can comprehend that attribute values $r_i (r_i \in [a, b]; i = 1, 2, \dots, n)$, given by experts, approach a fixed number. The values r_i are followed the normal distribution with a mean of μ and a standard deviation of σ , and the normal distribution with parameters μ and σ will be denoted $N(\mu, \sigma^2)$, where $\mu = \frac{1}{2}(a + b)$. For any normal distribution with parameters μ and σ , the interval $\mu \pm 3\sigma$ covers 99.73% of the distribution: $p(|r_i - \mu| \leq 3\sigma) \approx 0.9974$, and according to the 3σ principle of the normal distribution, then we have $\sigma = \frac{1}{6}(b - a)$.

Definition 1. If $\tilde{\alpha} = [a, b]$ is an interval number and subjects to the normal distribution $N(\mu, \sigma^2)$, the attribute value $r \in [a, b]$, then $[a, b]$ is called the normal distribution interval number, denoted by $\tilde{\beta} = \{\mu, \sigma\}$. According to the 3σ principle of the normal distribution, mean μ and standard deviation σ can be get as follows:

$$\mu = \frac{1}{2}(a+b) \quad (1)$$

$$\sigma = \frac{1}{6}(b-a) \quad (2)$$

Definition 2. Supposed that $\tilde{\beta}_1 = \{\mu_1, \sigma_1\}$, $\tilde{\beta}_2 = \{\mu_2, \sigma_2\}$ be two normal distribution interval numbers, then we have

$$(1) \tilde{\beta}_1 \oplus \tilde{\beta}_2 = \left\{ \mu_1 + \mu_2, \sqrt{\sigma_1^2 + \sigma_2^2} \right\};$$

$$(2) \lambda \tilde{\beta}_1 = \{\lambda \mu_1, \lambda \sigma_1\}, \lambda \geq 0.$$

It can be seen that the normal distribution interval numbers satisfied the following properties.

$$(1) \tilde{\beta}_1 \oplus \tilde{\beta}_2 = \tilde{\beta}_2 \oplus \tilde{\beta}_1;$$

$$(2) \lambda(\tilde{\beta}_1 \oplus \tilde{\beta}_2) = \lambda \tilde{\beta}_1 \oplus \lambda \tilde{\beta}_2;$$

$$(3) \lambda(\tilde{\beta}_1 \oplus \tilde{\beta}_2) = \lambda \tilde{\beta}_1 \oplus \lambda \tilde{\beta}_2.$$

Definition 3. If $\tilde{\beta}_1 = \{\mu_1, \sigma_1\}$, $\tilde{\beta}_2 = \{\mu_2, \sigma_2\}$ be two normal distribution interval numbers, then: (1) If $\mu_1 < \mu_2$, then we have $\tilde{\beta}_1 < \tilde{\beta}_2$; (2) If $\mu_1 > \mu_2$, then we have $\tilde{\beta}_1 > \tilde{\beta}_2$; (3) If $\mu_1 = \mu_2$, then ①if $\sigma_1 = \sigma_2$, then we have $\tilde{\beta}_1 = \tilde{\beta}_2$; ②if $\sigma_1 < \sigma_2$, then we have $\tilde{\beta}_1 > \tilde{\beta}_2$; ③if $\sigma_1 > \sigma_2$, then we have $\tilde{\beta}_1 < \tilde{\beta}_2$;

In multiple attribute decision making, the elements in the normal distribution interval numbers have some correlations with each other, and thus it is necessary to consider this issue.

Let $m(x_i) (i=1, 2, \dots, n)$ be the weight of the element $x_i \in X (i=1, 2, \dots, n)$, where m is a fuzzy measure, defined as follows:

Definition 4. A fuzzy measure m on the set X is a set function $m: \theta(x) \rightarrow [0, 1]$ satisfying the following axioms:

$$(C) \int \tilde{\beta} dm = NDINCOA(\tilde{\beta}_1, \tilde{\beta}_2, \dots, \tilde{\beta}_n) = \left\{ \sum_{j=1}^n (m(B_{\tau(j)}) - m(B_{\tau(j-1)})) \mu_{\tau(j)}, \sqrt{\sum_{j=1}^n (m(B_{\tau(j)}) - m(B_{\tau(j-1)}))^2 \sigma_{\tau(j)}^2} \right\} \quad (4)$$

Whose aggregated value is also a normal distribution interval number.

Especially, if $m(\{x_{\tau(j)}\}) = m(B_{\tau(j)}) - m(B_{\tau(j-1)})$, $j=1, 2, \dots, n$, then NDINCOA operator reduces to

NDINWAA operator. If $m(B) = \sum_{j=1}^{|B|} \omega_j$, for all $B \subseteq X$,

$$(1) m(\phi) = 0, m(X) = 1;$$

$$(2) A \subseteq B \text{ implies } m(A) \leq m(B), \text{ for all } A, B \subseteq X;$$

$$(3) m(A \cup B) = m(A) + m(B) + \rho m(A)m(B), \text{ for all } A, B \subseteq X \text{ and } A \cap B = \phi, \text{ where}$$

$$\rho \in (-1, \infty).$$

Especially, if $\rho = 0$, then the condition (3) reduces to the axiom of additive measure:

$$m(A \cup B) = m(A) + m(B), \text{ for all } A, B \subseteq X \text{ and } A \cap B = \phi. \text{ If all the elements in } X \text{ are independent,}$$

$$\text{and we have } m(A) = \sum_{x_i \in A} m(\{x_i\}), \text{ for all } A \subseteq X.$$

Based on Definition 4, in what follows we use the well-known Choquet integral [9] to propose an operator for aggregating the normal distribution interval numbers with correlative weights:

Definition 5. Let m be a fuzzy measure on X , and $\tilde{\beta}(x_j) = \{\mu(x_j), \sigma(x_j)\} (j=1, 2, \dots, n)$

be n normal distribution interval numbers, then we call

$$(C) \int \tilde{\beta} dm = NDINCOA(\tilde{\beta}(x_1), \tilde{\beta}(x_2), \dots, \tilde{\beta}(x_n)) = \bigoplus_{j=1}^n (m(B_{\tau(j)}) - m(B_{\tau(j-1)})) \tilde{\beta}_{\tau(j)} \quad (3)$$

the normal distribution interval number Choquet ordered averaging (NDINCOA) operator, where

$(C) \int \tilde{\beta} dm$ denotes the Choquet integral,

$(\tau(1), \tau(2), \dots, \tau(n))$ is a permutation

of $(1, 2, \dots, n)$ such that $\tilde{\beta}(x_{\tau(1)}) \geq \tilde{\beta}(x_{\tau(2)}) \geq$

$\dots \geq \tilde{\beta}(x_{\tau(n)})$, $B_{\tau(k)} = \{x_{\tau(j)} | j \leq k\}, k \geq 1$ and

$$B_{\tau(0)} = \phi.$$

The NDINCOA operator (3) can be transformed into the following form by induction on n :

where $|B|$ is the number of the elements in the set B ,

then $\omega_j = m(B_{\tau(j)}) - m(B_{\tau(j-1)})$, $j=1, 2, \dots, n$, where

$$\omega = (\omega_1, \omega_2, \dots, \omega_n)^T, \omega_j \geq 0, j=1, 2, \dots, n,$$

and $\sum_{j=1}^n \omega_j = 1$. Then, NDINCOA operator reduces to

NDINOWA operator.

III. THE MULTIPLE ATTRIBUTE DECISION MAKING BASED ON THE NDINCOA OPERATOR

In this section, we shall develop an approach to the multiple attribute decision making with correlative weights which the attribute values are given in terms of normal distribution interval numbers as follows. Let $A = \{a_1, a_2, \dots, a_m\}$ be a discrete set of alternatives, and $X = \{x_1, x_2, \dots, x_n\}$ be the set of attributes; m be a fuzzy measure on X , where $0 \leq m(\{x_1, \dots, x_j\}) \leq 1, m(\{X\}) = 1, m(\{\phi\}) = 0$.

Supposed that $\tilde{\alpha}_{a_i}(x_j) = [a_{a_i}(x_j), b_{a_i}(x_j)]$ be the attribute value in the attribute set x_j with respect to the alternative a_i which given by experts.

Step 1. We use the method which is proposed by Xu [10] to standardize interval number $\tilde{\alpha}_{a_i}(x_j)$, and then convert interval number $\tilde{\alpha}_{a_i}(x_j)$ into normal distribution interval number $\tilde{\beta}_{a_i}(x_j)$ with the equation(1) and equation(2), where $\tilde{\beta}_{a_i}(x_j) = \{\mu_{a_i}(x_j), \sigma_{a_i}(x_j)\}$, we would get a normal distribution interval number decision making

matrix $\tilde{R} = (\tilde{\beta}_{a_i}(x_j))_{m \times n}, i = 1, 2, \dots, m, j = 1, 2, \dots, n$.

And then, we rearrange the normal distribution interval numbers corresponding to each project in descending order by using the method presented in Definition 3.

Step 2. With the NDINCOA operator, we calculate the overall evaluation information corresponding to each project.

$$z_i = NDINCOA(\tilde{\beta}_{a_i}(x_1), \tilde{\beta}_{a_i}(x_2), \dots, \tilde{\beta}_{a_i}(x_n)) \\ = \bigoplus_{j=1}^n (m(B_{\tau(j)}) - m(B_{\tau(j-1)})) \tilde{\beta}_{a_i}(x_{\tau(j)}), i = 1, 2, \dots, m \quad (5)$$

Step 3. We rank the above the normal distribution interval numbers by using the method presented in Definition 3:

Step 4. The ranking of the alternatives can be gained and the best one can be find out.

IV. NUMERICAL EXAMPLE

Let us suppose there is an investment company, which wants to invest a sum of money to eight possible projects. The investment company must take a decision according to the following three attributes: x_1 is the risk analysis; x_2 is the growth analysis; x_3 is the environmental impact analysis. The eight possible alternatives are to be evaluated using the normal distribution interval numbers by the experts under the attributes, as shown in the following table.

Table 1: The attribute values of each attribute with respect to eight enterprises given by experts.

projects	Attribute (x_1)	Attribute (x_2)	Attribute (x_3)
a_1	{75.5, 0.7}	{85.3, 0.8}	{86.5, 0.9}
a_2	{82.5, 0.6}	{79.5, 0.5}	{83.5, 0.7}
a_3	{78, 0.3}	{82.5, 0.2}	{81.5, 0.8}
a_4	{89.5, 0.8}	{83.5, 0.6}	{80.5, 0.5}
a_5	{82.5, 0.6}	{85, 0.3}	{89, 0.2}
a_6	{81.5, 0.5}	{86, 0.4}	{90, 0.8}
a_7	{83, 0.4}	{84.5, 0.2}	{88.5, 0.3}
a_8	{82, 0.2}	{85.5, 0.3}	{89.5, 0.5}

The experts evaluate the projects $a_i (i = 1, 2, 3, \dots, 8)$ in relation to the attributes $x_j (j = 1, 2, 3)$, and give more importance to x_1 and x_2 than to x_3 , but, on the other hand, the experts give some advantage to the enterprises that are good both in x_3 and in either of x_1 and x_2 . Let

$$m(\phi) = 0, m(X) = m(\{x_1, x_2, x_3\}) = 1 \\ m(\{x_3\}) = 0.3, m(\{x_1\}) = m(\{x_2\}) = 0.4 \\ m(\{x_1, x_2\}) = 0.6, m(\{x_1, x_3\}) = m(\{x_2, x_3\}) = 0.8$$

Step 1. We rearrange the normal distribution interval numbers corresponding to each project in descending order, by using the method presented in Definition 3.

$$\begin{aligned} \tilde{\beta}_{a_1}(x_{\tau(1)}) &= \{86.5, 0.9\}, \tilde{\beta}_{a_1}(x_{\tau(2)}) = \{85.3, 0.8\} \\ \tilde{\beta}_{a_1}(x_{\tau(3)}) &= \{75.5, 0.7\}, \tilde{\beta}_{a_2}(x_{\tau(1)}) = \{83.5, 0.7\} \\ \tilde{\beta}_{a_2}(x_{\tau(2)}) &= \{82.5, 0.6\}, \tilde{\beta}_{a_2}(x_{\tau(3)}) = \{79.5, 0.5\} \\ \tilde{\beta}_{a_3}(x_{\tau(1)}) &= \{82.5, 0.2\}, \tilde{\beta}_{a_3}(x_{\tau(2)}) = \{81.5, 0.8\} \\ \tilde{\beta}_{a_3}(x_{\tau(3)}) &= \{78.0, 0.3\}, \tilde{\beta}_{a_4}(x_{\tau(1)}) = \{89.5, 0.8\} \\ \tilde{\beta}_{a_4}(x_{\tau(2)}) &= \{83.5, 0.6\}, \tilde{\beta}_{a_4}(x_{\tau(3)}) = \{80.5, 0.5\} \\ \tilde{\beta}_{a_5}(x_{\tau(1)}) &= \{89.0, 0.2\}, \tilde{\beta}_{a_5}(x_{\tau(2)}) = \{85.0, 0.3\} \\ \tilde{\beta}_{a_5}(x_{\tau(3)}) &= \{82.5, 0.6\}, \tilde{\beta}_{a_6}(x_{\tau(1)}) = \{90.0, 0.8\} \\ \tilde{\beta}_{a_6}(x_{\tau(2)}) &= \{86.0, 0.4\}, \tilde{\beta}_{a_6}(x_{\tau(3)}) = \{81.5, 0.5\} \\ \tilde{\beta}_{a_7}(x_{\tau(1)}) &= \{88.5, 0.3\}, \tilde{\beta}_{a_7}(x_{\tau(2)}) = \{84.5, 0.2\} \\ \tilde{\beta}_{a_7}(x_{\tau(3)}) &= \{83.0, 0.4\}, \tilde{\beta}_{a_8}(x_{\tau(1)}) = \{89.5, 0.5\} \\ \tilde{\beta}_{a_8}(x_{\tau(2)}) &= \{85.5, 0.3\}, \tilde{\beta}_{a_8}(x_{\tau(3)}) = \{82.0, 0.2\} \end{aligned}$$

Step 2. With the NDINCOA operator (5), we calculate the overall evaluation information corresponding to each project.

$$\begin{aligned} z_1 &= NDINCOA(\tilde{\beta}_{a_1}(x_1), \tilde{\beta}_{a_1}(x_2), \tilde{\beta}_{a_1}(x_3)) = \{83.7, 0.5025\}; \\ z_2 &= NDINCOA(\tilde{\beta}_{a_2}(x_1), \tilde{\beta}_{a_2}(x_2), \tilde{\beta}_{a_2}(x_3)) = \{82.2, 0.3796\}; \\ z_3 &= NDINCOA(\tilde{\beta}_{a_3}(x_1), \tilde{\beta}_{a_3}(x_2), \tilde{\beta}_{a_3}(x_3)) = \{81.2, 0.3353\}; \\ z_4 &= NDINCOA(\tilde{\beta}_{a_4}(x_1), \tilde{\beta}_{a_4}(x_2), \tilde{\beta}_{a_4}(x_3)) = \{84.7, 0.3960\}; \\ z_5 &= NDINCOA(\tilde{\beta}_{a_5}(x_1), \tilde{\beta}_{a_5}(x_2), \tilde{\beta}_{a_5}(x_3)) = \{85.7, 0.2012\}; \\ z_6 &= NDINCOA(\tilde{\beta}_{a_6}(x_1), \tilde{\beta}_{a_6}(x_2), \tilde{\beta}_{a_6}(x_3)) = \{86.3, 0.3280\}; \\ z_7 &= NDINCOA(\tilde{\beta}_{a_7}(x_1), \tilde{\beta}_{a_7}(x_2), \tilde{\beta}_{a_7}(x_3)) = \{85.4, 0.1565\}; \\ z_8 &= NDINCOA(\tilde{\beta}_{a_8}(x_1), \tilde{\beta}_{a_8}(x_2), \tilde{\beta}_{a_8}(x_3)) = \{86, 0.21587\}. \end{aligned}$$

Step 3. We rank the above the normal distribution interval numbers by using the method presented in Definition 3:

$$Q(z_1) = 0.9574; Q(z_2) = 1.1538; Q(z_3) = 1.5596; \\ Q(z_4) = 1.0123; Q(z_5) = 1.7323; Q(z_6) = 1.1726.$$

Step 4. The ranking of the alternatives can be gained:

$$z_6 \succ z_8 \succ z_5 \succ z_7 \succ z_4 \succ z_1 \succ \\ z_2 \succ z_3, a_6 \text{ is the best one.}$$

V. CONCLUSION

In this paper, we have developed the normal distribution interval number Choquet ordered averaging (NDINCOA) operator, which is used to discuss the correlative normal distribution interval numbers. Furthermore, we have developed an approach to multiple attribute decision making with correlative weights which the attribute values are given in terms of normal distribution interval numbers based on the NDINCOA operator. Finally, an illustrative example is given.

ACKNOWLEDGEMENTS

This research was supported by the Humanities and Social Sciences Foundation of Ministry of Education of the People's Republic of China under Grant No. 14XJCZH002, and the construction plan of scientific research innovation team for colleges and universities in Sichuan Province (15TD0004).

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