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Mathematical Model of Stability Analysis of a Disturb System

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***Abstract* – This model deals with the stability of a system of a fluid flow through a horizontal channel with porous walls subject to a disturbance.**

The system is formulated through partial differential equations and investigated through a mathematical methods and matrices to separate the governing partial differential equations into two parts steady state and disturb partial differential equations.

After a suitable mathematical manipulation, the neutral line between the stable and unstable regions is obtained which include the relation between the physical numbers. It was found that

If $\frac{8X_1^2+Y_1^2}{8X_1\sqrt{X_1}} \ldots. -D_2 > 0$ **, The system is unstable.**

$\frac{8X_1^2+Y_1^2}{8X_1\sqrt{X_1}} \ldots. -D < 0$ **, The system is stable.**

Also:

If $-\frac{8X_1^2+Y_1^2}{8X_1\sqrt{X_1}} \ldots. -D_2 > 0$ **, The system is unstable.**

$-\frac{8X_1^2+Y_1^2}{8X_1\sqrt{X_1}} \ldots. -D_2 < 0$ **, The system is stable.**

These are represented the separation between the regions of stable and unstable fluid.

***Keyword* – Modeling, Fluid, Disturbance, Stability, Unstable, Partial Differential Equations.**

I. Introduction

Any system whatsoever is bound to be disturbed. A basic question therefore is, "Will the disturbance gradually die down or will it grow in amplitude in such a way that the system does not return to its original state?" In the field of fluid mechanics the broad study of stability is concerned, in part, with the determination of the critical values, if any, of the flow parameters which distinguish the two different regimes associated with the answer to the above question.

In this research project a mathematical model of a heat and a dissipative fluid flowing in horizontal channel with porous walls are considered. This subject has an important application in engineering and science in general and specially in astrophysical and aeronautical problems in addition to geophysical applications. Most of such applications systems are subject to be disturbed and then the need is required to analysis that case for stability or otherwise, as it will follow in the next chapters. This project is concerned with the stability analysis of the whole system.

Stability analysis has been recently studies by many authors and it is of great interest because of the growing industrial applications importance. The analysis of stability of a disturb system seems to be an active field of research, Davey[1],Stuart [2],Look[3] and Sagalakov [4] discussed the effect of magnetic field upon stability of corresponding MHD flow. It was found that the magnetic field has a powerful stabilizing influence on the disturb system.

More recently Mosa[5] analysis the stability of heat transfer through an MHD porous media and found the parameters critical values separating the two regimes of the flow.

Mosa and Ibrahim[6] analysis the stability of a model of laminar flow with natural convection heat transfer from a thin vertical cylinder and found that the large value of the radius from the center increase tendency toward the neutral stability curve.

Also Mosa & Ali [7] presented a model of a stability analysis for fluid flow between two infinite parallel plates and showed that the disturbance with large wave number grows faster than that with smaller wave number. The effect of optically thick limit and buoyancy forces on the stability of MHD ekman layer was investigated, Bragdi & Mosa [8], and showed the stable influences on the system.

Elena Tamaševi ̌ci ¯ ut˙e, Ar ¯unasTamaševi ̌cius[9], found that the stabilization is achieved with vanishing small perturbations. The feedback does not change the system, but changes its stability properties.

D. Gonze& M. Kaufman[10], Discussed the stability of linear stability in their article the theory of non-linear dynamical systems and found that " the behavior of the system around the steady state and resort to the linearized stability principle. They describe this method first for a one-variable and subsequently for a two-variable system and showed that, this theory can be generalized to an N-variable system".

II. Methodology

1-The mathematical model, governing differential equations, boundary conditions, non-dimensional forms of the governing differential equations and boundary conditions are presented.

2- The primary velocity, temperature and concentration (The functions in the model) are considered to have superimposed on them two – dimensional infinitesimal disturbances.

3- Steady state quantities are indicated with suffix 1 whilst perturbations are denoted by suffix 2, such that

$F(x,y,t) = F_1(y) + F_2(x,y,t)$, where x,y are the dimensional spatial coordinates and t is the time.

4- Substitutions of the form in 3 above in the non-separate to two systems one for pure steady state and the other is for the disturbance with the relative boundary conditions.

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5- Solve the steady state system and then the disturb system through the form $F_2(x, y, t) = Ae^{\alpha t + i(k_1 x + k_2 y)}$. The assumed form of the disturbance imply a spatial periodic waves where $\alpha = \alpha_1 + i\alpha_2$ is the wave speed which is a complex number. A positive or negative α_1 implies growth or decay of the disturbance. The k_1 and k_2 are non-dimensional wave numbers in the x and y directions, respectively and are therefore a real quantities, A is a constant.

6- This study is concentrated on the change of sign of α_1 to separate the different regimes of the fluid behavior.

III. The Model and Governing Differential Equations

Consider the fully developed laminar flow of fluid between two horizontal parallel porous walls, distance h apart, and choose the coordinate system such that the x-axis is parallel to the channel and along the direction of the flow. The y-axis is taken as the vertical coordinate measured position upwards whilst the z-axis is in the direction mutually orthogonal to the other two axes. The governing equations are well established; see for example Jassim and Ahmed [11], such that

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad \ldots (2.2.1)$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \vartheta\nabla^2 v + g\beta(T - T_1) + g\beta^*(C - C_1) - \frac{\vartheta}{k}u \quad \ldots (2\text{-}2.2)$$

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\nabla^2 T + \frac{\mu}{Pc_p}[\left(\frac{\partial u}{\partial y}\right)^2] \quad \ldots (2.2.3)$$

$$\frac{\partial C}{\partial t} + u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D\nabla^2 C \quad \ldots (2.2.4)$$

Where u and v are the velocity component, t is the time and $T, g, \beta, \beta^*, \vartheta, k, \alpha, \mu, P, c_P, D$ are the temperature, gravitational acceleration, thermal expansion coefficient, concentration diffusivity, dynamical viscosity, density, specific heat at constant pressure , mass with the following boundary conditions,

$$\left.\begin{array}{l} u = v = 0.0 \\ T = T_0, T_1 \\ C = C_0, C_1 \end{array}\right\} \quad y = 0, h \quad \ldots (2.2.5)$$

and h is the width of the channel.

Non-dimensional form:

To solve governing equation (2.2.1)-(2.2.4) with the boundary conditions (2.2.5), we need to introduce the following non-dimensional quantities [4, 3],

$$\left.\begin{array}{l} X = \frac{x}{h},\ Y = \frac{yGr^{1/4}}{h},\ U = \frac{uhGr^{-1/2}}{\vartheta},\ V = \frac{vhGr^{-1/4}}{\vartheta} \\ t = \frac{t\vartheta Gr^{1/2}}{h^2},\ pr = \frac{\vartheta}{\alpha},\ \theta = \frac{T - T_0}{T_0 - T_1},\ C = \frac{C - C_1}{C_0 - C_1} \\ Gr = \frac{g\beta h^3(T_0 - T_1)}{\vartheta^2},\ Gr^* = \frac{g\beta^* h^3(C_0 - C_1)}{\vartheta^2} \end{array}\right\} \quad \ldots (3.1)$$

Substituting these quantities onto equations (2.2.1)-(2.2.4), the governing equations becomes.

$$\left[\frac{\sqrt{Gr}\vartheta}{h^2}\right]\frac{\partial U}{\partial X} + \left[\frac{\sqrt{Gr}\vartheta}{h^2}\right]\frac{\partial V}{\partial Y} = 0 \quad \ldots (3.1a)$$

$$\left[\frac{Gr\vartheta^2}{h^3}\right]\frac{\partial U}{\partial t} + \left[\frac{Gr\vartheta^2}{h^3}\right]U\frac{\partial U}{\partial X} + \left[\frac{Gr\vartheta^2}{h^3}\right]V\frac{\partial U}{\partial Y} = \left[\frac{\sqrt{Gr}\vartheta^2}{h^3}\right]\frac{\partial^2 U}{\partial X^2} + \left[\frac{Gr\vartheta^2}{h^3}\right]\frac{\partial^2 U}{\partial Y^2} + g\beta(T - T_\infty) + g\beta^*(C - C_\infty) - \frac{\sqrt{Gr}\vartheta^2}{Kh}U \quad \ldots (3.1b)$$

$$\left[\frac{(T_0 - T_1)\sqrt{Gr}\vartheta}{h^2}\right]\frac{\partial U}{\partial t} + \left[\frac{(T_0 - T_1)\sqrt{Gr}\vartheta}{h^2}\right]U\frac{\partial\theta}{\partial X} + \left[\frac{(T_0 - T_1)\sqrt{Gr}\vartheta}{h^2}\right]V\frac{\partial\theta}{\partial Y} = \left[\frac{\alpha(T_0 - T_1)}{h^2}\right]\frac{\partial^2 U}{\partial X^2} + \left[\frac{\alpha(T_0 - T_1)\sqrt{Gr}\vartheta}{h^2}\right]\frac{\partial^2\theta}{\partial Y^2} + \frac{\mu}{pc_p}\left[\frac{(Gr\sqrt{Gr}\vartheta^2}{h^4}\right]\left(\frac{\partial U}{\partial t}\right)^2 \quad \ldots (3.1c)$$

$$\left[\frac{(C_0 - C_1)\sqrt{Gr}\vartheta}{h^2}\right]\frac{\partial\emptyset}{\partial t} + \left[\frac{(C_0 - C_1)\sqrt{Gr}\vartheta}{h^2}\right]U\frac{\partial\emptyset}{\partial X} + \left[\frac{(C_0 - C_1)\sqrt{Gr}\vartheta}{h^2}\right]V\frac{\partial\emptyset}{\partial Y} = D\left[\left(\frac{(C_0 - C_1)}{h^2}\right)\frac{\partial^2\emptyset}{\partial X^2} + \left(\frac{(C_0 - C_1)\sqrt{Gr}}{h^2}\right)\frac{\partial^2\emptyset}{\partial XY^2}\right] \quad \ldots (3.1d)$$

Simplifying the above equations, the governing equations under these non-dimensional quantities becomes.

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \quad \ldots (3.2)$$

$$\frac{\partial U}{\partial t} + U\frac{\partial U}{\partial X} + V\frac{\partial U}{\partial Y} = \frac{1}{\sqrt{Gr}}\frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2} + \theta + \frac{Gr^*}{Gr}\emptyset - \frac{1}{\sqrt{Gr}Da}U \quad \ldots (3.3)$$

$$\frac{\partial\theta}{\partial t} + U\frac{\partial\theta}{\partial X} + V\frac{\partial\theta}{\partial Y} = \frac{1}{\sqrt{Gr}pr}\frac{\partial^2 U}{\partial X^2} + \frac{1}{pr}\frac{\partial^2\theta}{\partial Y^2} + \varepsilon\left[\left(\frac{\partial U}{\partial Y}\right)^2\right] \ldots (3.4)$$

$$\frac{\partial\emptyset}{\partial t} + U\frac{\partial\emptyset}{\partial X} + V\frac{\partial\emptyset}{\partial t} = \frac{1}{Sc}\frac{1}{\sqrt{Gr}}\frac{\partial^2\emptyset}{\partial X^2} + \frac{\partial^2\emptyset}{\partial Y^2} \quad \ldots (3.5)$$

Where

Gr = Grashof number for heat transfer

$\varepsilon = \frac{g\beta h}{C_p}$ = dissipation

pr = Prandtl number

Gr^* = Grashof number for mass transfer

$Sc = \frac{\vartheta}{D}$ = Schmidt number

D = the mass diffusion coefficient

$Da = \frac{K}{h^2}$ = Darcy number

and the boundary conditions (2.2.5) in the non-dimensional form become,

$$\left.\begin{array}{ll} U = V = 0 & at\ Y = 0,1 \\ \theta = 0.0 & at\ Y = 0 \\ \theta = 10.0 & at\ Y = 1 \\ \emptyset = 0.0 & at\ Y = 0 \\ \emptyset = 1.0 & at\ Y = 1 \end{array}\right\} \quad \ldots (3.6)$$

The differential equations can be rewritten as follow:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad \ldots (1)$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}$$

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$$= \frac{1}{\sqrt{Gr}}\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \theta + \frac{Gr^*}{Gr}\phi - \frac{1}{\sqrt{GrDa}}u \qquad \ldots\ldots (2)$$

$$\frac{\partial\theta}{\partial t} + u\frac{\partial\theta}{\partial x} + v\frac{\partial\theta}{\partial y} = \frac{1}{\sqrt{Gr}Pr}\frac{\partial^2\theta}{\partial x^2} + \frac{1}{Pr}\frac{\partial^2\theta}{\partial y^2} + \epsilon\left(\frac{\partial u}{\partial y}\right)^2 \qquad \ldots\ldots (3)$$

$$\frac{\partial\phi}{\partial t} + u\frac{\partial\phi}{\partial x} + v\frac{\partial\phi}{\partial y} = \frac{1}{Sc}\left\{\frac{1}{\sqrt{Gr}}\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2}\right\} \qquad \ldots\ldots (4)$$

$B.C.^s$

$$\left.\begin{array}{lll} u = v = 0 & at & y = 0,1 \\ \theta = 0 & at & y = 0 \\ \theta = 10 & at & y = 1 \\ \phi = 0 & at & y = 0 \\ \phi = 1 & at & y = 1 \end{array}\right\} \qquad \ldots\ldots (5)$$

Let

$$\left.\begin{array}{l} u(x,y,t) = u_1(y) + y_2(x,y,t) \\ v(x,y,t) = v_1(y) + v_2(x,y,t) \\ \theta(x,y,t) = \theta_1(y) + \theta_2(x,y,t) \\ \phi(x,y,t) = \phi_1(y) + \phi_2(x,y,t) \end{array}\right\} \qquad \ldots\ldots (6)$$

Substitute (6) into (1),(2),(3),(4) and (5), such that

$$\frac{\partial(u_1+u_2)}{\partial x} + \frac{\partial(v_1+v_2)}{\partial y} = 0$$

$$\frac{\partial u_1}{\partial x} + \frac{\partial u_2}{\partial x} + \frac{\partial v_1}{\partial y} + \frac{\partial v_2}{\partial y} = 0$$

$$\frac{\partial u_2}{\partial x} + \frac{\partial v_1}{\partial y} + \frac{\partial v_2}{\partial y} = 0$$

Separate:

$$\frac{\partial v_1}{\partial y} = 0 \implies v_1 = constant \implies v_1 = 0 \text{and}$$

$$\frac{\partial u_2}{\partial x} + \frac{\partial v_2}{\partial y} = 0 \ldots\ldots\ldots (7)$$

$$\frac{\partial(u_1+u_2)}{\partial t} + (u_1+u_2)\frac{\partial(u_1+u_2)}{\partial x} + (v_1+v_2)\frac{\partial(u_1+u_2)}{\partial y}$$
$$= \frac{1}{\sqrt{Gr}}\left(\frac{\partial^2(u_1+u_2)}{\partial x^2}\right) + \left(\frac{\partial^2(u_1+u_2)}{\partial y^2}\right) + (\theta_1+\theta_2) + \frac{Gr^*}{Gr}(\phi_1+\phi_2) - \frac{1}{\sqrt{GrDa}}(u_1+u_2)$$

$$\frac{\partial u_1}{\partial t} + \frac{\partial u_2}{\partial t} + u_1\frac{\partial u_1}{\partial x} + u_1\frac{\partial u_2}{\partial x} + u_2\frac{\partial u_2}{\partial x} + u_2\frac{\partial u_2}{\partial x} + v_1\frac{\partial u_1}{\partial y} + v_1\frac{\partial u_2}{\partial y} + v_2\frac{\partial u_1}{\partial y} + v_2\frac{\partial u_2}{\partial y}$$
$$= \frac{1}{\sqrt{Gr}}\frac{\partial^2 u_1}{\partial x^2} + \frac{1}{\sqrt{Gr}}\frac{\partial^2 u_2}{\partial x^2} + \frac{\partial^2 u_1}{\partial y^2} + \frac{\partial^2 u_2}{\partial y^2} + (\theta_1+\theta_2) + \frac{Gr^*}{Gr}\phi_1 + \frac{Gr^*}{Gr}\phi_2 - \frac{u_1}{\sqrt{GrDa}} - \frac{u_2}{\sqrt{GrDa}}.$$

Then:

$$\frac{d^2u_1}{\partial y^2} + \phi_1 + \frac{Gr^*}{Gr}\phi_1 - \frac{u_1}{\sqrt{GrDa}} = 0 \qquad \ldots\ldots\ldots (8)$$

$$\frac{\partial u_2}{\partial t} + u_1\frac{\partial u_2}{\partial y} + v_2\frac{du_1}{dy} + v_2\frac{\partial u_2}{\partial y} = \frac{1}{\sqrt{Gr}}\frac{\partial^2 u_2}{\partial x^2} + \frac{\partial^2 u_2}{\partial y^2} + \theta_2 + \frac{Gr^*}{Gr}\phi_2 - \frac{u_2}{\sqrt{GrDa}} \qquad \ldots\ldots (9)$$

$$\frac{\partial(\theta_1+\theta_2)}{\partial t} + (u_1+u_2)\frac{\partial(\theta_1+\theta_2)}{\partial x} + (v_1+v_2)\frac{\partial(\theta_1+\theta_2)}{\partial y}$$
$$= \frac{1}{\sqrt{Gr}Pr}\frac{\partial^2(\theta_1+\theta_2)}{\partial x^2} + \frac{1}{Pr} + \frac{\partial^2(\theta_1+\theta_2)}{\partial y^2} + \epsilon\left(\frac{\partial u_1}{\partial y} + \frac{\partial u_2}{\partial y}\right)^2$$

Then:

$$\frac{\partial\theta_1}{\partial t} + \frac{\partial\theta_2}{\partial y} + u_1\frac{\partial\theta_1}{\partial x} + u_1\frac{\partial\theta_2}{\partial x} + u_2\frac{\partial\theta_1}{\partial x} + u_2\frac{\partial\theta_2}{\partial x} + v_1\frac{\partial\theta_1}{\partial y} + v_1\frac{\partial\theta_2}{\partial y} + v_2\frac{\partial\theta_1}{\partial y} + v_2\frac{\partial\theta_2}{\partial y}$$
$$= \frac{1}{\sqrt{Gr}Pr}\frac{\partial^2\theta_2}{\partial x^2} + \frac{1}{Pr} + \frac{d^2\theta_1}{dy^2} + + \frac{1}{Pr} + \frac{d^2\theta_2}{dy^2} + \epsilon\left(\frac{du_1}{dy}\right)^2 + 2\epsilon\frac{du_1}{dy}\frac{\partial u_2}{\partial y} + \epsilon\left(\frac{\partial^2 u_2}{\partial y_2}\right)^2$$

Separate one can obtain:

$$\frac{1}{Pr}\frac{d^2\theta_1}{dy_2} + \epsilon\left(\frac{du_1}{dy}\right)^2 = 0 \qquad \ldots\ldots\ldots (10)$$

$$\frac{\partial\theta_2}{\partial t} + u_1\frac{\partial\theta_2}{\partial x} + v_2\frac{d\phi_1}{dy} = \frac{1}{\sqrt{Gr}Pr}\frac{\partial^2\theta_2}{\partial x^2} + \frac{1}{Pr}\frac{\partial^2\theta_2}{\partial y^2} + 2\epsilon\frac{du_1}{dy}\frac{\partial u_2}{\partial y} \qquad \ldots\ldots\ldots (11)$$

$$\frac{\partial(\phi_1+\phi_2)}{\partial t} + (u_1+u_2)\left(\frac{\partial(\phi_1+\phi_2)}{\partial x}\right) + (v_1+v_2)\frac{\partial(\phi_1+\phi_2)}{\partial y}$$
$$= \frac{1}{Sc}\left(\frac{1}{\sqrt{Gr}}\frac{\partial^2(\phi_1+\phi_2)}{\partial x^2} + \frac{\partial^2(\phi_1+\phi_2)}{\partial y^2}\right).$$

Then,

$$v_1\frac{\partial\phi_1}{\partial y} + v_1\frac{\partial\phi_2}{\partial y} + v_2\frac{\partial\phi_1}{\partial y} + v_2\frac{\partial\phi_2}{\partial y}$$
$$= \frac{1}{Sc}\left\{\frac{1}{\sqrt{Gr}}\frac{d^2\phi_1}{dx^2} + \frac{1}{\sqrt{Gr}}\frac{\partial^2\phi_2}{\partial x^2} + \frac{d^2\phi_1}{dy^2} + \frac{d^2\phi_2}{\partial y^2}\right\}.$$

That is

$$\frac{d^2\phi_1}{dy^2} = 0 \qquad \ldots\ldots\ldots (12)$$

$$\phi_1 = c_1y + c_2 \implies \phi = 1 \quad , \quad \frac{d\phi_1}{dy} = 1$$

$$\frac{\partial\phi_2}{\partial t} + u_1\frac{\partial\phi_2}{\partial x} + v_2\frac{d\phi_1}{dy} = \frac{1}{Sc}\left\{\frac{1}{\sqrt{Gr}}\frac{\partial^2\phi_2}{\partial x^2} + \frac{\partial^2\phi_2}{\partial x^2}\right\} \ldots\ldots\ldots (13)$$

The system of steady state are equations: 8, 10 &12.
Disturbance system are: 7, 9, 11, and 13
Let:

$$\left.\begin{array}{l} u_2 = A_1 e^{\propto t + i(k_1x + k_2y)} \\ v_2 = A_2 e^{\alpha t + i(k_1x + k_2y)} \\ \theta_2 = A_3 e^{\alpha t + i(k_1x + k_2y)} \\ \phi_2 = A_4 e^{\alpha t + i(k_1x + k_2y)} \end{array}\right\} \qquad \ldots\ldots\ldots (14)$$

Substitute the form (14) into the equations 7, 9, 11 &13.

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Such that:

(7) Becomes

$A_1 k_1 + A_2 k_2 = 0$ ………. (15)

(9) Becomes:

$$\alpha A_1 e + A_1 u_1 i k_1 e + A_2 \frac{du_1}{dy} e$$

$$= \frac{1}{\sqrt{Gr}} e(-)k_1^2 A_1 + e(-1)k_2^2 A_2 + A_3 e + \frac{Gr^* e}{Gr} A_4 - \frac{A_1 e}{\sqrt{GrDa}}$$

$$A_1\left\{\alpha + iu_1k_1 + \frac{k_1^2}{\sqrt{GrDa}}\right\} + A_2\left\{\frac{du_1}{dy} + k_2^2\right\} - A_3 - \frac{Gr^*}{Gr}A_4 = 0$$

……… (16)

(11) Becomes:

$$A_2 e\alpha + u_1 A_3 e(ik_1) + A_2 e\frac{dA_1}{dy}$$

$$= \frac{A_3 e}{\sqrt{GrPr}}(-1)k_1^2 + \frac{A_3}{Pr}(-1)ek_2^2 + 2\epsilon\frac{du_1}{dy}(A_1 ik_2)$$

$$A_1\left\{-i2k_2\epsilon\frac{du_1}{dy}\right\} + A_2\frac{d\theta_1}{dy} + A_3\left\{\alpha + u_1 ik_1 + \frac{k_1^2}{\sqrt{GrPr}} + \frac{k_2^2}{Pr}\right\} = 0 \quad \text{…… (17)}$$

(13) Becomes:

$$A_4\alpha e + u_1 i A_4 k_1 e + A_2 e\frac{d\phi_1}{dy}$$

$$= \frac{1}{Sc}\left\{\frac{A_4 e}{\sqrt{Gr}}(-1)k_1^2 + A_4(-1)ek_2\right\}$$

Or:

$$A_2\frac{d\phi_1}{dy} + A_4\left\{\alpha + iu_1k_1 + \frac{k_1^2}{Sc\sqrt{Gr}} + \frac{k_2^2}{Sc}\right\} = 0 \quad \text{……… (18)}$$

These differential equations, 15-18, can be organized in a matrix form, such that

$$\begin{bmatrix} k_1 & k_2 & 0 & 0 \\ \alpha + iu_1k_1 + \frac{k_1^2}{\sqrt{Gr}} + \frac{1}{\sqrt{GrDa}} & \frac{du_1}{dy} + k_2^2 & (-1) & -\frac{Gr^*}{Gr} \\ 2ik_2\frac{du_1}{dy} & \frac{d\theta_1}{dy} & \alpha + iu_1k_1 + \frac{k_1^2}{\sqrt{GrPr}} & 0 \\ 0 & \sqrt{GrPr} & 0 & \alpha + iu_1k_1 + \frac{k_1^2}{SC\sqrt{Gr}} + \frac{k_2^2}{Sc} \end{bmatrix} = 0$$

Or

$$k_1\left\{\begin{vmatrix} \frac{du_1}{dy} + k_2^2 & -1 & -\frac{Gr^*}{Gr} \\ \frac{d\theta_1}{dy} & \alpha + iu_1k_1 + \frac{k_1^2}{\sqrt{GrPr}} + \frac{k_2^2}{Pr} & 0 \\ \frac{d\phi_1}{dy} & 0 & \alpha + iu_1k_1 + \frac{k_1^2}{SC\sqrt{Gr}} + \frac{k_2^2}{Sc} \end{vmatrix} - k_2\begin{vmatrix} \alpha + iu_1k_1 + \frac{k_1^2}{\sqrt{Gr}} + \frac{1}{\sqrt{GrDa}} & -1 & -\frac{Gr^*}{Gr} \\ -2ik_2\frac{du_1}{dy} & \alpha + iu_1k_1 + \frac{k_1^2}{\sqrt{GrPr}} + \frac{k_2^2}{Pr} & 0 \\ 0 & 0 & \alpha + iu_1k_1 + \frac{k_1^2}{Sc\sqrt{Gr}} + \frac{k_2^2}{Sc} \end{vmatrix}\right\}$$

$$= 0$$

Let

$$A^*{}_1 = \left(\frac{du_1}{dy} + k_2^2\right)\left[\left(\alpha + iu_1k_1 + \frac{k_1^2}{\sqrt{Grpr}} + \frac{k_2^2}{Pr}\right)\left(\alpha + iu_1k_1 + \frac{k_1^2}{Sc\sqrt{Gr}} + \frac{k_2^2}{Sc}\right)\right] + \left[\frac{d\theta_1}{dy}\left(\alpha + iu_1k_1 + \frac{k_1^2}{Sc\sqrt{Gr}} + \frac{k_2^2}{Sc}\right)\right] - \frac{Gr^*}{Gr}\left[-\frac{d\theta_1}{dy}\left(\alpha + iu_1k_1 + \frac{k_1^2}{\sqrt{GrPr}} + \frac{k_2^2}{Pr}\right)\right].$$

Also:

$\frac{du_1}{dy} + k_2^2 = a_1,\ \frac{k_1^2}{\sqrt{GrPr}} + \frac{k_2^2}{Pr} = a_2 \& b_1 = \frac{k_1^2}{Sc\sqrt{Gr}} + \frac{k_2^2}{Sc}$ and $\frac{d\phi_1}{dy} = 1$, from equation12.

Then

$$A^*{}_1 = k_1\{a_1[(\alpha + iu_1k_1 + a_2)(\alpha + iu_1k_1 + b_1) -]\} + \left[\frac{d\theta_1}{dy}(\alpha + iu_1k_1 + b_1)\right] - \frac{Gr^*}{Gr}(1)(\alpha + iu_1k_1 + a_2)$$

That is $A^*{}_1 = k_1\left\{a_1\left\{\alpha^2 + iu_1k_1\alpha + \alpha b_1 + iu_1k_1\alpha - u_1^2k_1^2 + iu_1k_1b_1 + \alpha a_2 + iu_1k_1a_2 + a_2b_1 + \alpha\frac{d\theta_1}{dy} + iu_1k_1\frac{d\theta_1}{dy} + b_1\frac{d\theta_1}{dy} + \frac{Gr^*}{Gr}u_1k_1 + \frac{Gr^*}{Gr}a_2\right\}\right.$

Or

$$A^*{}_1 = k_1\left\{a_1\alpha^2 + \left(iu_1k_1 + b_1 + iu_1k_1 + a_2 + \frac{d\theta_1}{dy} + \frac{Gr^*}{Gr}\right)\alpha - u_1^2k_1^2 + iu_1k_1b_1 + iu_1k_1a_2 + a_2b_1 + iu_1k_1\frac{d\theta_1}{dy} + b_1\frac{d\theta_1}{dy} + \frac{Gr^*}{Gr}k_1u_1 + \frac{Gr^*}{Gr}a_2\right\} \quad \text{……….. (19)}$$

$$A^*{}_2 = k_2\left\{(\alpha + iu_1k_1 + b_2)\left(\alpha + iu_1k_1 + \frac{k_1^2}{\sqrt{Gr}} + \frac{1}{\sqrt{GrDa}}\right) - 2ik_2\frac{du_1}{dy}\right\}$$

$$= k_2\left\{\alpha^2 + iu_1k_1\alpha + \frac{k_1^2}{\sqrt{Gr}} + \frac{\alpha}{\sqrt{GrDa}} + iu_1k_1\alpha - u_1^2k_1^2 + \frac{iu_1k_1^3}{\sqrt{Gr}} + \frac{iu_1k_1}{\sqrt{GrDa}} + \alpha b_1 + iu_1k_1b_1 + \frac{b_1k_1^2}{\sqrt{Gr}} + \frac{b_1}{\sqrt{GrDa}}\right\}$$

$$= k_2\left\{\alpha\left(\alpha^2 + \frac{k_1^2}{\sqrt{Gr}} + \frac{1}{\sqrt{GrDa}} + 2iu_1k_1 + b_1\right) + \frac{iu_1k_1^3}{\sqrt{Gr}} - u_1^2k_1^2 + \frac{iu_1k_1}{\sqrt{GrDa}} + iu_1k_1b_1 + \frac{b_1k_1^2}{\sqrt{Gr}} + \frac{b_1}{\sqrt{GrDa}}\right\} \quad \text{…… (20)}$$

Let

$$C_1 = \frac{k_1^2}{\sqrt{Gr}} + \frac{1}{\sqrt{GrDa}} + b_1,$$

$$C_2 = \frac{iu_1k_1^3}{\sqrt{Gr}} - u_1^2k_1^2 + \frac{iu_1k_1}{\sqrt{GrDa}} + iu_1k_1b_1 + \frac{b_1k_1^2}{\sqrt{Gr}} + \frac{b_1}{\sqrt{GrDa}}$$

$$= i\{B_1\} + B_2$$

Where

$$B_1 = \frac{u_1k_1^3}{\sqrt{Gr}} + \frac{u_1k_1}{\sqrt{Gr}} + u_1k_1b_1$$

$$B_2 = \frac{b_1k_1^2}{\sqrt{Gr}} - u_1^2k_1^2 + \frac{b_1}{\sqrt{GrDa}}.$$

Also $A^*{}_2 = k_2\{\alpha^2 + (c_1 + 2u_1k_1)\alpha + iB_1 + B_2\}$ …. (21)

Equation (19) becomes:

$$A^*{}_1 = k_1\{a_1\alpha^2 + (E_1 + 2iu_1k_1)\alpha + E_2 + iE_3\} \quad \ldots\ldots (22)$$

Where

$$E_1 = b_1 + a_1 + \frac{d\theta_1}{dy} + \frac{Gr^*}{Gr}$$

$$E_2 = a_2b_1 + u_1^2k_1^2 + b_1\frac{d\theta_1}{dy} + \frac{Gr^*}{Gr}a_2$$

$$E_3 = u_1k_1b_1 + u_1k_1a_2 + u_1k_1\frac{d\theta_1}{dy} + \frac{Gr^*}{Gr}u_1k_1.$$

Since

$A^*{}_1k_1 = A^*{}_2k_2$, that is

$$\{a_1\alpha^2 + (E_1 + 2iu_1k_1)\alpha + E_2 + iE_3\}k_1 - k_2\{\alpha^2 + (c_1 + 2u_1k_1)\alpha + iB_1 + B_2\} = 0$$

Then:

$$\alpha^2(\alpha_1k_1 - k_2) + \alpha(E_1 + 2iu_1k_1 - k_2c_1 - 2iu_1k_1k_2) + k_1E_2 + iE_3k_1 - iB_1k_2 - k_2B_2 = 0 \quad \ldots. (23)$$

Let:

$$D_1 = a_1k_1 - k_2$$

$$D_2 = E_1 - k_2c_1$$

$$D_3 = 2u_1k_1 - 2u_1k_1k_2$$

$$D_4 = k_1E_2 - k_2B_2$$

$$D_5 = E_3k_1 - B_1k_2$$

Give:

$$\alpha^2D_1 + \alpha(D_2 + iD_3) + D_4 + iD_5 = 0$$

$$\alpha = \frac{-(D_2 + iD_3) \mp \sqrt{(D_2 + iD_3)^2 - 4D_1(D_4 + iD_5)}}{2D_1}$$

$$= \frac{-(D_2 + iD_3) \mp \sqrt{D_2^2 - D_3^2 + 2iD_2D_3 - 4D_1D_4 - 4iD_1D_5}}{2D_1}$$

$$\alpha = \frac{-(D_2 + iD_3) \mp \sqrt{D_2^2 - D_3^2 + 4D_1D_4 + 2i(D_2D_3 - 2iD_1D_5)}}{2D_1}$$

Let,

$$X_1 = D_2^2 - D^2{}_3 - 4D_1D_4 \quad and \quad Y_1 = 2(D_2D_3 - 2D_1D_5)$$

Then,

$$\alpha = \frac{-(D_2 + iD_3) \mp \sqrt{X_1 + iY_1}}{2D_1}$$

$$\alpha = \alpha_1 + i\alpha_2$$

$$\alpha_1 = \frac{-D_2 \mp \sqrt{X_1} + \frac{Y_1^2}{8X_1\sqrt{X_1}} + \cdots}{2D_1} = 0,$$

That is:

Either $-D_2 + \frac{8X_1^2+Y_1^2}{8X_1\sqrt{X_1}} + \cdots = 0 \ldots. (24)$

Or $-D_2 - \frac{8X_1^2+Y_1^2}{8X_1\sqrt{X_1}} + \cdots = 0 \ldots\ (25)$

Then,

If $\frac{8X_1^2+Y_1^2}{8X_1\sqrt{X_1}} - D_2 > 0$, The system is unstable.

And $\frac{8X_1^2+Y_1^2}{8X_1\sqrt{X_1}} - D_2 < 0$, The system is stable.

Also:

If $-\frac{8X_1^2+Y_1^2}{8X_1\sqrt{X_1}} - D_2 > 0$, The system is unstable.

And $-\frac{8X_1^2+Y_1^2}{8X_1\sqrt{X_1}} - D_2 < 0$, The system is stable.

IV. Results and Discussion

The main object of the present analysis is to examine a disturb system of a diffusion of heat in fluid flow in a channel with porous walls. The stability or otherwise depends on the sign of the complex speed $\alpha = \alpha_1 + i\alpha_2$. A positive or negative α_1 implies growth or decay of the disturbance, respectively. This study is concentrated on the change of the sign of α_1 to separate the different regimes of fluid behavior. Also it must be well understood that the neutral stability is define by setting $\alpha_1=0$, which represent the line between stable and unstable regions through the relation between the parameter Gr, ε, Pr , Gr*, Sc , D and Da. After all the procedures mentioned in the methodology, chapter 3, the resulting relations between the parameters which represent the neutral line are found to be:

If $\frac{8X_1^2+Y_1^2}{8X_1\sqrt{X_1}} \ldots. -D_2 > 0$, The system is unstable.

$\frac{8X_1^2+Y_1^2}{8X_1\sqrt{X_1}} \ldots. -D < 0$, The system is stable.

Also:

If $-\frac{8X_1^2+Y_1^2}{8X_1\sqrt{X_1}} \ldots. -D_2 > 0$, The system is unstable.

$-\frac{8X_1^2+Y_1^2}{8X_1\sqrt{X_1}} \ldots. -D_2 < 0$, The system is stable.

These results represent the main object of the project and the separations between the two regions, the stable and unstable.

Conclusion

In this model of a project has been investigated by mathematical methods, treating a case of a disturb fluid flow in a horizontal channel with porous walls, which have been formulated by partial differential equations and then exposed to a disturbance leads to separate these equations to two system, steady and disturb systems. With some mathematical manipulations and matrices through which we have the neutral line. The final results show that there are four possibilities for the stability or otherwise to measure the separation between the two mentioned cases.
The major possible next devolvement of this model is to consider the heights of the waves to be variables, A1,A2,A3 and A4 .

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