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Frequency Identification of a Hammerstein Model

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***Abstract* – Hammersetin systems identification is studied in the presence of possibly infinite-order linear dynamics and nonparametric nonlinear element. The latter can be noninvertible and of arbitrary-shape. Using simple constant and periodic excitations, and getting benefit from model plurality, the problem identification problem is made. All estimators are shown to be consistent.**

***Keywords* – Hammerstein Systems, Nonlinear Systems, Fourier Series Expansions, Frequency Identification.**

I. Introduction

The Hammerstein model is a series connection of a memoryless nonlinearity and a linear dynamic bloc (Fig. 1). Black-box nonlinear system identification is a very wide research area [1]. The considerable diversity of nonlinear systems has led to a large variety of identification problems and a proliferation of identification approaches and methods.

In this paper, the problem of identifying Hammerstein systems is addressed.

Hard nonlinearities of known type have been considered in [2] and [4].

Unlike many previous works e.g. [4], the model structure of the linear subsystem is entirely unknown. Furthermore, the system nonlinearity is of arbitrary-shape and can be noninvertible. In most previous works devoted to Hammerstein system identification, the nonlinear element is supposed to be continuous. Moreover, this latter is generally assumed to be a (truncated) polynomial or Fourier series in the variable e.g. [5]-[9].

The present strategy is allowed to interest a wide range of the system nonlinearity. The identification problem amounts to determining an accurate estimate of the (nonparametric) frequency response $G(j\omega)$, for a set of frequencies $(\omega_1 \ldots \omega_m)$, and the nonlinearity. The present identification method is a two-stage: the system nonlinearity is identified first, using simple constant inputs, and based upon in the second stage to identify the linear subsystem using the Fourier decomposition.

The paper is organized as follows: the identification problem is formulated in Section 2; the nonlinear operator identification is coped with in Section 3; the linear subsystem frequency response determination is investigated in Section 4.

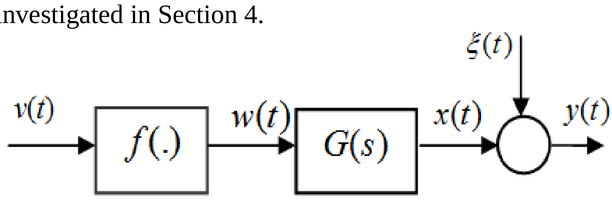

Fig.1. Wiener model structure

II. Identification Problem Statement

Standard Wiener systems consist of a memoryless nonlinear element $f(.)$ followed in series by a linear dynamic subsystem $G(s)$ (Fig. 1). This model is analytically described by the following equations:

$$w(t) = f\left(v(t)\right). \tag{1a}$$

$$x(t) = g(t) * w(t). \tag{1b}$$

$$y(t) = x(t) + \xi(t) = g(t) * w(t) + \xi(t). \tag{1c}$$

where $g(t) = L^{-1}(G(s))$ is the inverse Laplace transform of $G(s)$; the symbol * refers to the convolution operation; $w(t)$ is the internal signal; $x(t)$ is the undisturbed output. The only measurable signals are the system input $v(t)$ and output $y(t)$. The equation error $\xi(t)$ is a zero-mean stationary sequence of independent random variables; it accounts for external noise, it is supposed to be ergodic (so that arithmetic averages can be substituted to probabilistic means whenever this is necessary).

Because the system identification is carried out in open loop (Fig. 1), the linear block $G(s)$ must satisfy the stability asymptotically. Except for this assumption, the linear subsystem is arbitrary, particularly it may be arbitrary and unknown structure. The system nonlinearity $f(.)$ is of arbitrary-shape and is allowed to be noninvertible.

We aim at designing an identification scheme that is able to provide a model estimate $\left(\hat{f}(.), \hat{G}(j\omega_k)\right)$ that represents well the system when. Since $w(t)$ and $x(t)$ are not measurable, the system identification should be fully based upon measurements of the input $v(t)$ and the output system $y(t)$. Therefore, the considered identification problem does not have a unique solution: if the model $(f(v), G(s))$ represents a solution then, any model of the form $(f(v)/k, kG(s))$ is also a solution (where k is any nonzero real). This naturally leads to the question: what particular model should we focus on? This question will be answered later. Such a lack of uniqueness, will be exploited (in Section 3) to cope with the uncertainty on the amplitude of the internal signals $w(t)$ and $x(t)$.

III. Identification of the Nonlinearity

In this section, we aim establish an identification scheme that is able to provide an accurately estimate of the system nonlinearity $f(.)$. In Section 2 it was shown that, if k is any nonzero real, so any model of the form $(f(v)/k, kG(s))$ is representative of the system. Accordingly, the system to be identified is described by the transfer function:

$$\bar{G}(s)=\frac{G(s)}{G(0)}. \quad (2a)$$

and the nonlinearity:

$$\bar{f}(v)=G(0)f(v). \quad (2b)$$

It is readily seen that, this model to be identified is the only that satisfies the property: $\bar{G}(0)=1$ (i.e. the linear subsystem is of unit static gain).

To avoid multiplication of notations, the model of interest will still be denoted $(f(.),G(s))$ but the description (1a)-(1c) is completed with the following property:

$$G(0)=1. \quad (3)$$

Then, the considered system described by (1a)-(1c) and satisfying the property (3) is excited by simple constant inputs:

$$v(t)=V_j \text{ for } j=1\ldots N. \quad (4)$$

where the number N is arbitrarily chosen by the user. Then, it follows from (1a) and (4) that, the internal signal $w(t)$ is constant and can be expressed as follows:

$$w(t)=W_j=f\left(V_j\right) \text{ for } j=1\ldots N. \quad (5)$$

Accordingly, as the linear subsystem $G(s)$ is asymptotically stable, it follows that the steady-state of the output undisturbed signal $x(t)$ is constant i.e. $x(t)\underset{t\to\infty}{\to}X_j$, and is written using (1b), (3) and (5):

$$X_j=W_j=f\left(V_j\right) \text{ for } j=1\ldots N. \quad (6)$$

This result means that if the input system is constant $v(t)=V_j$ ($j=1\ldots N$) then the steady-state undisturbed output $x(t)$ depends only on the input value and the system nonlinearity $f(.)$.

On the other hand, notice that the steady-state undisturbed output X_j ($j=1\ldots N$) can simply be estimated using the fact that $\xi(t)$ is zero-mean and (1c). Specifically, X_j can be recovered by averaging $y(t)$ on a sufficiently large interval as follows:

$$\hat{X}_j(M)=\frac{1}{M}\sum_{i=0}^{M-1}y(i). \quad (7)$$

for some (large enough) integer *M*. Finally, a number of points of the nonlinear function $f(.)$ can thus be accurately estimated by repeating the above experiment successively for V_1 to V_N.

Then, the set of obtained couples $\left(V_j,\hat{X}_j(M)\right)$ with $j=1\ldots N$, are estimates of N points all belonging to the system nonlinearity $f(.)$.

Furthermore, the larger the parameter *M* is, the better the estimation accuracy.

IV. Linear Subsystem Identification

In this section, an identification method is proposed to obtain estimates of the complex gain corresponding to the linear subsystem $G(s)$ at the frequencies $k\omega$ ($k=0,1,\ldots$) whatever $\omega>0$. Following the procedure of Section 3, one gets estimates of a set of *N* different points belonging to the system nonlinearity $f(.)$.

To get profit from this result, the system is excited by a periodic input signal $v(t)$, with period $T=2\pi/\omega$, that only takes the values V_j:

$$v(t)\in\left\{V_j;\ j=1\ldots N\right\}. \quad (8)$$

Accordingly, the latter needs not to take all values V_j ($j=1\ldots N$), involved in the procedure of Section 3. Doing so, it is clear from Section 3 that, the working point $(v(t),u(t))$ occupies the positions $\left(V_j,X_j\right)$ ($j=1\ldots N$), estimated in Section 3. Accordingly, the internal signal $w(t)$ turns out to be perfectly known. Specifically, $w(t)$ is in turn periodic, with period *T*, and:

$$w(t)\in\left\{X_j;\ j=1\ldots N\right\}. \quad (9)$$

In fact, ω is only imposed by the user through the choice of the period *T* of the excitation signal defined by (8).

On the other hand, since the internal signal $w(t)$ is periodic (of period *T*) and known, it can be developed in Fourier series:

$$w(t)=\sum_{k=0}^{\infty}s_k\sin(k\omega t+\psi_k). \quad (10)$$

where the Fourier parameters s_k and ψ_k can be exactly determined ($w(t)$ is perfectly known). Then, it follows from (1b) and (10) that:

$$w(t)=\sum_{k=0}^{\infty}s_k\left|G(jk\omega)\right|\sin\left(k\omega t+\psi_k+\varphi(k\omega)\right). \quad (11)$$

Finally, one immediately gets from (1c) and (11) that:

$$y(t)=\sum_{k=0}^{\infty}s_k\left|G(jk\omega)\right|\sin\left(k\omega t+\psi_k+\varphi(k\omega)\right)+\xi(t). \quad (12)$$

Knowing that the undisturbed output signal $x(t)$ is periodic, having the same period *T* of input, it can be developed also in Fourier series:

$$x(t)=\sum_{k=0}^{\infty}A_k\sin(k\omega t+\alpha_k). \quad (13)$$

Equations (11) and (13) show how to obtain the complex amplitudes $G(jk\omega)$ $(k=1,2,\ldots)$. This is performed noticing that the right side of (12) is nothing other than the Fourier series expansion of the output signal $y(t)$, up to noise. Accordingly, one immediately gets from (11) and (13):

$$\left|G(jk\omega)\right|=A_k/s_k \text{ for } k=1,2,\ldots \quad (14a)$$

$$\varphi(k\omega)=\alpha_k-\psi_k \text{ for } k=1,2,\ldots \quad (14b)$$

Given that all deterministic terms on the right side of (12) are periodic, with common period *T*, and $\xi(t)$ is a zero-mean ergodic white noise, the effect of the latter can be filtered considering the following trans-period averaging of the output:

$$\hat{x}_L(t) = \frac{1}{L}\sum_{j=1}^{L} y\left(t+(j-1)T\right), \quad 0 \le t < T . \qquad (15)$$

for some (large enough) integer L. Then, the Fourier parameters in (13) can be obtained using the estimate $\hat{x}_L(t)$ of $x(t)$ in (15).

Finally, knowing that the Fourier parameters s_k and ψ_k of $w(t)$ are well-known, and using the Fourier parameters A_k and α_k of $x(t)$ (obtained by (15)), it readily follows from (14a-b) that the complex amplitudes $G(jk\omega)$ $(k = 0,1,\ldots)$ are obtained (i.e. the modulus gain $|G(jk\omega)|$ and the phase $\varphi(k\omega)$ at the frequencies $k\omega$).

V. Conclusion

The problem of system identification is addressed for Hammerstein systems where the linear subsystem may be parametric or not, finite order or not. The system nonlinearity is of arbitrary-shape and can be noninvertible. The proposed identification scheme performs separately the estimation of the linear subsystem and the nonlinear element.

The identification problem is dealt with using a two-stage approach combining frequency. Data acquisition in presence of constant inputs is performed in the first stage following the procedure of Section 3. Then, an accurate estimate of a set of points belonging to the nonlinearity can be accurately estimated.

Finally, the transfer function response is identified in the second stage using the algorithm described Section 4. To the author's knowledge, unlike many of previous study, the present method has solved the identification problem for a large class of Hammerstein systems. Furthermore, the proposed approach involves easily generated excitation signals and simple Fourier series decomposition based.

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