International Journal of Electronics Communication and Computer Engineering
Volume 6, Issue 3, ISSN (Online): 2249–071X, ISSN (Print): 2278–4209

Effects of Radiation, Heat Source/Sink, Viscous Dissipation on Unsteady MHD Fluid Flow over a Stretching Sheet

Dr. V. Dhanalaxmi
Department of Mathematics, University College of Technology,
Osmania University, Hyderabad, 500007, India.
Email: v_laxmi_sudha@yahoo.co.in

***Abstract* – The aim of this paper is to present the unsteady magneto hydrodynamic (MHD) boundary layer flow and heat transfer of a fluid over a stretching sheet in the presence of radiation and viscous dissipation. By similarity transformation, the governing partial differential equations are transformed to set of partial differential equations. The resultant equations are then solved numerically using Runge-Kutta fourth order method along with shooting technique. Effects of various physical parameters on dimensionless velocity and temperature profiles were depicted graphically and analyzed in detail. The numerical predictions have been compared with already published papers and good agreement is obtained. Finally, numerical values of physical quantities such as the skin friction coefficient and the local Nusselt number are presented in tabular form.**

***Keywords* – Unsteady, Thermal Radiation, Stretching Sheet, Viscous Dissipation.**

I. Introduction

The study of hydrodynamic flow and heat transfer over a stretching surface has gained considerable attention due to its applications in industries and important bearings on several technological processes. The quality of final product depends on the rate of heat transfer at the stretching surface during the manufacturing processes. The flow and heat transfer problems for stretching surfaces have received attention by numerous researchers [1]-[10]. Sakiadis [11] investigated the boundary layer behavior on continuous solid surfaces.

Unsteady problems due to a stretching surface received less attention. The unsteady aspects become interesting in certain practical problems, where the motion of the stretching surface may start impulsively from rest. Elbashbeshy and Bazid [12] presented similarity solutions for unsteady flow and heat transfer over a stretching surface. They examined effects of unsteady parameter (A) and Prandtl number (Pr) on the flow and heat transfer characteristics. They observed that the unsteady parameter and Prandtl number increase heat transfer rate at the surface. These results were supported by Azid [13] and Ishak [14] and they obtained the exact solution of unsteady mixed convection boundary layer flow and heat transfer. The results show that the buoyancy parameter increases the heat transfer rate at the surface. Bachok et al. [15] investigated the effect of material parameter of the unsteady laminar flow of an incompressible micro polar fluid. They found that the skin friction coefficient decreases as the material parameter increases and the micro polar fluid reduces drag compared to viscous fluid. Liu and Anderson [16] studied the thermal characteristics of a viscous film on an unsteady stretching surface.

Ishak [17] studied the MHD flow and heat transfer characteristics over an unsteady stretching surface. Zanariah et al. [18] extended Ishak's [17] work by introducing the effect of radiation for the MHD flow and heat transfer over an unsteady stretching surface. Radiation is energy that comes from a source and travels through some material or through space. Light, heat and sound are types of radiation. Radiation is considered in his study due to the fact that thermal radiation effect might play a significant role in controlling heat transfer process in polymer processing industry. In many new engineering processes such as fossil fuel combustion energy processes, solar power technology, astrophysical flows, gas turbines and the various propulsion devises for aircraft, missiles, satellites, and space vehicle re-entry occur at high temperature. So knowledge of radiation plays a very important role and hence, its effect cannot be neglected. Also thermal radiation is of major importance in many processes in engineering areas which occur at a high temperature for the design of many advance energy conversion systems and pertinent equipment. The Rosseland approximation is used to describe the radioactive heat flux in the energy equation.

The objective of the present study is to extend the works of Zanariah and Aziz [18] by introducing effects of viscous dissipation and heat source in the energy equation. The irreversible process by means of which the work done by a fluid on adjacent layers due to the action of shear forces is transformed into heat is defined as viscous dissipation. Viscous dissipation is of interest for many applications: significant temperature rises are observed in polymer processing flows such as injection molding or extrusion at high rates. Aerodynamic heating in the thin boundary layer around high speed aircraft raises the temperature of the skin. Boundary layer flows with internal heat generation over a stretching sheet continues to receive attention because of its many practical applications in a broad spectrum of engineering systems.

II. Mathematical Formulation

We consider the unsteady two dimensional laminar boundary layer flow on a continuously stretching sheet immersed in an incompressible electrically conducting fluid. It is assumed that the unsteady flow and heat transfer start at time t=0. Keeping the origin fixed, the surface is stretched with velocity U_w (x,t) along the x-axis.

We assume that stretching velocity U_w (x,t) and the surface temperature T_w (x,t) are of the form:

$$U_w\,(x,t) = \frac{ax}{1-ct} \text{ and } T_w(x,t) = T_\infty + \frac{bx}{1-ct}, \qquad (1)$$

Where a, b, and are constants with a>0, b>0 and c > 0 (with ct<1) and both a and b have dimension time $^{-1}$.

The two dimensional governing equations for the flow are:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad (2)$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \upsilon\frac{\partial^2 u}{\partial y^2} - \frac{\sigma B^2}{\rho}u \qquad (3)$$

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} - \frac{1}{\rho c_p}\frac{\partial q_r}{\partial y} + \frac{\mu}{\rho c_p}\left(\frac{\partial u}{\partial y}\right)^2 + \frac{Q}{\rho c_p}(T - T_\infty), \qquad (4)$$

subject to the boundary conditions:

$$U = U_w,\ V = 0,\ T = T_w \text{ at } y = 0$$
$$U \to 0 \quad T \to T_\infty \text{ as } y \to \infty, \qquad (5)$$

Where u and v are velocity components along the x- axis and y-axis, respectively. T is the fluid temperature in the boundary layer, t is time, V is the kinematic viscosity, ρ is the fluid density, α is the thermal diffusivity, C_p is the specific heat at constant pressure, μ is the coefficient of viscosity, Q is the volumetric heat generation/ absorption rate and q_r is the radiative heat flux. To obtain similarity solution for equations (2) to (5), the variable magnetic field B and heat generation/absorption Q are of the form: $B = \frac{B_0}{\sqrt{1-ct}}$ and $Q = \frac{Q_0}{1-ct}$, where B_0 and Q_0 are constants.

The radioactive heat flux q_r, under Rosseland approximation, has the form

$$q_r = -\frac{4\sigma^*}{3k^*}\frac{\partial T^4}{\partial y} \qquad (6)$$

where σ* is the Stefan Boltzmann constant and k* is the absorption coefficient. Temperature differences in the flow are assumed to be sufficiently small such that T^4 may be expressed as a linear function of temperature. Expanding T^4 about T_∞ in Taylor's series and neglecting higher orders yields:

$$T^4 \cong 4T_\infty{}^3 T - 3T_\infty{}^4. \qquad (7)$$

Substituting (6) and (7) in equation (4) gives

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \left(\alpha + \frac{16\sigma^* T_\infty{}^3}{3k^*}\right)\frac{\partial^2 T}{\partial y^2} - \frac{1}{\rho c_p}\frac{\partial r_r}{\partial y} + \frac{\mu}{\rho c_p}\left(\frac{\partial u}{\partial y}\right)^2 + \frac{Q}{\rho c_p}(T - T_\infty) \qquad (8)$$

The equation of continuity is satisfied for the stream function Ψ(x,y) such that:

$$u = \frac{\partial\psi}{\partial y} = \frac{ax}{1-ct}f'(\eta),$$

$$v = -\frac{\partial\psi}{\partial x} = -\left[\frac{\upsilon a}{1-ct}\right]^{\frac{1}{2}} f(\eta) \qquad (9)$$

The momentum and energy equation can be transformed into the corresponding ordinary differential equations by introducing the following dimensionless functions f and θ, and the similarity variable η as:

$$\eta = y\left(\frac{U_w}{\upsilon x}\right)^{1/2}, \psi = [U_w \upsilon x]^{\frac{1}{2}} f(\eta),$$
$$\theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty} \qquad (10)$$

The transformed ordinary differential equations are:

$$f''' + ff'' - f'^2 - Mf' - A\left[f' + \frac{1}{2}\eta f''\right] = 0 \qquad (11)$$

$$(1+R)\theta'' + \mathrm{Pr}(f\theta' - f'\theta) - PrA(\theta + \frac{1}{2}\eta\theta') + \mathrm{Pr}(ME_c(f'')^2 + \gamma\theta) = 0 \qquad (12)$$

and corresponding boundary conditions are:

$$f(0) = 0, f'(0) = 1, \theta(0) = 1 \text{ at } \eta = 0$$
$$f'(\eta) \to 0,\ \theta(\eta) \to 0 \text{ as } \eta \to \infty \qquad (13)$$

where primes denote differentiation with respect to η, and R $= \frac{16\sigma^* T_\infty{}^3}{3k^* k}$ is the radiation parameter and k is thermal conductivity, A $= \frac{c}{a}$ (Unsteady parameter), $M = \frac{\sigma B_0{}^2}{\rho a}$ (Magnetic parameter), $Ec = \frac{U_w{}^2}{c_p(T_w - T_\infty)}$ (Eckert Number), $\gamma = \frac{Q_0}{\rho c_p a}$ (Heat source parameter) and $Pr = \frac{\upsilon}{\alpha}$ (Prandtl number).

The physical quantities of interest are the skin friction coefficient

$$C_f = \frac{2\tau_w}{\rho U_w{}^2} \qquad (14)$$

and the local Nusselt number

$$Nu_x = \frac{xq_w}{k(T_w - T_\infty)}, \qquad (15)$$

where the surface shear stress τ_w and the surface heat flux q_w are defined by

$$\tau_w = \mu\left(\frac{\partial u}{\partial y}\right)_{y=0} \text{ and } q_w = -k\left(\frac{\partial T}{\partial y}\right)_{y=0} \qquad (16)$$

Using non - dimensional variables (9) in (16) we obtain

$$\frac{1}{2}C_f\sqrt{Re_x} = f''(0) \text{ and } \frac{Nu_x}{\sqrt{Re_x}} = -\theta'(0) \qquad (17)$$

where $Re_x = \frac{U_w x}{\upsilon}$ is the local Reynolds number.

III. Numerical Solution

Since equations (11) and (12) are highly nonlinear, it is difficult to find the closed form solutions. Thus, the solutions of these equations with the boundary conditions (13) are solved numerically using the Runge-Kutta fourth order method with a systematic guessing of $f'(0) = 0$ and $\theta'(0) = 0$ by the shooting technique until the boundary conditions $f'(\eta) \to 0$, $\theta(\eta) \to 0$ as $\eta \to \infty$ decay exponentially to zero. From the process of numerical computation, the Skin friction coefficient and Nusselt number which are respectively proportional to $f''(0)$ and $-\theta'(0)$ are presented in tabular form.

IV. Results And Discussion

The nonlinear ordinary differential equations (11) and (12) subject to the boundary condition (13) is solved numerically by using the Runge-Kutta method along with the shooting technique.

International Journal of Electronics Communication and Computer Engineering
Volume 6, Issue 3, ISSN (Online): 2249–071X, ISSN (Print): 2278–4209

To verify the validity and accuracy of the result obtained, numerical results for the values of heat transfer rate at the surface are compared with Ishak [17] and Zanariah et al. [18] for the case R=0 and steady state case (A=0). The numerical values as shown in table 1 are in excellent agreement with the values of Ishak [17] and Zanariah [18]. As it is shown in the table, the heat transfer coefficient increases with an increase of Prandtl number. This is true, because by definition, Prandtl number is the ratio of kinematic viscosity to thermal diffusivity. An increase in the values of Prandtl number implies that momentum diffusivity dominates thermal diffusivity. Hence, the rate of heat transfer at the surface increases with increasing values of Prandtl number.

Table 1: Values of –θ' (0) for various values of A, M, R and Pr

Parameters				–θ' **(0)**		
A	M	R	Pr	Ishak [17]	Zanariah [18]	Present
0	0	0	0.72	0.8086	0.808631	0.8086308
			1	1.0000	1.000000	1.0000000
			3	1.9237	1.923683	1.9237161
			6.7	3.0003	3.000272	3.0003220
0	1	0	0.7	0.6897	0.689712	0.6897110
			1	0.8921	0.892147	0.8921452
			10	3.6170	3.616992	3.6170717
1	0	0	0.7	1.0834	1.083386	1.0832785
			7	3.7682	3.768235	3.7645541
1	1	0	0.7	1.0500	1.049986	1.0499175
			7	3.7164	3.716467	3.7136611
1	1	1	0.7	-------	0.708645	0.7086420
			1	-------	0.867918	0.8678333
			3	-------	1.608920	1.6081314
			7	-------	2.561119	2.5596874

Effects of unsteady parameter (A), Magnetic parameter (M), Radiation parameter (R), Prandtl number (Pr), Eckert number (Ec) and Heat source parameter (γ) on Skin friction coefficient $f''(0)$ and Nusselt number $-\theta'(0)$ are shown in Table-2. It is observed that as the magnetic parameter M increases, the magnitude of both the skin friction coefficient $f''(0)$ and Nusselt number $-\theta'(0)$ decrease. It is found that as the unsteadiness parameter (A) increases, the magnitude of the skin friction coefficient $f''(0)$ decreases, where as the magnitude of the Nusselt number increases. It is noticed that as the radiation parameter (R) or Eckert number (Ec) increases, the magnitude of the Nusselt number $-\theta'(0)$ decreases. It is observed that as the Prandtl number (Pr) increases, the magnitude of the Nusselt number $-\theta'(0)$ increases. It is found that as the heat source parameter (γ) increases, the magnitude of the Nusselt number $-\theta'(0)$ decreases.

Table 2: Numerical values of the skin friction and Nusselt number for various values of A, M, R, Pr, Ec, γ

A	**M**	**R**	**Pr**	**Ec**	γ	$-\boldsymbol{\theta}'(\mathbf{0})$	$\boldsymbol{f}''(\mathbf{0})$
0.5	0.5	0.5	0.72	0.1	0.1	0.662541	-1.365633
				0.5		0.558067	-1.365633
				1.0		0.427475	-1.365633
					0.2	0.612222	-1.365633
					0.3	0.552430	-1.365633
			1.0	0.1	0.1	0.807784	-1.365633
			3.0			1.532200	-1.365633
	1.0		0.72	0.1	0.1	0.636583	-1.538442
	1.5					0.615585	-1.693472
1.0	0.5					0.800362	-1.497214
1.5						0.911575	-1.619752
0.5		1.0				0.556132	-1.365633
		1.5				0.485385	-1.365633

Fig.1 and Fig.2 depict temperature profiles for various values of Prandtl number (Pr) and Unsteadiness parameter (A). It is noticed that an increase in (Pr) results a decrease of the thermal boundary layer thickness and in general lowers average temperature within the boundary layer. The reason is that smaller values of (Pr) are equivalent to increasing the thermal conductivities, and heat is able to diffuse away from the heated plate more rapidly than for higher values of (Pr). Hence in the case of smaller Prandtl numbers as the boundary layers is thicker and the rate of heat transfer is reduced. Both figures show that the thermal boundary layer thickness decrease as Pr or A increase with increasing temperature gradient at the surface. Thus, the heat transfer rate at the surface increases with increasing values of Pr or A.

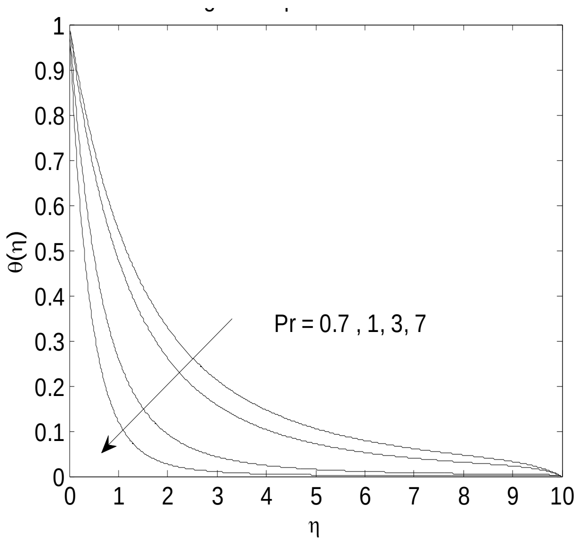

Fig.1. Temperature profiles for different values of (Pr)

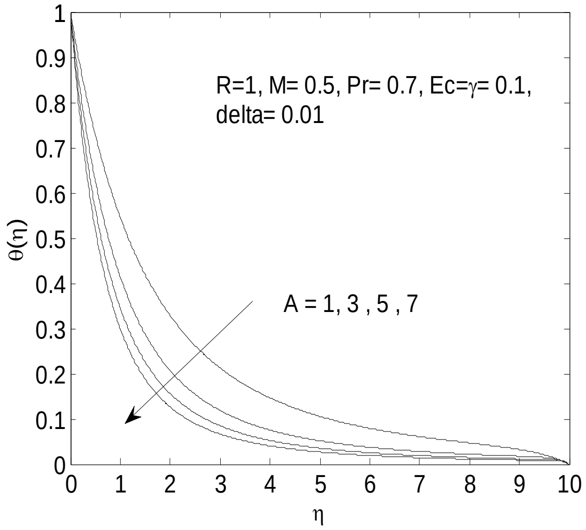

Fig.2. Temperature profiles for different values of (A)

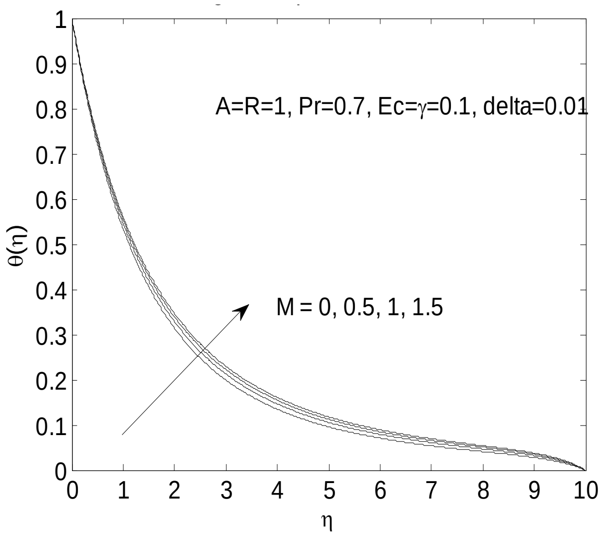

Fig.3 Temperature profiles for different values of (M)

The temperature profile for various values of magnetic parameter M, radiation parameter R, Eckert number ***L***, heat source parameter are presented in Fig-3, 4, 5 and 6 respectively. Figures -3 and 8 show effects of M on the temperature and velocity profiles, respectively. As expected, the temperature θ(η) and the velocity ***f'***. profiles increase with the increase in M. When M increases, it will also increase the Lorentz force which opposes the flow and leads to enhance deceleration of the velocity profiles. Fig-4 shows the effect of thermal radiation R on temperature. As R increases the temperature profile also increases. The radiation parameter R is responsible to thickening the thermal boundary. This enables the fluid to release the heat energy from the flow region and causes the system to cool. This is true because the Rosseland approximation results in an increase in temperature. Fig-5 shows the effect of viscous dissipation parameter Ec on temperature profile. The Eckert number Ec expresses the relationship between the kinetic energy in the flow and the enthalpy. It embodies the conversion of Kinetic energy into internal energy by work done against the viscous fluid stresses. Greater viscous dissipative heat causes a rise in the temperature profile. we notice that an increase in the Eckert number Ec is to increase the temperature distribution. This is in conformity with the fact that the energy stored in the fluid region as a consequence of dissipation due to viscosity and elastic deformation. Effect of A and M on the velocity profiles are shown in Fig. 7 and Fig. 8 respectively. The momentum boundary layer thickness decreases with M or A, hence, induces an increase in the absolute value of the velocity gradient at the surface.

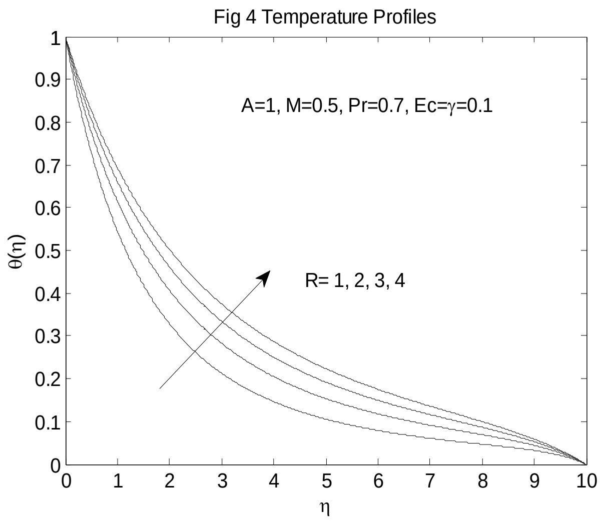

Fig.4 Temperature profiles for different values of (R)

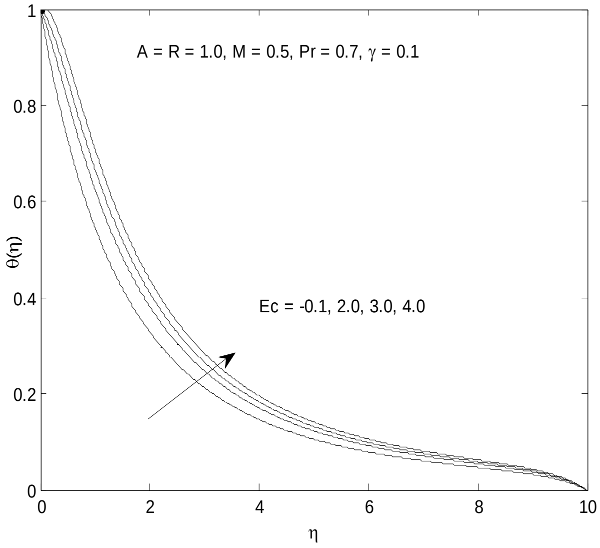

Fig.5 Temperature profiles for different values of (Ec)

International Journal of Electronics Communication and Computer Engineering
Volume 6, Issue 3, ISSN (Online): 2249–071X, ISSN (Print): 2278–4209

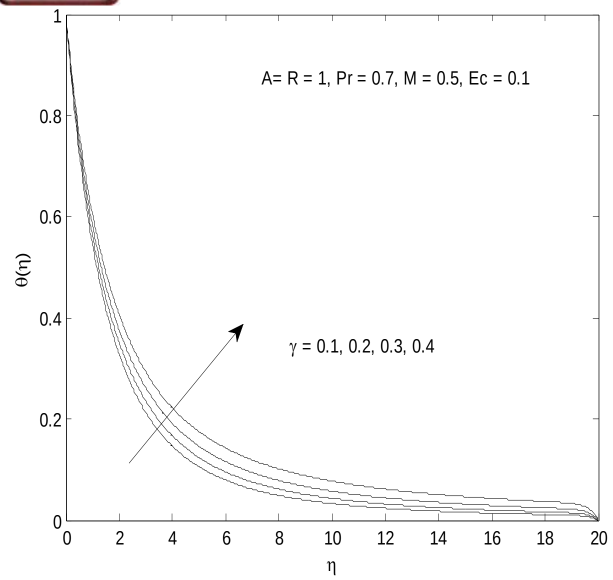

Fig.6 Temperature profiles for different values of (ϒ)

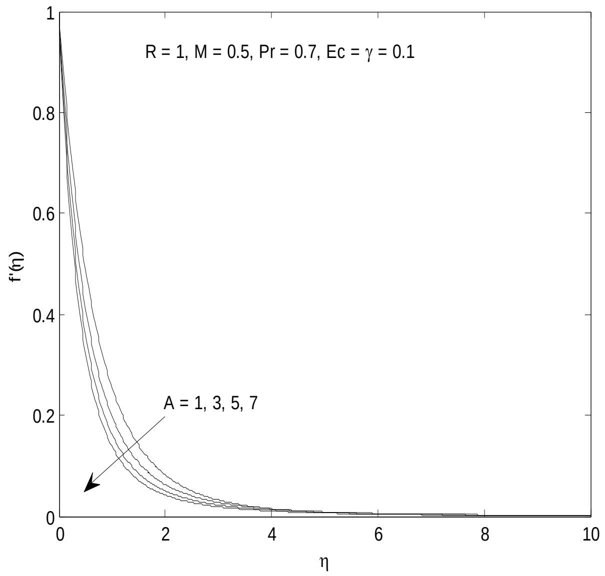

Fig.7 Temperature profiles for different values of (A)

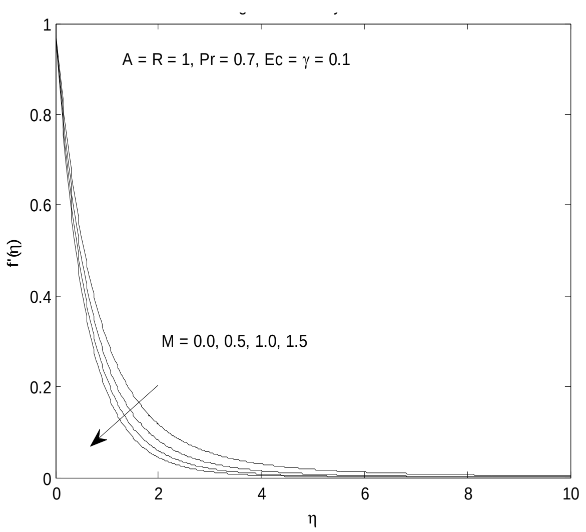

Fig.8 Temperature profiles for different values of (M)

V. Conclusion

In this work, the effect of radiation, Heat source/sink and viscous dissipation on unsteady MHD fluid flow over a stretching sheet has been analyzed in this paper. Effect of radiation parameter, magnetic parameter, unsteady parameter, Eckert number, Prandtl number and heat source parameters on velocity profile, temperature profile, skin friction coefficient and the rate of wall heat transfer characteristics are observed. From numerical results, the following conclusions may be drawn. The heat transfer rate at the surface increases with increasing values of unsteadiness parameter(A) and Prandtl number(Pr).

The heat transfer rate at the surface decreases with increase in radiation parameter(R), magnetic parameter(M), Eckert number(Ec) and heat source parameter (ϒ) .

Temperature distribution increases as the radiation parameter R or Eckert number Ec increases.

Velocity profiles decreases with increase in magnetic parameter (M) or unsteadiness parameter (A). As the magnetic parameter (M) increases both the magnitude of the skin friction coefficient $f''(0)$ and Nusselt number $-\theta'(0)$ decreases.

As the unsteadiness parameter increases, the magnitude of the skin friction coefficient decreases whereas the magnitude of the Nusselt number increases. As the radiation parameter R or Eckert number Ec increases the magnitude of the Nusselt number $-\theta'(0)$ decreases. As the Prandtl number Pr increases, the magnitude of the Nusselt number $-\theta'(0)$ increases. As the heat source parameter γ increases, the magnitude of the Nusselt number $-\theta'(0)$ decreases.

References

[1] Salleh M.Z., Nazar R. & Pop I. (2010), Boundary layer flow and heat transfer over a stretching sheet with Newtonian heating. Journal of the Taiwan Institute of Chemical Engineers, 41, pp. 651-655.

[2] Abbas, Z, & Hayat, T. (2008), Radiation effects on MHD Flow in a porous space. International Journal of Heat and Mass transfer, 51, pp. 1024-1033.

[3] Ghaly, A.Y. (2002), Radiation effects on a certain MHD free convection flow Chaos, soliton and Fractals, 13, pp. 1843-1850.

[4] Subhashini, S.V., Samuel,N.,& Pop, I. (2011), Effects of Buoyancy assisting and opposing flows on mixed convection boundary layer flow over a permeable vertical surface. International Communications in Heat and Mass Transfer, 38, pp. 499-503.

[5] Raptis, A. (2004), Effect of thermal radiation on MHD flow. Applied Mathematics and Computation, 153, pp. 645-649.

[6] Chaim, T.C. (1995), Hydro magnetic flow over a surface stretching with a power-law velocity. International Journal Engineering Sciences, 33(3), pp. 429-435.

[7] Chen, C.H., (2010), On the analytic solution of MHD flow and heat transfer for two types of viscoelastic fluid over a stretching sheet with energy dissipation, internal heat source and thermal radiation. International Journal of Heat and Mass Transfer, 53, pp. 4264-4273.

[8] Magyari, E. & Keller, B. (200), Exact solutions for self simila0r boundary layer flows induced by permeable stretching walls. Eur.J.Mech, B-fluids, 19, pp. 109-122.

[9] Fang, T.Zhang, J.,& Zhong, Y. (2012). Boundary layer flow over a stretching sheet with variable thickness. Applied mathematics and Computation, 218, pp, 7241-7252.

[10] Liao S. (2010), A new branch of solutions of boundary layer flows over an impermeable stretched plate. International Journal of Heat and Mass Transfer, 48, pp. 2529-2539.
[11] Sakiadis, B.C. (1961), Boundary layer behavior on continuous solid surfaces. I,Boundary layer equations for two dimensional and asymmetric flow , AICHE, J, 7, pp. 26-28.
[12] Elbashbeshy, E.M.A., & Bazid, M.A.A. (2004), Heat transfer over an unsteady stretching surface. Heat and Mass Transfer, 41, pp. 1-4.
[13] E.I-Aziz, M.A., (2010), Unsteady fluid and heat flow induced by a stretching sheet with mass trans chemical reaction. Chemical Engineering Communication, 197, pp. 1261-1272.
[14] Ishak. A. Nazar, R, & Pop, I (2009), Boundary layer flow and heat transfer over an unsteady stretching vertical surface, Mechanics, 44, pp. 369-375.
[15] Bachok, N., Ishak, A.,& Nazar, R,(2011), Flow and heat transfer over an unsteady stretching sheet in a micropolar fluid. Mechanics, 46, pp. 935-942.
[16] Liu, I.C., & Anderson, H.J.,(2008), Heat transfer in a liquid on an unsteady stretching sheet. International Journal of Thermal Sciences, 47, pp. 766-772.
[17] Ishak, A. (2010), Unsteady MHD flow and heat transfer over a stretching plate. Journal of Applied Sciences, 10, pp. 2127-2131.
[18] Zanariah Mohd Yusof, Siti Khuzaimah Solid, Ahmad Sukri Abd Aziz, and Seripah Awang Kechil, (2012), Radiation Effect on Unsteady MHD flow over a stretching sheet, World Academy of Science, Engineering and Technology, Vol.6, 2012-12-27.

AUTHOR'S PROFILE

V. Dhanalaxmi
was born in Andhra Pradesh, India in 1966. She received M.Sc degree from R.E.C, Warangal, A.P. India, M.Phil, M.Ed degree from Osmania University, Hyderabad, A.P.India, M.Tech in Computer Science from JNTU, Hyderabad, A.P. India.

Since May 2007, Whe has been an Assistant Professor at the Department of Mathematics, Osmania University, Hyderabad, A.P., India. She has been working on viscoelastic fluid flows towards Ph.D programme in Applied Mathematics.