International Journal of Electronics Communication and Computer Engineering
Volume 5, Issue 6, ISSN (Online): 2249–071X, ISSN (Print): 2278–4209

Study on Calculation Method of Minimum Non-Zero Total Float in CPM Network

WANG Jing-min, GUAN Xiao-yan

***Abstract* – Recently, it's universal phenomenon that large engineering projects like environmental protection projects are over budget and late. It puts plan management workers into trouble and wastes social resource largely. As a management way in modern project management, the minimum non-zero total float in CPM network plays an important role in time management and cost management. Therefore, this paper expounds time parameters and relationships between float time and the length of path firstly. And then this paper puts forward three kinds of calculation methods, which are calculating the total float of all the non-critical process, calculating the minimum non-zero float time of node and calculation method on the basis of minimum non-zero total float process distribution theorem. The second calculation method of minimum non-zero total float is the biggest innovation of this paper. This method reduces amount of calculation and improves the computational efficiency greatly and proves to be practical.**

***Keywords* – Project Management, CPM Network, Float Time, Non-Zero Total Float, Non-Critical Process.**

I. Introduction

In order to strengthen construction management, the United States Dupont Chemical Company put forward the critical path method (CPM) in 1956[1]. Being able to use programmed methods to calculate float time of each process and find the main contradiction of modern project effectively, it provided reliable data for decision-making managers and has been widely applied in the project management in countries around the world.

Afterwards, Battersby[2], Tomas[3], Elmaghraby[4] and other scholars put forward total float, safe float, free float , node float and other useful concepts, which makes the study and application of CPM vary widely. We can find out critical path and the key working procedure by using CPM[5]&[6], which are the main contradictions of modern project that decision-making managers need to consider carefully.

However, when the difference between secondary critical path and critical path is narrow, the secondary critical path will translate into critical path directly. Visibly, secondary critical path[7] is related to the safety of the project schedule management and it determines the time limit for a project of the biggest amount of effective compression. As a result, in order to reduce the risk of project, managers also need to pay attention to the secondary critical path. Because minimum non-zero total float in CPM network is equal to the difference between the length of secondary critical path and critical path. Thus, calculation of minimum non-zero total float is one of the ways to find out secondary critical path and it plays an important role in time management and cost management.

However, there are few literatures about calculation methods of minimum non-zero total float or the calculation methods are complex. According to the defects of the existing literature, this paper puts forward three kinds of calculation methods and the second one of them is the biggest innovation of this paper. This method reduces amount of calculation and improves the computational efficiency greatly and proves to be practical. So the research of this paper has important meaning in learning value and practice application.

II. Time Parameters used Commonly in CPM Network

According to literatures [8] and [10], the time parameters used commonly in CPM network are as follows.

A. The earliest time parameters

The earliest start time of process (ij) is denoted as ES_{ij}; the earliest finish time of process (ij) is denoted as EF_{ij}; the duration of process (ij) is denoted as T_{ij}; the earliest start time of node (i) is denoted as ES_i .The relations between them are as follows.

ES_{ij} is equal to the maximum earliest end time of immediate front process , that is

$$ES_{ij}=\max\left\{EF_{k_1},EF_{k_2},\cdots,EF_{k_n}\right\};\ EF_{ij}=ES_{ij}+T_{ij};$$

$$ES_{ij}=ES_i.$$

B. The latest time parameters

The latest start time of process (ij) is denoted as LS_{ij}; the latest finish time of process (ij) is denoted as LF_{ij}; the latest finish time of node (j) is denoted as LF_j .The relations between them are as follows.

LF_{ij} is equal to minimum latest start time that immediately after the process,
that is

$$LF_{ij}=\min\left\{LS_{jr_1},LS_{jr_2},...,LS_{jr_n}\right\};\ LS_{ij}=LF_{ij}-T_{ij};$$

$$LF_{ij}=LF_j.$$

C. The float time parameters

1) Total float (TF_{ij}) of the process (ij)

Without affecting the total duration of a project, the maximum float time that can be used by the process (ij) is call total float of the process (ij). It is denoted as TF_{ij}, and $TF_{ij}=LS_{ij}-ES_{ij}=LF_{ij}-EF_{ij}$.

International Journal of Electronics Communication and Computer Engineering
Volume 5, Issue 6, ISSN (Online): 2249–071X, ISSN (Print): 2278–4209

2) Float time of the node (TF_i)

The difference between the latest finish time (LF_i) of node (i) and the earliest start time (ES_i) of it is called float time of the node. It is denoted as TF_i and $TF_i = LF_i$ - ES_i.

3) Interference floats time of the later process

The maximum amount the process (ij) can make the float time of immediate later process reduce when it is using float time is equal to the float time (TF_j) of the end node (j). Therefore, interference float time of the later process is denoted as TF_j.

4) Free float (FF_{ij}) of the process (ij)

Without affecting the float time of the immediate later process, the maximum amount of the float time that the process (ij) can use is called free float of the process (ij). It is denoted as FF_{ij}, and $FF_{ij} = TF_{ij}$ - TF_j.

5) Interference floats time of the front process

The maximum amount the process (ij) can make the float time of immediate front process reduce when it is using float time is equal to the float time (TF_i) of the start node (i). Therefore, interference float time of the front process is denoted as TF_i.

6) Safe float (SF_{ij}) of the process (ij)

Without affecting the float time of the immediate front process, the maximum amount of the float time that the process (ij) can use is called safe float of the process (ij). It is denoted as SF_{ij}, and $SF_{ij} = TF_{ij}$ - TF_i.

D. Node type of the important time parameters[5]

$$TF_{ij} = LF_j - ES_i - T_{ij}$$
$$FF_{ij} = ES_j - ES_i - T_{ij}$$
$$SF_{ij} = LF_j - LF_i - T_{ij} \quad (1)$$

II. Relationships Between Float Time and the Length of Path

According to the literature [8] to [11], the professor Qi Jianxun studied on the relationships between float time and length of the path. The relationships this paper will use are as follows.

(1) The formula for the length of path between any node (i) and source point (s) is as follows.

$$ES_i - \bar{\mu}_{s-i} = \sum_{(uv)\in\mu_{s-i}} FF_{uv} \quad (2)$$

In the formula, when $\sum_{(uv)\in\mu_{s-i}} FF_{uv}$ decreases, $\bar{\mu}_{s-i}$ will increase. When $\sum_{(uv)\in\mu_{s-i}} FF_{uv}$ =0, $\bar{\mu}_{s-i}$ reaches maximum; then we can get $\bar{\mu}_{s-i} = \bar{\mu}^{\nabla}_{s-i} = ES_i$. It can be seen from the above analysis that all the free float of the process on the longest path is zero.

(2) The total amount of the safe float of every process on any path $(\mu_{s-e} = \mu)$ from source point (s) to end point (e) is equal to the total amount of its free float and the amount is equal to the difference between the length of critical path ($\bar{\mu}^{\nabla}$) and the length of this path ($\bar{\mu}$). The formula is as follows.

$$\sum_{(uv)\in\mu} SF_{uv} = \sum_{(uv)\in\mu} FF_{uv} = \bar{\mu}^{\nabla} - \bar{\mu} \quad (3)$$

(3) The total amount of the safe float of every process on the path (μ_{i-e}) from any node (i) to end point (e) is equal to the difference between the length of critical path ($\bar{\mu}^{\nabla}$) and the sum of the latest finish time of the node and the length of this path (μ_{i-e}).The formula is as follows.

$$\sum_{(uv)\in\mu_{i-e}} SF_{uv} = \bar{\mu}^{\nabla} - (LF_i + \bar{\mu}_{i-e}) \quad (4)$$

In the formula, when $\sum_{(uv)\in\mu_{i-e}} SF_{uv}$ decreases, $\bar{\mu}_{i-e}$ will increase. When $\sum_{(uv)\in\mu_{i-e}} SF_{uv}$ =0, $\bar{\mu}_{i-e}$ reaches maximum and it is equal to ($\bar{\mu}^{\nabla} - LF_i$).

(4) Total float (TF_{ij}) of any process (ij) is equal to the difference $\left(\bar{\mu}^{\nabla} - \bar{\mu}^{\nabla}_{s-i}\right)$ between the length of critical path ($\bar{\mu}^{\nabla}$) and the length of the longest path that goes through this process (ij). The formula is as follows.

$$TF_{ij} = \left(\bar{\mu}^{\nabla} - \bar{\mu}^{\nabla}_{s-i}\right) \quad (5)$$

(5) The relationship between float time of the node (TF_i) and the length of path

The formula is as follows.

$$TF_i = \bar{\mu}^{\nabla} - \bar{\mu}^{\nabla}_i \quad (6)$$

In the formula, $\bar{\mu}^{\nabla}$ means the length of critical path and $\bar{\mu}^{\nabla}_i$ means the length of the longest path that goes through this node (i).

III. Calculation Method of Minimum Non-zero Total Float In CPM Network

This paper puts forward three kinds of calculation methods, which are calculating the total float of all the non-critical process, calculating the minimum non-zero float time of node and calculation method on the basis of minimum non-zero total float process distribution theorem. The solving process and the advantages and disadvantages of each method are as follows, and calculation example is given to illustrate the correctness of each method.

E. Calculate the total float of all the non-critical process

According to fig.1 and the above formulas of the time parameters, this paper calculates the earliest start time and the latest finish time of every node. Then, we can get the critical path is $(1)\to(3)\to(6)\to(8)\to(11)\to(14)$. On the basis of formula (1), this paper calculates the total float

International Journal of Electronics Communication and Computer Engineering
Volume 5, Issue 6, ISSN (Online): 2249–071X, ISSN (Print): 2278–4209

of every non-critical process. Therefore, the minimum non-zero total float is six and the process is $(8)\rightarrow(12)$. This calculation method of minimum non-zero total float is simple and direct, while the processes and nodes of project network are too many realistically. Since the calculation of this method is complex and it is liable to make mistakes, so this calculation method of minimum non-zero total float is not practical.

In the meantime, there are few literatures about simple and fast calculation method of minimum non-zero total float. However, the secondary critical path that is found quickly by calculating the minimum non-zero total float is used widely in engineering projects, for example, construction optimum management [12].

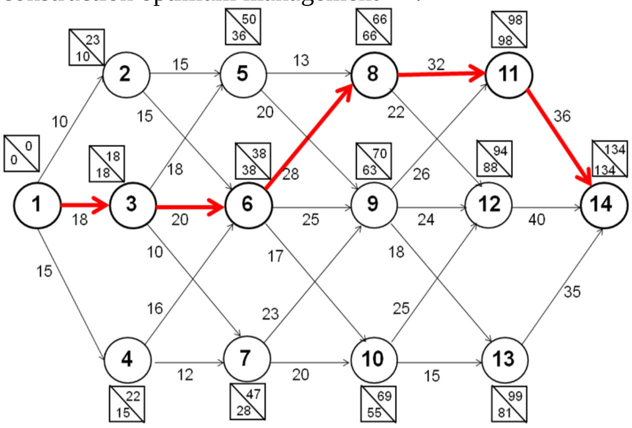

Fig.1. CPM network

F. Calculate the minimum non-zero float time of node

On the basis of the relationships between float time and the length of path, we can deduce that the minimum non-zero total float is equal to the difference between the length ($\bar{\mu}^{\nabla}$) of critical path and the length ($\bar{\mu}^{[1]}$) of secondary critical path. The formula is as follows.

$$\bar{\mu}^{\nabla}-\bar{\mu}^{[1]}=\min\left\{TF_{ij}/(ij)\in G\right\} \quad (7)$$

It can be seen from the formula that if we find the secondary critical path we can calculate the minimum non-zero total float. According to formula (6), the secondary critical path is the longest path that goes through the node whose non-zero float time is minimum. What is more, the difference length between the longest path and the critical path is equal to float time of node.

Therefore, this paper draws that the minimum non-zero float time of node is equal to the minimum non-zero total float of process. The formula is as follows.

$$\min\left\{TF_i/(i)\in G\right\}=\min\left\{TF_{ij}/(ij)\in G\right\} \quad (8)$$

According to the definition of free float of process, this paper concludes the minimum non-zero total float process is the immediate front process that is on the longest path that goes through the node whose non-zero floats time is minimum. This calculation method of minimum non-zero total float and conclusion is the biggest innovation of this paper.

On the basis of formula $TF_i = LF_i - ES_i$, we compute the float time of non-critical node. Then we get that the minimum non-zero float time of node is six and the node is (12). Therefore, the minimum non-zero total float is six and the result of this method is same with the first method. However, compared with other methods, this method reduce amount of calculation and improve the computational efficiency and effect greatly.

In order to conduct security management of the project's progress, secondary critical path can be found easily. According to relationships between float time and the length of path, it can be seen that the length of the longest path from node (12) to start point is equal to the earliest start time of this node, that is $ES_{12}=88$. The free float of all the process on the way of this longest path is zero and this longest path is

$(1)\rightarrow(3)\rightarrow(6)\rightarrow(8)\rightarrow(12)$ which is shown in fig.1. The length of the longest path from node (12) to end point is equal to the difference between the length of critical path and the latest finish time of this node, that is $\bar{\mu}^{\nabla}-LF_{12}$=40. The safe float of all the process on the way of this longest path is zero and this longest path is $(12)\rightarrow(14)$ which is shown in fig.1. Therefore, the secondary critical path is

$(1)\rightarrow(3)\rightarrow(6)\rightarrow(8)\rightarrow(12)\rightarrow(14)$ and the length of it is128 (88+40). What is more, the minimum non-zero total float process is the immediate front process of node (12), which is $(8)\rightarrow(12)$ shown in fig.1. According to formula (7), it can be seen the minimum non-zero total float is six as well. Thus, it confirms the correctness of the conclusion that the minimum non-zero float time of node is equal to the minimum non-zero total float.

G. Calculation method on the basis of minimum non-zero total float process distribution theorem

The literatures [8] and [10] put forward minimum non-zero total float process distribution theorem: In CPM network, all minimum total float non-critical processes distribute on the longest path that goes through those minimum total float non-critical processes which are connected to the key nodes and their total float are all the same.

On the basis of the distribution theorem, the total float of minimum total floats non-critical processes which are connected to the key nodes is the minimum non-zero total float in the network. Therefore, in immediate front processes and immediate latter processes of critical nodes we can find out minimum total floats non-critical processes. And the longest path that goes through this minimum total float non-critical process is the secondary critical path in the network. In order to find out minimum total floats non-critical processes, this paper simplifies the network of figure 1. Simplified network is shown in fig.2 (It's non-standard network, only for calculating conveniently).

According to fig.2 and the above formulas of the time parameters, this paper calculates the total float of immediate front processes and immediate latter processes of every critical node. Then, we can get the minimum total float is six ($TF_{8-12}=6$), that is the minimum non-zero total float is six in the network. This calculation method of minimum non-zero total float is simple and the amount of calculation is small. When the network is too complex and the processes are too many, the advantage of this method is more prominent.

The longest path that goes through process $(8)\rightarrow(12)$ is the secondary critical path in the network. The approach to get the path is same with the second method, so it is no longer described here. After calculation, this paper obtains $\mu^{\nabla}_{8\text{-}12}=(12)\rightarrow(14)$, $\mu^{\nabla}_{1\text{-}8}=(1)\rightarrow(3)\rightarrow(6)\rightarrow(8)$ and [1]$=\mu^{\nabla}_{8,12}=(1)\rightarrow(3)\rightarrow(6)\rightarrow(8)\rightarrow(12)\rightarrow(14)$.

We calculate the length of secondary critical path is 128 further. According to formula (7), it can be seen the minimum non-zero total float is six as well. Thus, it confirms the correctness of this calculation method of minimum non-zero total float.

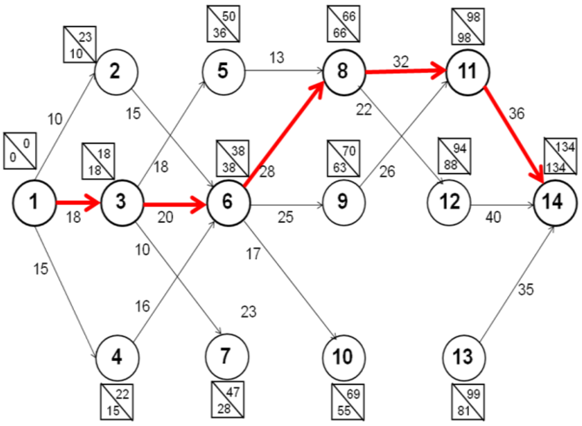

Fig.2. Simplified network

IV. Conclusions

In order to calculate the minimum non-zero total float, this paper puts forward three kinds of calculation methods, which are calculating the total float of all the non-critical process, calculating the minimum non-zero float time of node and calculation method on the basis of minimum non-zero total float process distribution theorem. The second calculation method of minimum non-zero total float is the biggest innovation of this paper. This method conclude that the minimum non-zero float time of node is equal to the minimum non-zero total float of process and minimum non-zero total float process is the immediate front process that is on the longest path goes through the node whose non-zero floats time is minimum. This method reduces amount of calculation and improves the computational efficiency greatly and proves to be practical.

Acknowledgements

This research is sponsored by:
(1) The Fundamental Research Funds for the Central Universities (2014XS86).
(2) Funding of the Energy Economic Development Strategy Research Base Project at Hebei Soft Science Research Base.
(3) The special funds of basic research activities of central universities in North China Electric Power University (12MS137).

References

[1] Elmaghraby S E. On criticality and sentivity in process networks [J].European Journal of Operational Research, 2000,127: 220-238.
[2] Battersby A. Network analysis for planning and scheduling [M].New York, St Matin's Press, 1970.
[3] Warren T. Four floats measures for critical path scheduling [J]. Journal of Industrial Engineering 1969,10:19-23.
[4] Elmaghraby S E. Process networks project planning and control by network models [J]. New York, John Wiley& Sons Inc,1977,18-22.
[5] Li Xingmei. The theory and method of the super-large type network simplification on time-cost tradeoff problem [D]. Bei Jing: North China Electric Power University, 2008.
[6] Sun Dedong. Research on time-cost tradeoff problem based on properties of network [D]. Bei Jing: North China Electric Power University, 2013.
[7] Jia Zhengyuan, Gong Lihua. Study on optimization of time-cost based on particle swarm optimization with considering secondary critical path [J]. Technology Economics, 2008, 27(10):69-73.
[8] Qi Jianxun, Zhang Lihui ,Li Xingmei. Maneuvering time characteristics of network plan management theory and its application [M].Bei Jing, Science press,2009.
[9] Qi Jianxun. New theory of network planning optimization and techno-economic decision-making [M].Bei Jing, Science press,1997.
[10] Zhang Lihui ,Qi Jianxun. Method study on using total float looking for secondary critical path in CPM network [J].Operations Research and Management Science, 2008, 17(4):79-83.
[11] Zhang Lihui, Qi Jianxun. Properties of Node slack and applications in CPM networks [J]. Chinese Journal of Management Science,2008,16(5):128-133.
[12] Wang Haichao,Yang Guoxi. Applicability-studies of the secondary critical path planning in the construction optimum management [J]. Journal of Yanshan University,2000,24(1):40-42.

Author's Profile

WANG Jing-min
Brief Introduction: Doctoral tutor, School of Economics and Management, North China Electric Power University, The research direction: Information management and decision analysis, Management science and application, Project assessment and risk management.
Email:jingminwang126@126.com

GUAN Xiao-yan
Brief Introduction: Master graduate student, School of Economics and Management, North China Electric Power University, The research direction: Management science and application, Project assessment and risk management.
Email: hdguanxiaoyan@126.com