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Non-Lubricated Bevel Gear Train Optimization using Heuristic Genetic Approach

S.K. Rajesh Kanna
Professor, Rajalakshmi Institute of Technology, Chennai, India
Email: skrkanna@gmail.com

Jaisree
Student, Anna University, Chennai, India

***Abstract* – This research investigated the problem of optimizing bevel gear design in a non-lubricated condition to transmit power between the perpendicular shafts at high speed. The bevel gear design problems can be viewed as a kind of NP hard problem, with standardized parameter values and several additional constraints. In this research, heuristic genetic evolutionary approach has been adopted in two phases for solving this NP-hard design problem. The first phase consists in building good initial solutions followed by genetic operations. The second phase improves the obtained genetic solution by checking with the constraints and the standard values as per the design standards. Results obtained on real data sets are compared with the traditional method and found the results are at par. The special criteria taken for consideration are that there is no lubricating oil and noise should be least possible.**

***Keywords* – Bevel Gear, Constraints, Evolutionary Algorithm, Genetic Approach, Optimization.**

I. INTRODUCTION

In industries, power transmission means transmitting the rotational motion from the prime mover to various parts of the machine tools to do some useful work from a set of mechanisms, with the objective of minimizing the power loss and obtaining positive transmission. The positive power transmissions can be achieved by using gears. Also these gears are used to amplify the speed and torques as required for the smooth operation of machine tools. Sometimes the rotational power needs to be transmitted between the inclined shafts. It often takes between the perpendicular shafts to transmit power at right angles through bevel gear pair.

In machine shops, special purpose machines require power transmission at right angles from the prime mover to the destination. Each machine has its own requirements such as speed of rotation, torque, load carrying capacity, etc. So for the given set of conditions with limited capacities, the goal is to design the bevel gear pair by satisfying the constraints and the design standards. As the weight and size of the bevel gears play a major role in transmission losses, it became essential to design the optimal gear as efficiently as possible. Bevel gears either at the design stage or at the manufacturing stage are usually standardized. Thereby a necessity arises to design the bevel gear based on the manufacturing criteria and standardization to avoid the lateral modifications. Also while evaluating the new design or finding the compatible dimensions based on the application, it required to check with the stress constraints. Thus the problem became non-linear and more complex to solve.

Genetic Algorithm (GA) is one among the evolutionary algorithm and currently developing computerized tools to perform these complex and combinatorial operations in less time. In order to reduce the complexity of the task, it have been split into two sub-problems: Genetic programming is the first stage used to identify the optimal solution from the larger solution space and the bevel gear design by considering the constraints in the second stage. The purpose of this research is to develop a software module for solving the above mentioned problems using genetic approach.

II. LITERATURE SURVEY

The relation between the line action and the sliding velocity had been derived and proved that the sliding velocity will be zero at pitch line. The sliding velocity increases if the conjugated teeth of contact line, rotates away from pitch line on either directions [1]. In general, sliding of gear teeth's under load condition will results in friction and the sliding friction will results in heat energy. The amount of heat generation will be proportional to the contact pressure and sliding velocity. The equation for calculating the hear generation and variations in temperature along the line of action are calculated by Blok's contact temperature theory. This research work considers the non-lubricated condition, so the change and variations in temperature need to be calculated for optimal design.

The revolution in the digital computing steers lot of researches towards developing gear design using computer programs. Thus it helps the researchers to build interactive computer modules to design a required type of gear under the given set of specified working conditions [13]. The researchers also portrayed computer-aided method for design and to check for the safe conditions of the automated gearbox design [12]. Similarly an interactive software module had also been formulated [10] to optimize a three-stage spur gear reduction unit. Some of the researchers also used the concept of expert system to design the gears suitable for manufacturing and that can be used in gearbox units [4]. With reference to the above literatures, this research also focused in developing an interactive programming module for bevel gear based on the given user defined conditions.

The application of genetic algorithm and the usage of genetic algorithm to optimize the conflicting parameters were introduced by John Holland [9]. Multi-objective optimization of gear box with different speeds was also solved using genetic algorithm [6, 17]. The particle swarm approaches also used to resolve different gear designs

[14]. In advance to this, particle swarm optimization (PSO) along with simulated annealing (SA) was also used to optimize the gears with the main objective of minimizing the weight of a spur gear train [16]. GA had also been utilized to minimize the volume of two-stage helical gear train by reducing the gear size [7]. Evolutionary algorithm had also been used to generate an automated optimal design for two-stage helical gear reducer [18]. So this research uses GA to optimize the bevel gear parameters in a non-lubricated environment. In this research the gear standards are considered based on the American Gear Manufacturers Association (AGMA) procedures [3, 11].

III. The GA-Bevel Gear Problem

The GA module have been developed in Visual basic software module and simulated in 3.10 GHz digital machine. The various stages of the developed module are explained as follows.

A. Data

The basic information for this problem is composed of two sets: primary parameter set and secondary parameter set. Primary parameters considered are power transmitting capability, number of teeth's in driver wheel, thickness of gear and the module. Secondary parameters considered are Coefficient of friction (μ), Thermal conductivity (α), Density (ρ), Specific heat (θ), Material of gear and pinion (m_i), Gear ratio (i), Young's modulus (E), Pressure Angle (α), Tool dedendum coefficient (h_f), Minimum root clearance (c), Allowable bending $[\sigma_b]_{al}$ and Compressive stress $[\sigma_c]_{al}$.

B. Constraints

Constraints are considered as special items because they are varying from application to application. For safety reasons and ease in manufacturing, they must be satisfied by the gear design parameters [11]. Constraints exist both in static and dynamic nature. The various constraints considered in this work are as follows.

Stress Constraints

During engagements of gear, the top and bottom surface of the gear teeth will subject to bending and compressive stresses respectively. The conditions for the bending and compressive stresses are given in the Equation (1) to (5).

$$\sigma_b \le [\sigma_b]_{al} \quad (1)$$

Whereas, '$[\sigma_b]_{al}$' = Allowable bending stress in N/mm^2 – Secondary parameter.

'σ_b' = Induced bending stress in N/mm^2 and is given in Equation (2).

$$\sigma_b = \frac{\left(R\sqrt{i^\wedge 2+1}\right)}{(R-0.5\,b)^\wedge 2 * (b\, m_t\, y_v) * \cos\alpha} \times [M_t] \quad (2)$$

Where, 'R'= cone distance = 0.5 x m_t x Z_1 x $\sqrt{(i^2+1)}$

'y' = Form factor

'$[M_t]$' = Design twisting moment in Nmm, and is given in Equation (3).

$$[Mt] = M_t \times k \times k_d \quad (3)$$

'M_t' = Normal twisting moment by Pinion in Nmm

'K' & 'k_d' are Load Concentration & Dynamic load factor

$$\sigma_c \le [\sigma_c]_{al} \quad (4)$$

Where, '$[\sigma_c]_{al}$' = Allowable crushing stress in N/mm^2.

'σ_c' = Induced crushing stress in N/mm^2 and is given in Equation (5).

$$\sigma_c = \left(\frac{0.72}{R-0.5\,b}\right) \times \sqrt{\left[\left(\frac{(i^\wedge 2+1)^\wedge 3}{ib}\right) \times E \times [M_t]\right]} \quad (5)$$

Where, 'E'= Young's Modulus of the gear in N/mm^2

Module Constraint

Module plays a major role in the gear design and in manufacturing. The condition for module is given in the Equation (6).

$$m \ge m_{avg} \quad (6)$$

'm_{avg}' is the minimum module and is given in the Equation (7).

$$M_{avg} = 1.26 \times \sqrt[3]{\frac{[M_t]}{(y_v\, \sigma_b\, \Psi_m Z_1)}} \quad (7)$$

'Ψ_m' is the ratio between the gear pair thickness and module.

The obtained module value should be standardized to the 'R' series values which are stored in the database.

Standardization Constraint

Due to globalization and for easy replacement of parts, standardization became an essential parameter for every engineering design. Standardization is characterized by its values which makes ease in manufacturing and assembly [8]. Some of the standard values are the number of teeth, module, speed ratio, etc.

C. Bevel Gear Design

Bevel gears are used to transmit power between the inclined shafts. For experimentation purpose it had been considered that the shafts are at right angles. Its capacity depends on the type of load acting, speed and torque transmitted. The design procedure for the bevel gear is given as follows in the Equations 8 to 14

Gear Ratio (i)

The gear ratio is the ratio of speed of the pinion and the gear wheel. The formula for calculating gear ratio is given in the Equation (8).

$$i = \frac{Z_2}{Z_1} \text{ (or) } \frac{d_2}{d_1} = \frac{N_1}{N_2} \quad (8)$$

Cone Distance between pinion and gear (R)

The size of the gear wheels decides the cone distance and the formula for calculating the cone distance is given in the Equation (9).

$$R = \psi_y * \sqrt{(i^\wedge 2+1)} * \sqrt[3]{\left[\left(\frac{0.72}{(\psi-0.5)*[\sigma_c]}\right)^2 \times \left(\frac{E[M_t]}{i}\right)\right]} \quad (9)$$

'Ψ_y' is the ratio between gear thickness to center distance.

Gear Surface Temperature

In this research, the optimization is done for the gears which are rotating without lubricating oil, so more heat energy will be developed on the gear surface. The surface temperature of the gear wheels should be kept within the allowable limits while optimizing the design, because the gear life and the lubrication depend mainly on the amount of heat generated. The maximum contact temperature is obtained by Equation (10).

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$$\theta_{B\,max} = \theta_M + \theta_{fl\,max} \quad (10)$$

Whereas, 'θ_M' is tooth temperature,
'$\theta_{fl\,max}$' is maximum flash temperature along the line-of-action, which is calculated by Blok's relation given in the Equation (11).

$$\theta_{fl} = 31.62\ K\ \mu_m\ (X_r\ W_n / \sqrt{(b_H)})\ x\ \{(|V_{r1} - V_{r2}|) / [(B_{M1}\sqrt{(V_{r1})}) - (B_{M2}\sqrt{(V_{r2})})]\} \quad (11)$$

Whereas, 'K' is Hertzian distribution of frictional heat = 0.8;
'μ_m' is the Mean coefficient of friction;
'X_r' is the Load sharing factor;
'W_n' is the Normal unit load;
'B_{M1}' & 'B_{M2}' is thermal contact coefficients of the pinion and wheel;
'V_{r1}' & 'V_{r2}' is the Rolling tangential velocities in m/s of the pinion and wheel;

Gear Noise Calculation

Noises in the gear trains are due to vibration and transmission error caused by change in tooth topology, shaft deflections and mesh stiffness variation [2]. Transmission error along the line of action is given in the Equation (12).

$$TE = Rb\ \{\delta 2 - (i\ \delta 1)\} \quad (12)$$

Whereas, 'R_b' is gear base radius;
'δ_1', 'δ_2' are the angular rotation of pinion and gear;

D. Genetic Operations

GA have their own fleet which is operates with random numbers. Therefore, GA and its related operators can be deal with the complex problems easily with common operations. Note that in this problem, along with bevel gear design, constraints, standardization, surface temperature, noise generated and the non-lubricating oil conditions also need to be considered. Moreover, human solutions are not always feasible since they do not always verify all the given constraints [5]. Indeed, in genetic operations, all the above mentioned real time data are ignored and those data need to be converted to the genetic chromosomal format using encoding process.

Encoding

Bevel gear design is depends on the application and load conditions. So the bevel gear parameters are limited by lower and upper bounds. So the limits considered for this research is given in the Table 1. In this research the real values are used as such and no heuristic used to encode and decode the user data to avoid complexity and the computational time.

Table 1: Primary Bevel Gear Parameters

Parameter	Value	Parameter	Value
Power	10 to 30 Kw	Tooth Thick	10 to 100 mm
Module	01 to 20 mm	No. of Teeth	08 to 60

Initial Population Generation

Initially with the use of random numbers, a great number of feasible and attractive solutions have to be generated and each solution called as genetic chromosomes or parents [15]. The number of parents has been set to 20 in each generation. The subset from those chromosomes is the parameters which are going to be optimized such as power, module, tooth thickness and number of teeth which occupies one to four digits, fifth to eight digits, ninth to twelfth digit and last two digits respectively. Figure 1 shows a sample set of three parents generated randomly. The generated parents have been allowed for crossover operation to inherit the best properties from the parents.

Parent 1 /1	**30412022180912,**
Parent 2 /1	*10060812121222,*
Parent 3 /1	15071223222414,

Fig.1. Randomly Generated Parents

Crossover

In the crossover, the parents consist of collection of four subsets. To each set Ei is associated a fitness function value f_i. Crossover is the process of interchanging these subsets among the parents and thereby changes in the fitness function values. The intersecting point has to be generated randomly. As this research uses the parent sorting methodology, the crossover also should be in a sequential order. The sample crossover operation is given in the Figure 2.

P. No.	Initial Parent	C. Parent	C. Site	Offspring
1 /1	*30412022180912*	2	2	*30412022*121222
2 /1	10060812121222	1		10060812*180912*
3 /1	15071223222414	4	3	15071223220628
4 /1	12271853240628	3		12271853242414

Fig.2. Crossover

This heuristic was meant for problems in which each item is unique i.e. the crossover should not be within the strings instead subsets.

Mutation

Mutation is the process of changing a subset from the solution to avoid stagnation and repetition at the local optimal points. Furthermore, there exist lower and upper bounds on each type of the subsets, so the random mutant should lie within the bounds. The sample mutation operation is given in the figure 3.

P. No.	Offspring	M. Parent	M. Site	M. Offspring
1 /1	30412022121222	0		30412022121222
2 /1	10060812180912	0		10060812180912
3 /1	15071223220628	0		15071223220628
4 /1	12271853242414	1	3-12	12271853122414

Fig.3. Mutation Operation

Objective function

The aim of the problem is to minimize the weight and size of the bevel gear, subject to constraints. Multi-objective fitness function formulated in this research for optimizing the bevel gear pair is given in the Equation (13).

$$\text{Max. } F(x) = \{[(F_1/\text{max. } F_1) + (F_2/\text{min. } F_2) + (F_3/\text{max. } F_3)]/3\} \quad (13)$$

Whereas, F_1 is the maximization function for power transmission and is given in the Equation (14).

$$F_1 = P \qquad \text{where, } P^{(L)} \le P \le P^{(U)} \quad (14)$$

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'P' is Power transmitting capacity and lies between lower and upper limit.

F_2 is the minimization function for calculating gear weight [19] and is given in the Equation (15).

$$F_2 = \{[\frac{\pi}{4} \times d_1^2 \times b \times \rho] + [\frac{\pi}{4} \times d_2^2 \times b \times \rho]\} \quad (15)$$

'd_1', 'd_2' = Pitch circle diameter of pinion and gear in mm
'b' = Thickness of pinion and gear in mm
'ρ' = Density of the material in kg/mm^3

F_3 is the maximization function for efficiency calculation and is given in the Equation (16).

$$F_3 = 100 - P_L \quad (16)$$

'P_L'= Power loss which is calculated by the Equation (17).

$$P_L = \frac{50f}{\text{Cos}\Phi} \times \frac{(H_s^2 + H_t^2)}{(H_s + H_t)} \quad (17)$$

'H_s'= Specific sliding velocity at start of approach action
'H_t' = Specific sliding velocity at end of recess action
'f' = Coefficient of friction
' Φ ' = Pressure angle in degrees

'H_s' and 'H_t' are calculated by the Equations (18) & (19) respectively.

$$H_t = \frac{(i+1)}{i} \times \sqrt{\left(\left[\frac{r_0}{r}\right]^2 - \cos^2\Phi\right)} - \sin\Phi \quad (18)$$

$$H_s = (i+1) \times \sqrt{\left(\left[\frac{R_0}{R}\right]^2 - \cos^2\Phi\right)} - \sin\Phi \quad (19)$$

Whereas,
'R'& 'R_o' = Pitch and Outside circle radius of gear in mm.
'r' & 'r_o' = Pitch and Outside circle radius of pinion in mm
R_0 = R + one addendum

One addendum for 20^o full depth involute system = one module = m.

$$r_o = r + m = \frac{d_1}{2} + m$$

$$R_o = R + m = \frac{d_2}{2} + m$$

For power transmitting capability and the gear efficiency, the generated pattern is proportional to the upper bound of that type. Similarly for the weight, the generated pattern is proportional to the lower bound of that type. The combined objective function value has to be normalized as it involves different sub-functions and the aim is to maximize the COF by satisfying the design constraints.

Genetic Constraints

This research adapted separate heuristic to tackle additional constraints in genetic approach and are given as follows.

- The heuristic selects at each iteration a set of parameters not yet chosen.
- Repetition in the subset also identified and eliminated to form best population.
- At every generation, unfeasible subsets have been eliminated.
- After each run, the solution is analyzed and the items that repeat more number of times with lower bound values are moved down in the list by adding negative penalty value.

Termination Condition

Computational experiments have shown that the best results are obtained when sorting the items in decreasing order of their fitness value. After the sorting phase, the chromosomes are checked for the termination conditions as given below.

- Maximum number of generation is 100.
  Sensitivity analysis show that 100 iterations is enough for good results and also it had been found that there was a not drastic improvement in the result with other values.
- All the chromosomes having same fitness function value.
  By reaching the global spike or the saturated limit, the fitness function values remains constant.
- Repeated iterations no longer produce better results.

As the number of different items in the solution space is small i.e. 18,72,000 possible solutions, generating more feasible patterns leads to several occurrences of the same one.

Once the enough optimal patterns have been generated, then the subset of that pattern should be converted into manufacturing design documents and drawings. The sample result obtained from a random simulation is given in the following section.

IV. Results and Discussion

Consider the problem in which the items given in Table 1 had been used for experimentation. In this example, there are two types of items such as noise and gear surface temperature generated due to non-lubrication also taken into account. Without calculating the surface temperature, stress values cannot be calculated and in turn gear dimensions cannot be finalized. So the task force associated with the design criteria, constraints and also the other factors like temperature, etc. Table 1 gives the lower and upper bounds for each parameter. Note that the global bounds are not necessarily the sum of the destination bounds, other constraints may also apply. The developed GA is very fast and thus can be run to five times and the best result obtained among the iterations have been consider as the global best value. Because of the limitations and bounds, it is impossible to obtain the top bound values for all the parameters. It was found that the performance has been at par. 8% reduction of gear size and 1 % increase in efficiency were achieved with the optimized bevel gear design.

Note that once the selection is done, it may happen that the set of selected entities can be used directly for the manufacturing, because all the parameters were undergone for standardization. The developed GA can allow the user to perform many simulations by varying all the bevel gear parameters and genetic parameters. A sample result obtained from the developed algorithm is given as follows. Gear pair had been made of 40 Ni 2 Cr 1 Mo 28 (Cr-Mo) series steel of density 8.836×10^{-6} kg/mm^3 and surface hardened up to 56 HRC with the young's modulus values as $2.15x10^5$ N/mm^2. Tooth flanks were ground to achieve AGMA grade A4, Coefficient of friction is 0.05, Thermal conductivity is 48 W/ (m K), Specific heat as 544 J/ (kg K), Gear ratio is 2, Tool dedendum coefficient as 1.2, Backlash coefficient as 0.048, Minimum topland coefficient as 0.20, Minimum root clearance as 0.15, Bending and compressive stress as 400N/mm^2 and 1100N/mm^2.

V. Conclusion and Future Scope

In this paper, a special heuristic using genetic approach has been presented to improve the fast search for the bevel gear design problems. The duplication of the subsets in the chromosomes was avoided along with the elimination of unfeasible subsets. Thereby the genetic performance has been enhanced. Results obtained on real data sets are satisfactory and the experiments showed that result of the developed algorithms are better than the traditional methods both in the size and weight reduction with a slight improvement in the gear efficiency. Along with the noise generation and the temperature calculations, standardization also considered to make ease in manufacturing. The obtained results enforce all the declared constraints and thus may lead to best solutions.

This algorithm assumes that the cost is proportional to the weight of the gear and in turn didn't consider the factors influencing the cost. In the current version of the software, a set of bevel gear pair has been separated from the gear train. But for real time application, entire power transmission system need to be analysed, but integrating and solving the entire system simultaneously seems very hard.

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Author's Profile

Dr. S.K. Rajesh Kanna
Professor, Rajalakshmi Institute of Technology, Chennai, India. Born on 1980, Completed Masters with Distinction and Anna University Rank Holder, Gold medalist, completed Ph.D. in Anna University, Chennai. Worked in leading engineering colleges in Chennai. Published 57 papers and completed 07 funded projects. Organized 17 national level workshops and seminars. Having membership in 07 professional societies.
Email: skrkanna@gmail.com / +91 9884834301

A. D. Jaisree
completed bachelor degree in Computer Science and Engineering, Pavai Engineering College, Anna University, Chennai. Doing Master Degree in Mahindra Engineering College, Namakkal. Published various papers.