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Control of Fractional Order Uncertain Chaotic Unified Systems via Sliding Mode Control

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Abstract – **In this paper, a sliding mode control law is designed to control chaos in fractional order unified chaotic systems. Based on the sliding mode control method, the states of the fractional-order system have been stabled, even if the system with uncertainty is in the presence of external disturbance. In addition, chaos control is implemented in the fractional-order Lorenz system by utilizing this method. Simulation results confirm numerical results.**

Keywords – **Fractional-Order Chaotic Systems, Sliding Mode, Uncertainty, Control Chaos.**

I. INTRODUCTION

Chaos phenomenon can be considered as one of the hot issues in many applications such as medical [1-2], pharmaceutical [3], laser [4], and economic systems [5]. So, chaos can be considered as one of the most fascinating subjects that has attracted wide attention in recent years.

Fractional calculus has been introduced almost 300 years ago. Yet, in recent decades, it has been vastly studied [6-7]. Fractional-order chaotic systems occur as nonlinear phenomena in many scientific fields, such as chaotic behavior in financial systems [8] and many articles have been published in the field of fractional-order chaotic systems [9]. In two last decades, fractional calculus has been repeatedly exploited in chaos research such as investigation of chaotic behavior in fractional order Liu system [10].

Nowadays, controlling and synchronization of fractional order chaotic systems is a hot spot in research world. For example, Yin et al. implemented a fractional order sliding mode controller for unknown chaotic fractional order systems [11]. Also, Faieghi et al. studied chaotic synchronization of Genesio-Tesi system utilizing two strategies; active control and sliding mode [12]. The fractional order Liu system has a circuit realization as shown in Ref. [13].

In this paper, we first introduce fractional-order unified chaotic systems. Then, an sliding mode control law is proposed to control chaos in such fractional-order systems. The controller is developed to stabilize the unified fractional-order chaotic systems, even if the fractional-order systems with uncertainty and external disturbance. Numerical simulations show that the proposed method can easily eliminate chaos and stabilize the system on the sliding surface.

The paper is presented as follows: in Section 2, basic definitions of fractional calculus, notations and numerical algorithms are given. In Section 3, the general description of fractional-order chaotic systems is presented. Section 4 proposes the employment of the sliding mode control method to control chaos in the systems. Numerical simulation results are shown in Section 5. Finally, conclusion is addressed in section 6.

II. BASIC DEFINITION AND PRELIMINARIES

Definition 1: Fractional derivative and integral in the order of q is denoted by ${}_aD_t^q$ and this notation may include either integral concept or derivative concept, according to sign of q. It means:

$$ {}_aD_t^q = \begin{cases} \dfrac{d^q}{dt^q} & q > 0 \\ 1 & q = 0 \\ \int_a^t (dt)^{-q} & q < 0 \end{cases} \tag{1}$$

There are various definitions for fractional derivative and integral. The most common definitions are as Grunwald–Letnikov definition, Riemann–Liouville definition and Caputo definition. In the rest of this article Riemann-Liouville (RL) definition of fractional derivative is used. RL derivative in the order of q is explained below ([14]):

$$ {}_0D_t^q f(t) = D^q f(t) = \frac{d^q f(t)}{dt^q} = \frac{1}{\Gamma(m-q)} \frac{d^m}{dt^m} \int_0^t \frac{f(\tau)}{(t-\tau)^{q+m-1}} d\tau \tag{2}$$

where m is the first integer which is not less than q, i.e. $m-1 \le q < m$ and $\Gamma(.)$ is the well-known Euler's gamma function

$$\Gamma(P) = \int_0^\infty t^{p-1} e^{-t} dt \; ; \quad \Gamma(P+1) = P\Gamma(P) \tag{3}$$

Lemma 1 ([15]). The following autonomous system:

$$\frac{d^q x}{dt^q} = Ax, \qquad x(0) = x_0 \tag{4}$$

With $0 < q < 1, x \in R^n$ and $A \in R^{n\times n}$, is asymptotically stable if and only if

In this case, each component of $\left|\arg(eig(A))\right| > \dfrac{q\pi}{2}$ the states decays toward 0 like t^{-q}. Also, this system is stable if and only if $\left|\arg(eig(A))\right| > \dfrac{q\pi}{2}$ and those critical eigenvalues that satisfy

$\left|\arg(eig(A))\right| > \frac{q\pi}{2}$ have geometric multiplicity one.

III. SYSTEM DESCRIPTION

In this paper, we consider chaos synchronization for the unified fractional order chaotic system [16]. The mathematical model for the unified fractional order chaotic system is given by

$$\begin{cases} \frac{d^q x_1}{dt^q} = (25\alpha + 10)(x_2 - x_1) \\ \frac{d^q x_2}{dt^q} = (28 - 35\alpha)x_1 + (29\alpha - 1)x_2 - x_1 x_3 \\ \frac{d^q x_3}{dt^q} = x_1 x_2 - \frac{8+\alpha}{3} x_3 \end{cases} \tag{5}$$

where x_1, x_2, x_3 are state variables and parameter $\alpha \in [0,1]$. In fact, the above mentioned chaotic system has chaotic behavior for $\alpha \in [0,1]$. It is called general Lorenz system when $\alpha \in [0, 0.8)$. When $\alpha = 0.8$ it changes into general system of Lu and while, it is named as general system of Chen for $\alpha \in (0.8, 1]$. In what follows, with adding uncertainty, noise, and control signal to the system (5), we have:

$$\begin{cases} \frac{d^q x_1}{dt^q} = (25\alpha + 10)(x_2 - x_1) \qquad (6) \\ \frac{d^q x_2}{dt^q} = (28 - 35\alpha)x_1 + (29\alpha - 1)x_2 - x_1 x_3 + H(x_1, x_2, x_3) + d(t) + u(t) \\ \frac{d^q x_3}{dt^q} = x_1 x_2 - \frac{8+\alpha}{3} x_3 \end{cases}$$

The goal of proposed controller is design of sliding mode control which results in fractional order error system stability (6).

Remark 1. If $q_1 = q_2 = q_3 = q$, fractional-order system (5), is called a commensurate fractional-order system. Otherwise, it is called incommensurate fractional-order system.

IV. DESIGNING THE FRACTIONAL-ORDER SLIDING MODE CONTROL

To design a sliding mode controller, there are two steps: first, a sliding surface should be constructed that represents a desired system dynamics, and next, a switching control law should be developed such that a sliding mode exists on every point of the sliding surface, and any states outside the surface are driven to reach the surface in a finite time. As a choice for the sliding surface, one has:

$$s(t) = (D^{q_2} + \eta)\int_0^t x_2(\tau)d\tau \tag{7}$$

For the sliding mode method, the sliding surface and its derivative must satisfy

$$s(t) = 0, \quad \dot{s}(t) = 0 \tag{8}$$

which yields the following sliding mode dynamics

$$D^{q_2} x_2 = -\eta x_2 \tag{9}$$

By using Lemma 1, system (9) is asymptotically stable. As a result, the sliding mode surfaces (7) we have just constructed are appropriate for the control design. According to the sliding mode control theory and using (6) and (9), the equivalent control law is calculated as

$$\begin{aligned} u_{eq} &= -(28 - 35\alpha)x_1 - (29\alpha - 1)x_2 + x_1 x_3 \\ &\quad - H(x_1, x_2, x_3) - d(t) - \eta x_2 \end{aligned} \tag{10}$$

The next step to satisfy the sliding condition, the discontinuous reaching law is chosen as follow:

$$u_r(t) = -k_r sign(s) \tag{11}$$

where

$$sign(s) = \begin{cases} 1 & s > 0 \\ 0 & s = 0 \\ -1 & s < 0 \end{cases} \tag{12}$$

and k_r is the reach gain of the controller. Therefore, control law has been calculated from (14) and (15) as follows

$$\begin{aligned} u &= u_{eq} + u_r \\ &= -(28 - 35\alpha)x_1 - (29\alpha - 1)x_2 + x_1 x_3 \\ &\quad - H(x_1, x_2, x_3) - d(t) - \eta x_2 - k_r sign(s) \end{aligned} \tag{13}$$

Theorem 1. The fractional order unified chatic system (6) with the control law (13) is asymptotically stable if the controller gain $k_r > 0$.

Proof. Using the candidate Lyapunov function as

$$V = \frac{1}{2}s^2 \tag{14}$$

Taking time derivative gives

$$\begin{aligned} \dot{V} &= s\dot{s} = s\left[D^{q_2} x_2 + \eta x_2\right] \\ &= (28 - 35\alpha)x_1 + (29\alpha - 1)x_2 - x_1 x_3 + H(x_1, x_2, x_3) \\ &\quad + d(t) - (28 - 35\alpha)x_1 - (29\alpha - 1)x_2 + x_1 x_3 - \\ &\quad H(x_1, x_2, x_3) - d(t) - \eta x_2 - k_r sign(s) + \eta x_2 = k_r|s| < 0 \end{aligned} \tag{15}$$

Therefore, a Lyapunov function has been found that satisfies the conditions of the Lyapunov theorem $(V > 0, \dot{V} < 0)$. Thus, the closed-loop system in the presence of the controller (13) is globally asymptotically stable.

V. SIMULATION RESULTS

Lorenz's fractional-order chaotic system is a subsidiary of unified fractional-order chaotic systems in Equation (5) $(\alpha = 0)$. Chaotic behavior has been shown without uncertainty, external noise and input in Fig.1. Here, we have considered System (6) in the presence of uncertainty

$H(x_1,x_2,x_3)=0.7\sin x_1\cos x_2$ and external noise $d(t)=\sin 4t$ For simulation results, we have suggested $q=[0.995, 0.995, 0.995]$, controller $k_r=1.5$, $(x_1(0), x_2(0), x_3(0))=(-8,6,12)$ and for the fractional-order, and initial condition of fractional-order Lorenz system. Parameter in (7) are chosen as $\eta=4$. The states of the system (6) under the controller (13) and the sliding surface (7) are illustrated in Fig. 2, which shows that the sliding control law guarantees reaching the sliding surface and finally stabilization. Fig. 3 gives the trajectory of the control input.

VI. CONCLUSION

In this study, a class of fractional-order chaotic systems has been introduced. According to the Lyapunov stability theorem, a sliding mode control law has been designed to control chaos in the systems. Based on the sliding mode control method, the states of the fractional-order system have been stabilized. The proposed technique has also been used to the systems in the presence of uncertainty and external disturbance. Finally, numerical simulations are given to verify the effectiveness of the proposed control scheme.

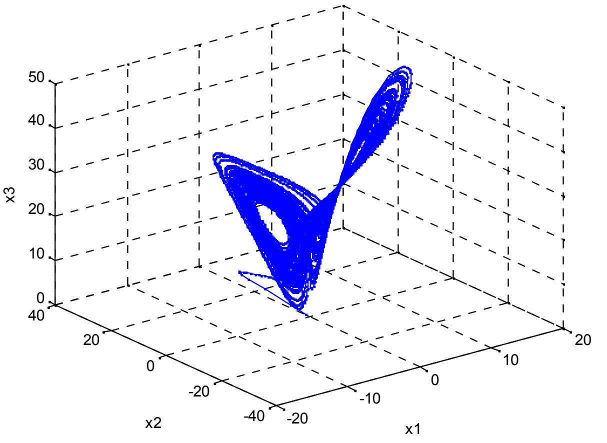
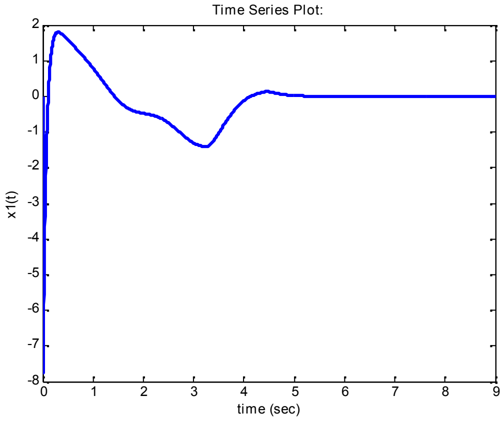


Fig1. Phase diagram of Lorenz system with fractional order *q*= [0.995, 0.995, 0.995]



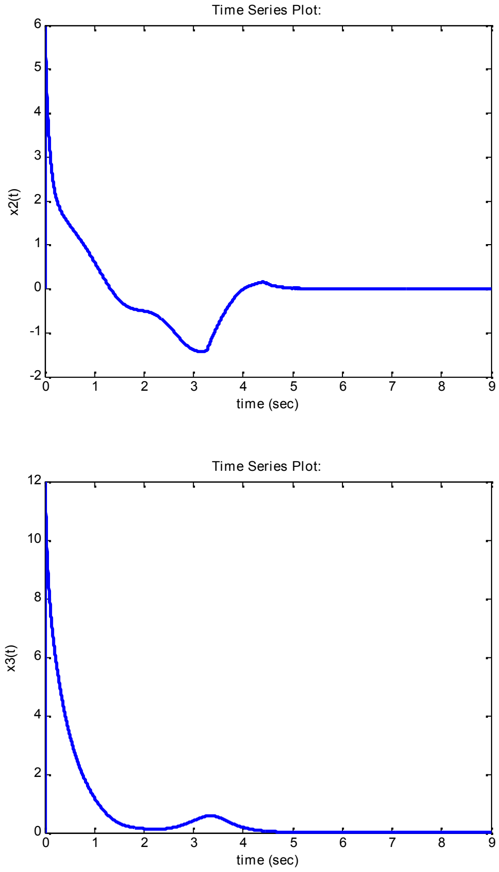


Fig.3. Result of control of the states variables

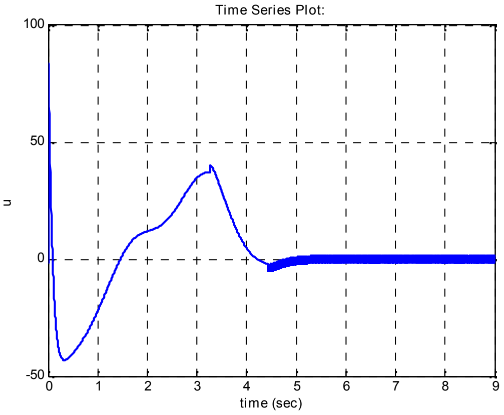


Fig.4. Control function

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