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Analysis of Crack Problem by using ADI-FDTD/MoMTD Hybrid Method

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Abstract – **In general, the hybrid method of moments in time domain (MoMTD) and finite difference in time domain (FDTD) is applied for problems that an antenna, especially wire antenna, is located near a multilayer scatterer. MoMTD and relative equations model antenna as a Huygens's surface. FDTD calculates fields in area that includes scatterer. When there is a small size - toward minimum wavelength- part in the scatterer, time step must be very small, because of the Courant condition. So the runtime of FDTD becomes long. In this paper we want to solve the time consuming problem of the hybrid MoMTD-FDTD. We used a newer kind of FDTD that called ADI-FDTD instead of FDTD. ADI-FDTD is unconditionally stable and there is no need for Courant condition anymore. As an ADI-FDTD problem, we chose to solve the problem of crack detection in a wall.**

Keywords – **ADI-FDTD, Crack, FDTD, MoMTD.**

I. INTRODUCTION

Appearing gap or crack in the industrial devices is a common occurrence. A crack in a part of device may cause malfunction or failure of the device, over time. Nondestructive testing (NDT) methods are available to detect cracks. One of these methods uses electromagnetic waves. In this method an antenna and a detector scan device and by comparing the results, crack can be detected. This method can be categorized in microwave imaging methods of NDT [1]. This method is useful for nonmetallic surfaces that can be multilayer. In this paper we are going to solve this electromagnetic problem.

Our problem is an antenna which is located in the vicinity of a wall. There is a gap in the wall that we want to find it. To solve this electromagnetic problem, there is a Hybrid method of moments in time domain (MoMTD) and finite difference in time domain (FDTD). This hybrid method is widely applied for simulation of multilayer scatterer in the vicinity of an antenna [2]. In this way, the antenna is solved by the MoMTD and multilayered medium is solved by FDTD. In fact, this hybrid method combines the benefits of both MoMTD and FDTD.

If a structure, which is smaller than the minimum wavelength is present in the environment, to model the structure with FDTD, dimensions of the mesh also must be smaller than the minimum wavelength. In the conventional FDTD there is a stability condition which is called Courant condition. This condition forces us to decrease time step to remain stability of FDTD [3]. Small time steps means increasing in repeated loops and thus longer simulation time.

To solve this time consuming problem we replace conventional FDTD with a newer kind of FDTD which called Alternating-Direction Implicit FDTD or ADI-FDTD. The ADI is a technique first found in the works of Douglas, used to perturb and factorize the unconditionally stable Crank–Nicolson scheme approximation of partial differential equations, to render it computationally costless without losing its unconditional stability [4].

ADI-FDTD discretizes Maxwell equation in half implicit and half explicit form [5]. ADI-FDTD solution will remove Courant condition and becomes an unconditionally stable method [6]. By increasing time steps we can reduce simulation time. Staker formulated this reduction in time consuming [7]. Upon his research, simulation time in ADI-FDTD method is a function of the size of the problem space and number of iterations.

Another parameter that should be mentioned in using ADI-FDTD instead of FDTD is dispersion error. By increasing time steps in ADI-FDTD this error will increase [8]. Although it is possible to use high order ADI-FDTD to reduce this error but this error in our problem is small enough to use first order ADI-FDTD [9].

In this paper we assume a small structure as a crack in a multilayer wall and we solve a nondestructive evaluation to detect this small crack by using ADI-FDTD/MoMTD hybrid method.

II. FDTD-MOMTD HYBRID METHOD

The first hybrid method of moments and finite difference was presented by Taflove [10]. In general, there are two methods for finding the transient response for a differential integral equation: frequency domain and time domain methods. In frequency domain methods, equation is solved in the frequency range of the source and then transient response can be found by using the inverse Fourier transform. In this method, the formulation of the problem is simpler but in cases when source signal bandwidth is wide, analysis should be done for so many frequencies that make solution time consuming. In second method equation is solved in time domain from beginning. In this paper we use time domain solution.

FDTD-MoMTD hybrid method uses the surface equivalence theorem to separate the main problem into two sub problems. The area consists of a thin wire antenna solves with MoMTD and a region that contains multilayer scatterer solves with FDTD [2].

In this method, first, an imaginary Huygens's surface is considered around a thin wire antenna. MoMTD calculates electric and magnetic fields produced by wire antenna in free space on the Huygens's surface. By using surface equivalence theorem, equivalent electric and magnetic surface currents can be obtained as shown in (1).

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$$J_s = n \times E$$
$$M_s = H \times n \quad (1)$$

Where J_s, M_s are electric and magnetic surface currents and n is normal vector to Huygens's surface and E, H are electric and magnetic fields on Huygens's surface.

Then we use the algorithm of FDTD while the wire antenna previously removed from the space and we have replaced it with equivalent surface currents. FDTD solves the remaining part of the problem that is scatterer effect. FDTD gives us a time domain solution of electromagnetic fields in places of interest.

As stated; FDTD requires complying with Courant condition. If we suppose Δx, Δy, Δz as mesh dimensions and Δt as time step, then Courant condition can be written as in (2).

$$\Delta t_{FDTD} < \left(c\sqrt{\frac{1}{\Delta x^2} + \frac{1}{\Delta y^2} + \frac{1}{\Delta z^2}} \right)^{-1} \quad (2)$$

Where c is the speed of the light. This condition doesn't allow Δt to become larger than specific value. Small Δt means more iterations and more time to simulate the problem.

III. ADI-FDTD/MoMTD Hybrid Method

ADI-FDTD/MoMTD is similar to FDTD-MoMTD method. The MoMTD that is used in both methods is similar with minor changes. We took advantage of the MoMTD simulation software for both methods. Although ADI-FDTD and FDTD have similarities, there are also major differences. In this paper, we only compare these two methods and we ignore explanation of MoMTD.

FDTD method is based on direct solution of Maxwell's equation. These equations can be written as follows:

$$\nabla \times E = -\mu \frac{\partial H}{\partial t}$$
$$\nabla \times H = \varepsilon \frac{\partial E}{\partial t} \quad (3)$$

Where ε, μ are permittivity and permeability coefficients. For example, in relation to Cartesian coordinates, first curl equation can be found as follows:

$$\frac{\partial E_x}{\partial t} = \frac{1}{\varepsilon}\left(\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} \right) \quad (4)$$

Based on Yee algorithm, conventional FDTD gives approximation of this equation as shown in (5) [11]. In this equation n is time index and i, j, k are spatial indexes. Five other equations can be approximated same as this equation. Conventional FDTD method calculates total fields in the space by using these six equations. This equation calculates E_x at $n+1$ step explicitly from H_z, H_y, E_x at n'th step which are known values from pervious iteration.

$$\frac{E_x\Big|^{n+1}_{i+\frac{1}{2},j,k} - E_x\Big|^{n}_{i+\frac{1}{2},j,k}}{\Delta t} = \frac{1}{\varepsilon}\left[\frac{H_z\Big|^{n}_{i+\frac{1}{2},j+\frac{1}{2},k} - H_z\Big|^{n}_{i+\frac{1}{2},j-\frac{1}{2},k}}{\Delta y} - \frac{H_y\Big|^{n}_{i+\frac{1}{2},j,k+\frac{1}{2}} - H_y\Big|^{n}_{i+\frac{1}{2},j,k-\frac{1}{2}}}{\Delta z} \right] \quad (5)$$

In ADI-FDTD method transition time from n to $n+1$ is broken to two half step; one from n to $n+1/2$ and another from $n+1/2$ to $n+1$. First half step of (4) is written as in (6).

$$\frac{E_x\Big|^{n+\frac{1}{2}}_{i+\frac{1}{2},j,k} - E_x\Big|^{n}_{i+\frac{1}{2},j,k}}{\Delta t/2} = \frac{1}{\varepsilon}\left[\frac{H_z\Big|^{n+\frac{1}{2}}_{i+\frac{1}{2},j+\frac{1}{2},k} - H_z\Big|^{n+\frac{1}{2}}_{i+\frac{1}{2},j-\frac{1}{2},k}}{\Delta y} - \frac{H_y\Big|^{n}_{i+\frac{1}{2},j,k+\frac{1}{2}} - H_y\Big|^{n}_{i+\frac{1}{2},j,k-\frac{1}{2}}}{\Delta z} \right] \quad (6)$$

Note that how this equation calculates E_x at $n+1/2$ step; implicitly from H_z at same time and explicitly from H_y and E_x at n'th step. In second half step we have:

$$\frac{E_x\Big|^{n+1}_{i+\frac{1}{2},j,k} - E_x\Big|^{n+\frac{1}{2}}_{i+\frac{1}{2},j,k}}{\Delta t/2} = \frac{1}{\varepsilon}\left[\frac{H_z\Big|^{n+\frac{1}{2}}_{i+\frac{1}{2},j+\frac{1}{2},k} - H_z\Big|^{n+\frac{1}{2}}_{i+\frac{1}{2},j-\frac{1}{2},k}}{\Delta y} - \frac{H_y\Big|^{n+1}_{i+\frac{1}{2},j,k+\frac{1}{2}} - H_y\Big|^{n+1}_{i+\frac{1}{2},j,k-\frac{1}{2}}}{\Delta z} \right] \quad (7)$$

Also note that how this equation calculates E_x implicitly from H_y and explicitly from H_z and E_x.

Similarly, five other equations can be derived. Writing these equations in matrix form, forward us to following equations:

$$M_1 X^{n+\frac{1}{2}} = P_1 X^n$$
$$M_2 X^{n+1} = P_2 X^{n+\frac{1}{2}} \quad (8)$$

Where X^n is a column vector that contains all the fields in the n'th time step. P_1, P_2, M_1, M_2 are coefficient matrixes that their elements depends on the spatial and temporal conditions of the problem. In fact, these are two systems of linear equation that P_1X^n and $P_2X^{n+1/2}$ are known values and $X^{n+1/2}$, X^{n+1} are unknown values and M_1, M_2 are coefficient matrixes. Each row of M_1 and M_2 has at most three nonzero elements. This kind of matrix is called Tridiagonal matrix. In fact, the heart of ADI-FDTD method is solution of this matrix system. We solved this matrix system with the help of the technique that is described in [12]. Thus, whenever this matrix system is solved; fields will be found in one dimension. This process is done for all three dimensions and all the time.

There are several boundary conditions that can be applied to FDTD or ADI-FDTD such as perfect match layer (PML) or Mur absorbing boundary condition [13], [14]. We used Mur as an absorbing boundary condition. Mur absorbing boundary condition is simple and efficient condition. For the conventional FDTD this boundary condition is used explicitly while for ADI-FDTD this boundary condition should be used implicitly[15].

IV. Simulation Result

In order to compare FDTD-MoMTD and ADI-FDTD/MoMTD methods, we simulate an electromagnetic nondestructive method. We placed a dipole antenna near a wall. Dipole length is 20 mm and has wire radius of 0.1 mm. The excitation of the antenna is derivative of Gaussian pulse with 2 nsec width and 1 V amplitude. The frequency spectrum of excitation signal is spread from zero to 1.5 GHz that means its minimum wavelength is 20 cm. By using MoMTD, currents on the Huygens's surface obtained. Huygens's surface was supposed to be 10 mm ×10 mm × 40 mm cube around the antenna. Then these surface currents applied to FDTD and ADI-FDTD methods. Mesh dimension for both methods was chosen to be 2 mm × 2 mm × 2 mm.

First we examine Courant stability condition by taking time step slightly larger than maximum allowed value. From Courant condition, maximum allowed value of time step is 3.849 ps. We chose time step to be 3.9 ps. In this case FDTD becomes unstable. The z component of electric field is shown in Fig. 1.

For comparison FDTD with the ADI-FDTD, we add a wall that has a crack in it. Fig. 2 shows the transverse cut of this structure where crack is parallel to the antenna.

Antenna and the Huygens's surface are same as before. Distance from antenna to wall is 32 mm. The wall is composed of two materials with different permittivity. Thickness of the coating on both sides is 4 mm and their relative permittivities (ε_r) are 2. The thickness of the wall is 20 mm and its relative permittivity and conductivity (σ) is 20, 1 s/m, respectively. Point of view for the z component of the electric field is behind the wall at 4 mm distance from the wall towards the center of the antenna.

When there is no crack in the wall the electric field at point of view is shown in Fig. 3. In this figure time step for FDTD is 3.849 ps and for ADI-FDTD is 38.49 ps that means 10 times larger than FDTD time step. We can increase time step for ADI-FDTD but the results may become less accurate.

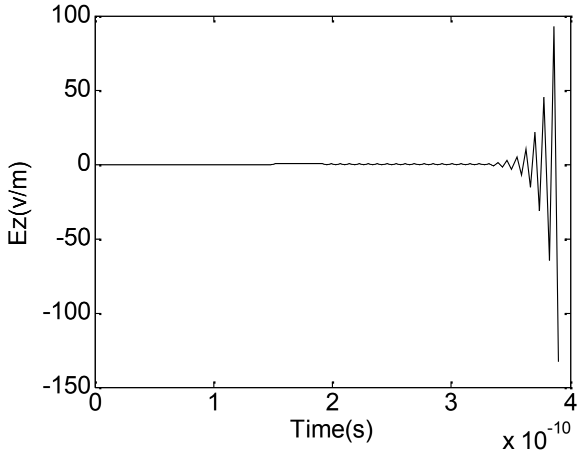

Fig.1. Instability of FDTD due to Courant condition

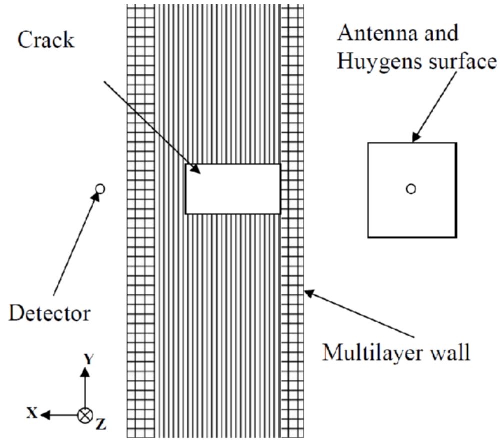

Fig.2. Transverse cut of problem space

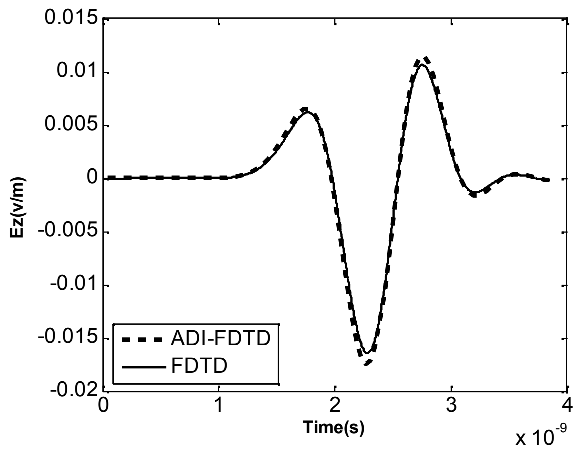

Fig.3.Electric field at detector point by FDTD and ADI-FDTD when no crack exists

Now we consider a crack with 14 mm width (in y direction) and 18 mm depth (in x direction) and long in z direction. In this case result is shown in Fig. 4.

When we put these two results; that calculated by ADI-FDTD, next to each other in one figure we can detect crack as shown in Fig. 5.

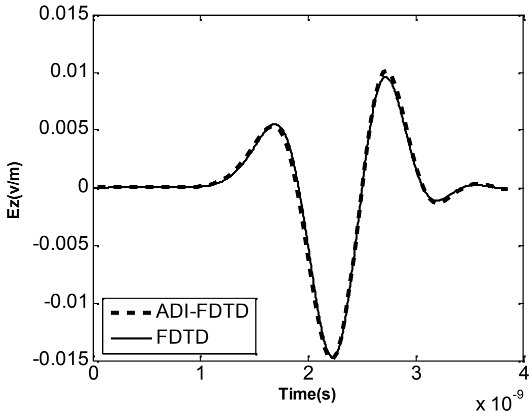

Fig.4. Electric field at detector point by FDTD and ADI-FDTD when crack exists

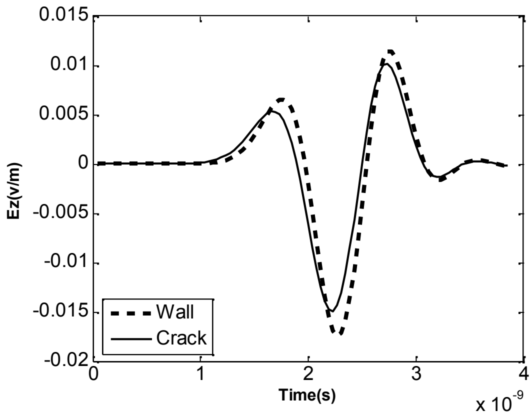

Fig.5. Electric field at detector point with crack and without crack in the wall calculated by ADI-FDTD/MoMTD

So when antenna is scanning the surface and such a change is seen in detector, antenna is crossing a crack. It is obvious that during scan time antenna and detector should be in fixed distance from the wall.

Table I shows simulation time and memory used by FDTD and ADI-FDTD for several time steps in this problem.

Table I: Run time and used memory comparison in FDTD versus ADI-FDTD

	Time step (ps)	Run time (s)	Used memory (MB)
FDTD	3.849	3280	42
ADI-FDTD	7.698	4590	79
	38.49	920	79
	76.98	459	79

Note that in small time steps ADI-FDTD run time is larger than FDTD run time because of more calculations that perform in one loop of ADI-FDTD in compare with FDTD. Also because of large matrixes in ADI-FDTD its used memory is more than FDTD.

V. Conclusion

In this paper, a hybrid method of MoMTD and ADI-FDTD was used to detect crack in multilayer wall. This method is suitable for simulation of electromagnetic structures, which are small compared to the minimum wavelength. ADI-FDTD is unconditionally stable and does not need to satisfy the Courant condition such as FDTD. So we were able to choose time step greater than the maximum value of the Courant condition and finally we reduce the computation time. Also error increases with increasing time step.

References

[1] R. Zoughi, “Microwave and millimeter wave nondestructive testing: A succinct introduction,” *Mater. Eval.*, vol. 53, no. 4, pp. 461–462,Apr. 1995.

[2] A. R. Bretones, R. Mittra, R. G. Martin, “A hybrid technique combining the MoM in the time domain and FDTD,” *IEEE Microwave and Guided Wave Letters.*, vol. 8, no. 8, pp. 281-283, Aug. 1998.

[3] R. Courant, K. Friedrichs, H. Lewy, “On the partial difference equations of mathematical physics,” *IBM Journal of Research and Development*, vol. 11, Issue 2, pp. 215–234, March 1967.

[4] J. Douglas, H. Rachford, “On the numerical integration of $u_{xx}+u_{yy}=u_t$ by implicit methods,” *J. Soc. Ind. Appl. Math,*. vol. 3, pp. 42–65, 1955.

[5] T. Namiki, “A new FDTD algorithm based on alternating-direction implicit method,” *IEEE Trans. Microwave Theory Tech.*, vol. 47, no. 10, pp. 2003–2007, Oct. 1999.

[6] F. Zheng, Z. Chen, and J. Zhang, “Toward the development of a three-dimensional unconditionally stable finite-difference time-domain method,” *IEEE Trans. Microwave Theory Tech.*, vol. 48, no. 9, pp. 1550–1558, Sep. 2000.

[7] S.W. Staker, C.L. Holloway, A.U. Bhobe, M. Piket,, “Alternating-direction implicit (ADI) formulation of the finite-difference time-domain (FDTD) method: Algorithm and material dispersion implementation,” *IEEE Trans. Microwave Theory Tech.*, vol.45, no.2, pp.156–166, May 2003.

[8] F. Zheng, Z. Chen, “Numerical dispersion analysis of the unconditionally stable 3-d ADI-FDTD method,” *IEEE Trans. Microwave Theory Tech.*, vol. 49, pp. 1006–1009, May 2001.

[9] J. Chen, Z. Wang, Y. Chen, “Higher-order alternative direction implicit FDTD method,” *Electron. Letter*, vol. 38, no. 22, pp. 1321-1322, Oct. 2002.

[10] A. Taflove, K. R. Umashankar, “A hybrid moment method-finite difference time-domain approach to electromagnetic coupling and aperture penetration into complex geometries,” *IEEE Trans. Antennas Propagation*, vol. 30, no. 4, pp. 617–627, July 1982.

[11] K. S. Yee, “Numerical solution of initial boundary value problems involving Maxwell’s equations in isotropic media,” *IEEE Trans. Antennas Propagation*, vol. 14, no. 5, pp. 302–307, May 1966.

[12] W.H. Press, S.A. Teukolsky, W.T. Vetterling, B.P. Flannery, *Numerical recipes in C: the art of scientific computing*, second edition, Cambridge University Press, New York, 1992, pp. 50–51.

[13] Shumin Wang a, F.L. Teixeira, “An efficient PML implementation for the ADI-FDTD method,” *IEEE Microwave and Wireless Components Letters*, vol. 13, no. 2, Feb. 2003.

[14] G. Mur, “Absorbing boundary conditions for the finite difference approximation of the time domain electromagnetic field equations,” *IEEE Trans. Electromagnetic Compatibility,* vol. 23, no. 4, pp. 377–382, Nov. 1981.

[15] T. Stefanski, T. D. Drysdale “Improved implementation of the Mur first order absorbing boundary condition in the ADI-FDTD method,” *Wiley Online Library, Microwave and Optical Technology Letters*, Volume 50, Issue 7, pages 1757–1761, Apr. 2008.

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