

# Trajectory Following of an Industrial Robot using Optimized Fractional Order PID Controller

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**Abstract** – In this paper, the multi-variable control of a two-link robot is considered. Industrial robots, such as robot of two degrees, are the highest acceptable robots in industry and academic environments. Because of the dynamic control of the new challenges the Multi-input, multi-output systems face, which can obtain robust control methods, this deficiency leads to excessive minimum value. In this paper an optimized fractional order PID controller by genetic algorithm and pattern search algorithm is used to control robot trajectory. As a result, the ability to have disturbance rejection system has a major impact on total control.

**Keywords** – Robotics, 2 Link Robot, Fractional Order PID, Genetic Algorithm, Pattern Search Algorithm.

## I. INTRODUCTION

The dynamics of the robot consist of two parts, the direct and inverse dynamics [1]. The purpose of direct dynamics is to gain the momentum, velocity and acceleration of the robot tool holding forces. Also torques applies to the joints or is irritates, but the inverse dynamic modeling with knowledge of routes, velocities and accelerations of the robot tool, or momenta forces driving the joints are calculated. Among classical methods for the calculation of dynamic robot models are Lagrange, D'Alembert method, Newton, Euler equations, virtual work and Hamilton [2]. Nowadays, most robots used need to work quickly and efficiently. Among uses of robots are use in assembly lines, medical, machining and many other applications mentioned. The use of industrial robots in the production process and automation industry has grown considerably in recent decades. Most machine tool spindle apparatus are used for the series chain kinematics. Due to the widespread use of dynamic structures, control is vital to the comment [3].

Cervantes, in 2001, the movement of the robot arm is done by a PID controller [4-7]. Lopez et al., considered in 2008, a proportional controller, derivative and integral on a six degree of freedom parallel robot [8]. Controller of proportional, derivative and integral are always used in different industrial lines.

Another branch of the robot controlled on the references [9-11] can be seen. One of the important issues in designing a robot controller design is resistantability to the uncertainty of such mass changes. Robust control theory, the sliding mode control law to form a simple procedure, could be brought into existence [10]. Because of the importance of using a non-linear sliding mode controller, it has long been paid attentioned to. Many systems have used this type of approach. In references, an example of this method which has widely been used in engineering

sciences as a controller a long with an estimation and optimization has been presented. One of the main difficulties is that sliding mode controller is on the way. These are chattering, through the use of strategies that have been proposed so far [12]. To reduce and eliminate the classical sliding, mode control can be used up. When the super twist algorithm comes, data entry doesn't need to change the sliding surface, and hence, the algorithm is suitable for many dynamic systems. In 1993, Levant introduced second order sliding mode in his paper [13].

This paper is organized as follows: In Part 2, the dynamic equations of the robot and the direct dynamic solution process has been explained. In Part 3, the Fractional order controller and optimization Algorithms are presented. In section 4, the simulation is done in MATLAB-SIMULINK by ode45 solver and finally in section 5, the conclusion has been made.

## II. DYNAMIC EQUATIONS AND SIMULATED ROBOTS

For a robot motion control, we need to know if the amount of torque is required to achieve the desired movement of the joint. This approach is so-called in robotics directs dynamics. A direct dynamic solution of the robot, because of differential operators, has been associated with different challenges. Robot modeling is used to model the S- Function in MATLAB-SIMULINK environment. It can be nonlinear model of the robot's environment to properly Simulink easily connected controllers to be used to it.

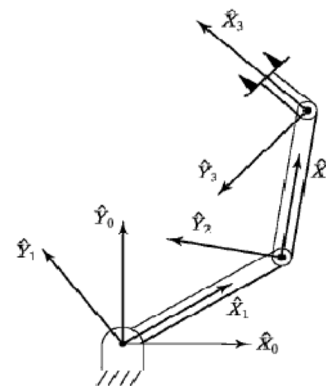


Fig.1. Two Link ROBOT [1]

In equation 1, the robot forward dynamics equation is demonstrated, Where,  $\tau$  is the torque of link and  $M$ ,  $V$  and  $G$  are the matrices related to robot parts. These matrices are described below.

$$\tau = M(\theta)\ddot{\theta} + V(\theta, \dot{\theta}) + G(\theta) \quad (1)$$

$$M = \begin{bmatrix} l_2^2 m_2 + 2l_1 l_2 m_2 c_2 + l_1^2 (m_1 + m_2) & l_2^2 m_2 + l_1 l_2 m_2 c_2 \\ l_2^2 m_2 + l_1 l_2 m_2 c_2 & l_2^2 m_2 \end{bmatrix} \quad (2)$$

$$G = \begin{bmatrix} m_2 l_2 g c_{12} + (m_1 + m_2) l_1 g c_1 \\ m_2 l_2 g c_{12} \end{bmatrix} \quad (3)$$

$$V(\theta, \dot{\theta}) = \begin{bmatrix} -m_2 l_1 l_2 s_2 \dot{\theta}_2^2 - 2m_2 l_1 l_2 s_2 \dot{\theta}_1 \dot{\theta}_2 \\ m_2 l_1 l_2 s_2 \dot{\theta}_1^2 \end{bmatrix} \quad (4)$$

In these equations,  $l_1$  and  $l_2$  are the length of the first and second arm respectively. Also  $m_1$  and  $m_2$  are the mass of links.  $\theta$  Shows the angle difference for each one. These formulas are derived from dynamics rule link NEWTON and etc.

In this paper, the nonlinear equations of robots, written in MATLAB environment and programming techniques, as it has been used in a block SIMULINK, is understandable. Also, with a view on the SIMMECHANICS and dynamics of its direct use, the reason for the current environment is made in the previous exercise, but the controllers are on. In figures 2 and 3, the main body of the space robot SIMULINK by the show's toolbox have been shown. Figures 4 and 5 show the simmechanics interaction windows.

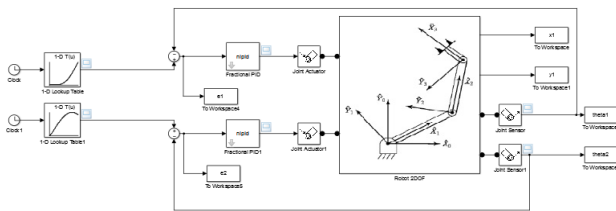


Fig.2. Schematic model of the series robot by SIMMECHANICS in SIMULINK

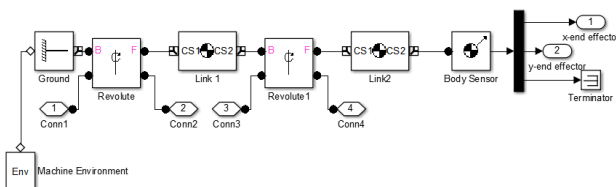


Fig.3. Joint modeling of multiple robots by means of SIMMECHANICS

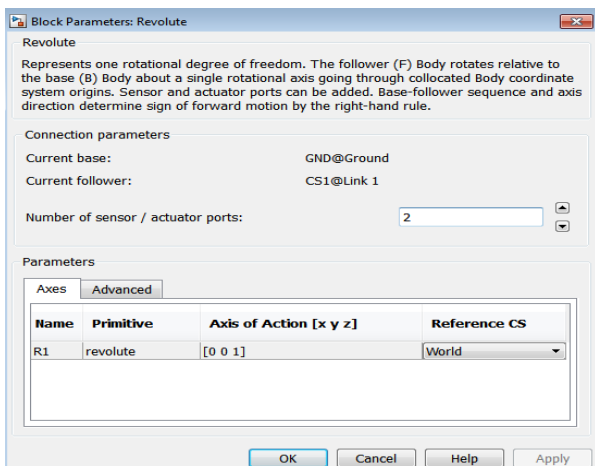


Fig.4. Joints schemes in simmechanics

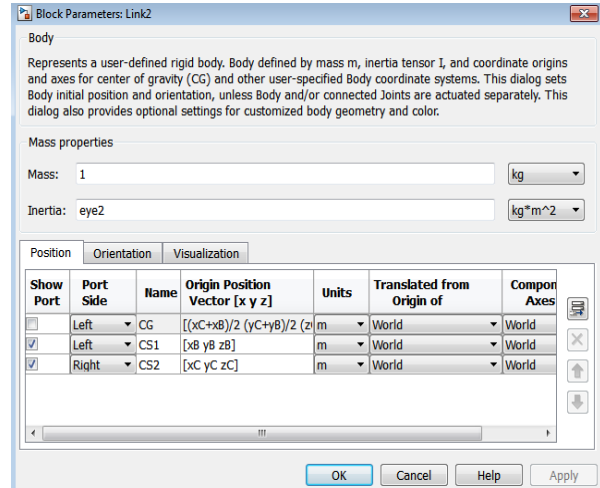


Fig.5. Body scheme in simmechanics

The nominal values used in the simulation are shown in table 1.

Table 1: Nominal values for robot parts

$m_1$	5 Kg
$m_2$	1Kg
$l_1$	1m
$l_2$	1m

### III. CONTROL METHODOLOGIES AND OPTIMIZATION MECHANISM

#### A. Fractional-order PID Controllers (FOPID)

The Fractional-order PID Controller is a generalization of the PID controller, where  $K_p$  is the proportional constant,  $K_I$  is the integration constant,  $k_D$  is the differentiation constant and integrator of order  $\mu$  and  $\mu$  can be any real numbers. The transfer function of such a controller has the form:

$$G_c(s) = K_p + \frac{K_I}{s^\lambda} + k_D s^\mu \quad (5)$$

The integrator term is  $s^{-\lambda}$  that is to say, on a semi-logarithmic plane, there is a line having slope  $-20$  dB./dec. The control signal  $u(t)$  can then be expressed in the time domain a

$$u(t) = k_p e(t) + k_I D^{-\lambda} e(t) + k_D D^\mu e(t) \quad (6)$$

Clearly, selecting  $\lambda = 1$  and  $\mu = 1$ , a classical PID controller can be recovered. The selections of  $\lambda = 1$ ,  $\mu = 0$ , and  $\lambda = 0$ ,  $\mu = 1$  respectively corresponds conventional PI & PD controllers. As shown in Fig.6, This Controller generalizes the classical PID controller and expands it from point to plane. This expansion can provide and add more flexibility to controller design in achieving control objectives and the real processes can be controlled more accurately.

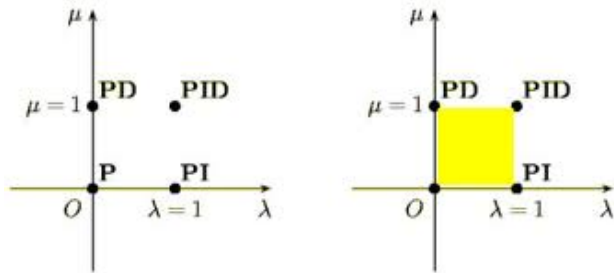


Fig.6. Overview of the classical and fractional controller

All these classical types of PID controllers are the special cases of the fractional  $PI^\lambda D^\mu$  controller given by (14). It can be expected that the  $PI^\lambda D^\mu$  controller may enhance the systems control performance. One of the most important advantages of the  $PI^\lambda D^\mu$  controller is the better control of dynamical systems, which are described by fractional order mathematical models. Another advantage lies in the fact that the  $PI^\lambda D^\mu$  controllers are less sensitive to changes of parameters of a controlled system [14]. This is due to the two extra degrees of freedom to better adjust the dynamical properties of a fractional order control system. However, all these claimed benefits were not systematically demonstrated in the literature.

To study the fractional order controllers, the starting point is of course the fractional order differential equations using fractional calculus. A commonly used definition of the fractional differ integral is the Riemann-Liouville definition [15].

$${}_a D_t^\alpha f(t) = \frac{1}{\Gamma(m-\alpha)} \left( \frac{d}{dt} \right)^m \int_0^t \frac{f(\tau)}{(t-\tau)^{1-(m-\alpha)}} d\tau \quad (7)$$

for  $m-1 < \alpha < m$  where  $\Gamma(\cdot)$  is the well-known Euler's gamma function. An alternative definition, based on the concept of fractional differentiation, is the Grunwald-Letnikov definition given by

$${}_a D_t^\alpha f(t) = \lim_{h \rightarrow 0} \frac{1}{\Gamma(\alpha)h^\alpha} \sum_{k=0}^{(t-a)/h} \frac{\Gamma(\alpha+k)}{\Gamma(k+1)} f(t-kh) \quad (8)$$

One can observe that by introducing notion of the fractional order operator  ${}_a D_t^\alpha f(t)$  the differentiator and integrator can be unified. Another useful tool is the Laplace transform. It has been shown in [15] that the Laplace transform of an nth ( $n \in \mathbb{R}^+$ ) derivative of a signal

$x(t)$  relaxed at  $t=0$  is given by  $L\{D^n x(t)\} = s^n X(s)$ . So, a fractional order differentia equation, provided both the signals  $u(t)$  and  $y(t)$  are relaxed at  $t=0$ , can be expressed in a transfer function form

$$\lambda \quad (9)$$

Where  $(a_m, b_m) \in \mathbb{R}^2, (\alpha_m, \beta_m) \in \mathbb{R}^2, \forall (m \in \mathbb{N})$

In this paper, from practical application point of view, we attempt to illustrate the benefits in a reproducible manner. It was pointed out in [16] that a band-limit implementation of fractional order controller is important in practice, and the finite dimensional approximation of the fractional order controller should be done in a proper range of frequencies of practical interest. This is true since

the fractional order controller in theory has an infinite memory and some sort of approximation using finite memory must be done.

### B. Genetic Algorithm

Genetic algorithm is a special type of evolutionary algorithms which uses reverted biology techniques such as inheritance and mutation. In fact, genetic algorithms utilize Darwin's principle of natural selection to find the optimal formula for predicting or matching patterns. Genetic algorithms are often a good option for prediction based on regression techniques. Briefly, genetic algorithm is a programming technique which employs genetic evolution as a problem-solving model. The problem, which has to be solved, is that the input and solutions are coded according to a pattern that is called fitness function. Each solution evaluates the candidate, while most of them are randomly selected. The flowchart of this algorithm is illustrated in Fig. 7; also the assigned variables to implement the method in MATLAB are available in Table 2.

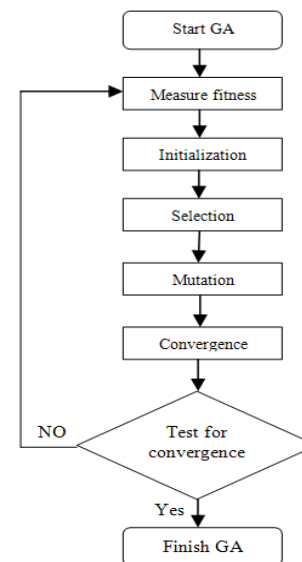


Fig.7. Biological genetic algorithm process flow.

Table 2: Properties of the conducted genetic algorithm

Option	Value
Crossover function	Heuristic
Crossover fraction	0.8
Elite number	2
Initial penalty	10
Mutation function	Adaptive feasible
Penalty factor	100
Population initial range	[-1,1]
Population size	100
Population type	Bit string
Selection function	Stochastic uniform

### C. Pattern Search Algorithm

Pattern search is a heuristic method that can be useful approximate solutions for some problem, but can fail on others. Outside of such classes, pattern search is not an iterative method that converges to a solution; indeed,

pattern search methods can converge to non-stationary points on some relatively tame problems.

#### IV. SIMULATION AND RESULTS

In this section, proposed fractional order PID controller is applied on the series manipulator shown in figure 2. The results are shown in appendix in two sections for the first and the second cost function.

These cost functions 10 and 11 consider the errors and errors plus links efforts respectively.

Each part is calculated by two genetic and pattern search Algorithms. In the appendix part, four figures have been shown, Path trajectory, two links angle based on degree, the error of links angle based on degree and the effort used for each links based on newton-meter.

All of results from two cost functions for FOPID controller are tabulated in table 3. As this table shows, the pattern search could find the values better than genetic algorithm.

The results of first cost function show a maximum effort for each link, whereas the second cost function considers the effort values and for this reason the result of this cost function shows a logical effort for moving the linkages.

$$J_1 = \int_0^{t_f} (e_1^2 + e_2^2) dt \quad (10)$$

$$J_2 = \int_0^{t_f} (e_1^2 + e_2^2 + ef_1^2 + ef_2^2) dt \quad (11)$$

Table 3: Reported value for FOPID Controller

		J1		J2	
		GA	PS	GA	PS
1st link FOPID gains	K <sub>P</sub>	11.15	114.90	4.71	9.34
	K <sub>I</sub>	10.42	26.73	1.90	0.15
	K <sub>D</sub>	4.60	0.11	2.27	1.52
	μ	0.27	0.10	0.35	0.35
	μ	0.34	0.26	0.12	0.62
2nd link FOPID gains	K <sub>P</sub>	5.64	33.08	0.79	2.04
	K <sub>I</sub>	5.31	1.11	1.36	0.11
	K <sub>D</sub>	1.65	16.61	0.32	0.32
	μ	0.85	0.67	0.90	0.90
	μ	0.28	0.33	0.10	0.10
J value		17	5	192	151

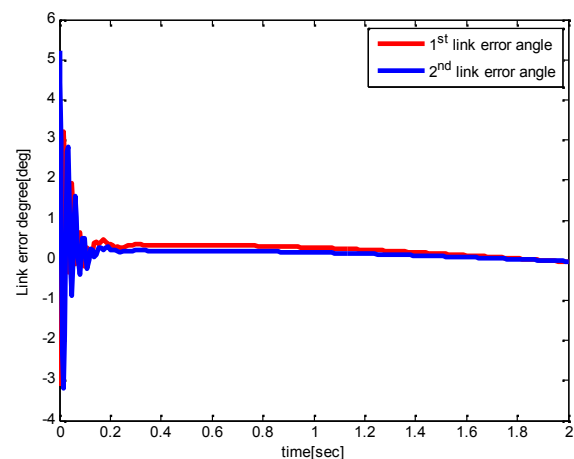
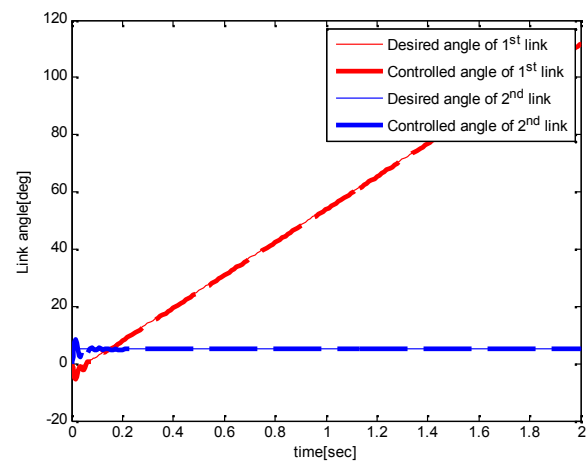
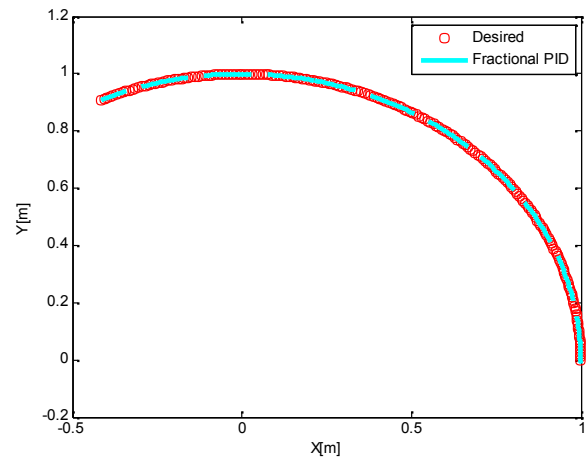
#### V. CONCLUSION

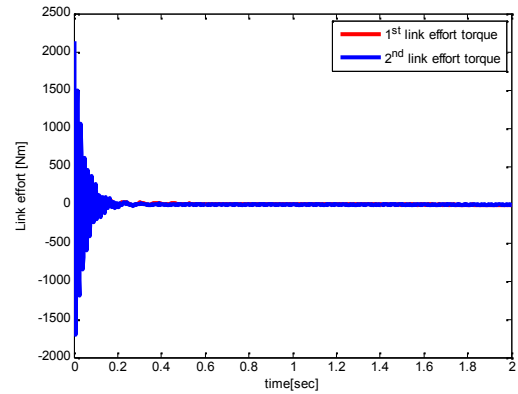
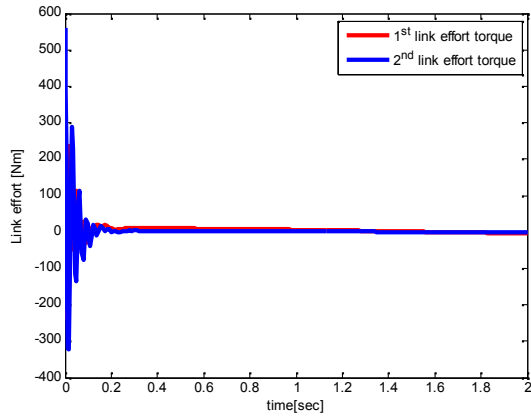
A trajectory control of a robot manipulator using optimized FOPID controller has been studied. Three PID control gains, K<sub>P</sub>, K<sub>I</sub>, K<sub>D</sub>, and two fractional order, λ and μ are adjustable parameters and will be updated offline with an adequate optimizing mechanism, genetic algorithm and direct search algorithm. The proposed control technique was analyzed for a series manipulator has different dynamic. (It is clear that robot dynamics have integer order derivative and just the controller has fractional order derivative). Simulation result verifies the significance of the fractional controller.

#### APPENDIX

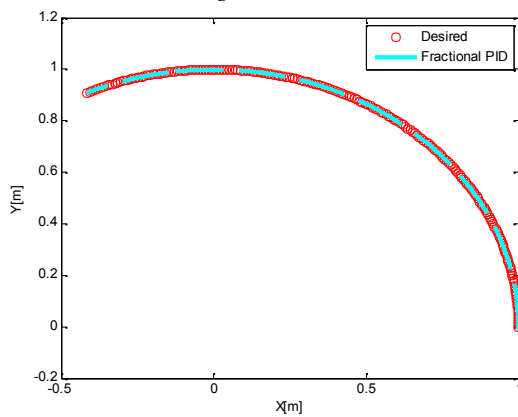
##### 1-First cost function

##### 1-1- Genetic Algorithm

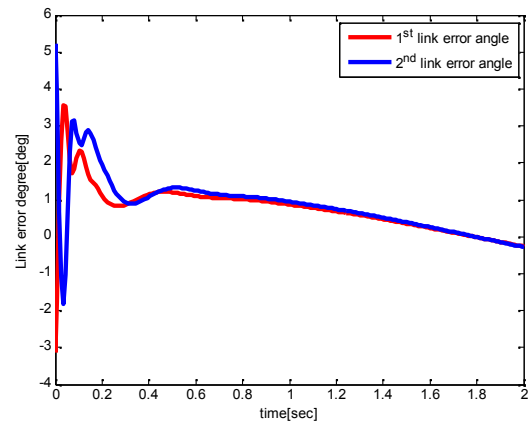
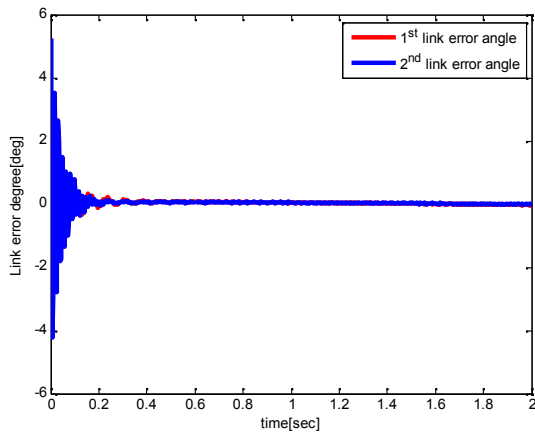
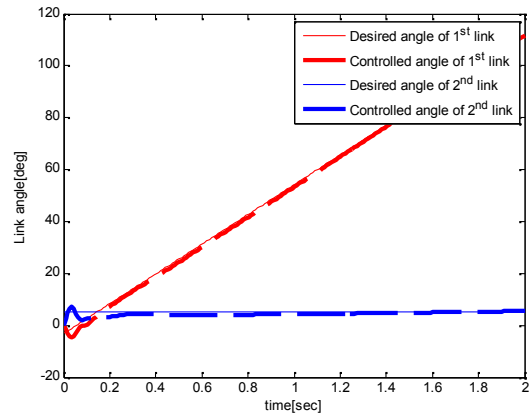
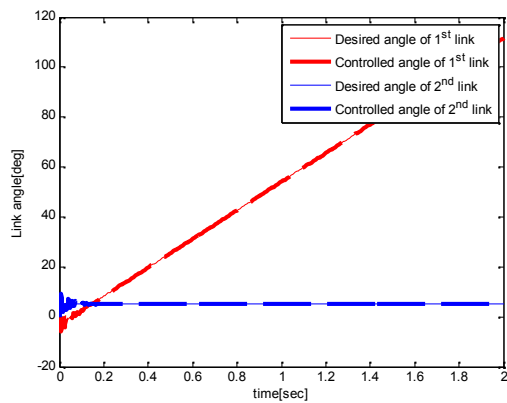
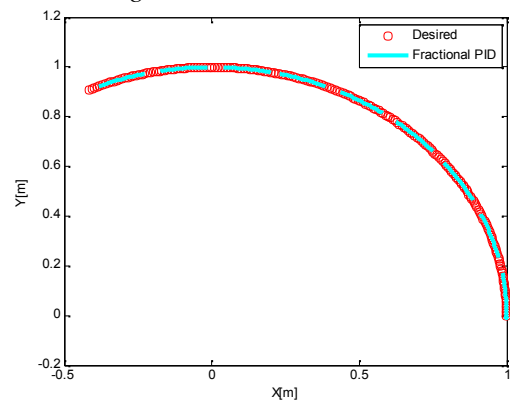


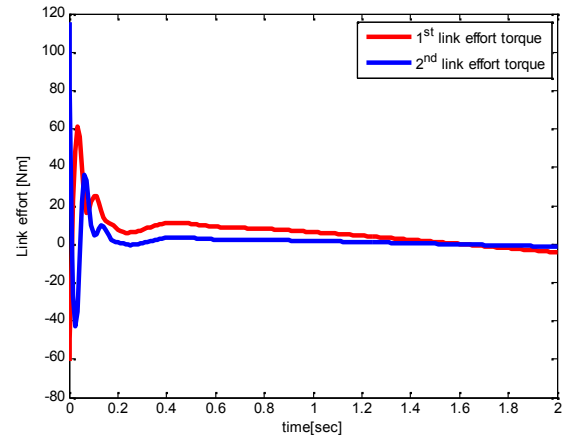
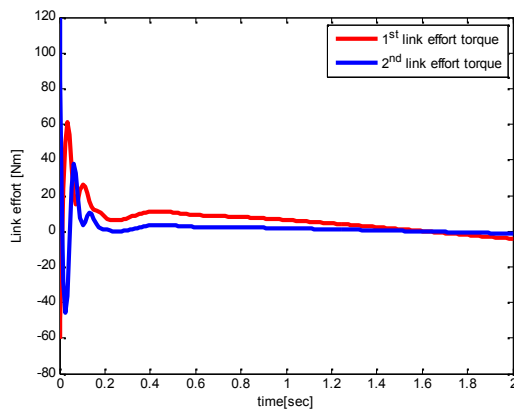


### 1-2- Pattern Search Algorithm

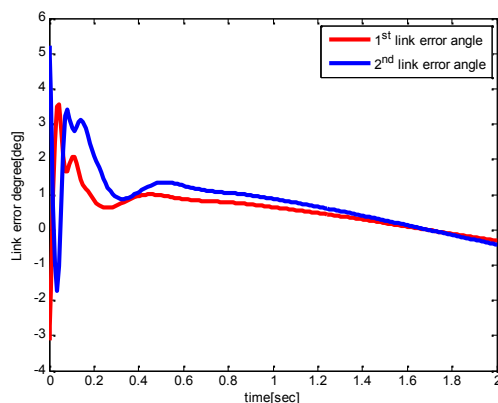
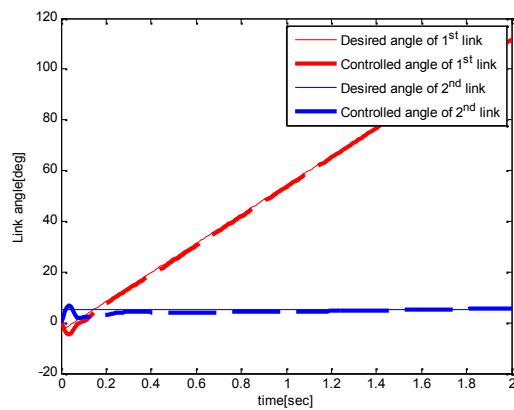
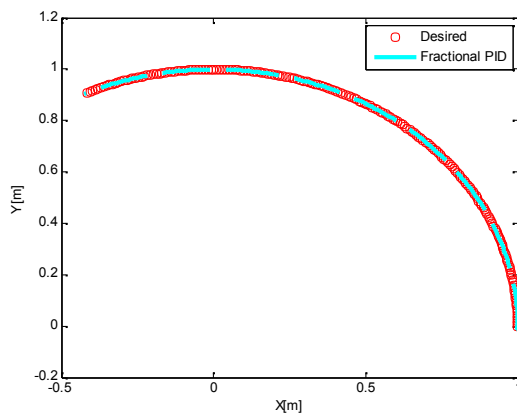


### 2-Second Cost Function 2-1- Genetic Algorithm





## 2-2- Pattern Search Algorithm



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