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Rough Properties of Soft Topological Space

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Abstract – **In this paper, the rough upper (lower) rough approximation is defined on the soft topological space over the rough soft formal context. The rough properties of the soft some rough properties are discussed over the soft topological.**

Keywords – **Rough Soft Formal Context, Upper (Lower) Rough Approximation, Topological Space, Soft Open (Closed) Set, Soft Interior Point.**

I. INTRODUCTION

Formal concept context[1] (see [1-5]), rough set[6] (see [6-11]), soft set[12] (see [12-25]) are extensively applied in the uncertainty reasoning, the uncertainty of data modeling the problems in engineering, physics, computer sciences, economics, social sciences, medical science and many other diverse fields. These may be due to the uncertainties of natural environmental phenomena, such as vagueness or uncertainty in the boundary between states or between urban and rural areas and so on. More and more researchers study these uncertainties and the related research results are flourishing and the related research results are flourishing.

In [26], authors defined the rough formal context, and discussed its properties. In [27], Shabir and Naz initiated the study of soft topological spaces. In [28], authors continued investigating the properties of soft topological spaces. In[29] , author defined the rough soft formal context, and discussed the rough properties of rough formal context in soft set. In [30], author discussed the topological structure of the rough soft formal context.

In this paper, we discuss the rough properties of topological structure based on the rough soft formal context. The rest of this paper is organized as following in section 2, we review some basic concepts and properties of rough concept formal, soft sets and soft topology space. In section 3, we define the rough approximation operations over the soft topological space and discuss rough properties of the rough lower (supper) approximations based on the soft topology over the rough soft formal context. Conclusions are given in section 4.

II. BASIC KNOWLEDGE

Definition 2.1[26]: Let (G,M,R) be an information system, where $G=\{a_1,\cdots,a_m\}$ is an object set, $M=\{x_1,\cdots,x_n\}$ is an attribute (property) set, R is an equivalent relation on G, $\forall A\subseteq G$, we can define the upper and the lower approximation of A about R:

$\overline{A_R}=\cup\{Y\in G/R\mid Y\cap A\neq\emptyset\}$

$=\{x\in G\mid [x]_R\cap A\neq\emptyset\}$;

$\underline{A_R}=\cup\{Y\in G/R\mid Y\subseteq A\}=\{x\in G\mid [x]_R\subseteq A\}$.

$\overline{A_R}$, $\underline{A_R}$ are called $R-$ upper approximation and $R-$ lower approximation of A. $\overline{A}$, $\underline{A}$ are denoted simply respectively. If $\overline{A}=\underline{A}$, we say that A is definable, otherwise, A is rough.

Similarly, we can define the upper and lower approximation of attributes set $B\subseteq M$ about an equivalent relation on M.

Definition 2.2[26]: Let (G, M, R) is a rough formal context, G is objects set, is also called the universe, M is attributes set. A pair (F,B) is a soft set over G, where $B\subsetneq M$, and $F:B\rightarrow\wp(G)$ is a set-value mapping over G, furthermore, the lower and upper rough approximations of pair (F,B) are denoted by $\underline{R}(F,B)=(\underline{F},B)$;$\overline{R}(F,B)=(\overline{F},B)$, which are soft sets over G with the set-valued mappings given by $\underline{F}(x)=\underline{B}(F(x))$ and $\overline{F}(x)=\overline{B}(F(x))$, where $x\in B$. The operators $\underline{R}$, $\overline{R}$ are called the lower and upper rough approximation operators on soft set (F,B).

If $\underline{R}=\overline{R}$, we say that the soft set (F;B) is definable, otherwise, (F;B) is rough. we call such quadruple tuple (G;M;R; F) as rough soft formal context, and, such soft set (F;B) on the rough soft formal context (G;M;R; F) which is called rough soft formal set.
Obviously, $\forall x\in B\subseteq M$; $F(x)\subseteq G$ is a parameterized family of subsets of G, and $F(x)$ is the set of x-approximate elements in (G, M, R, F).

Definition 2.3[12]: Let U be an initial universe set and E be a set of parameters. Let $\wp(U)$ denotes the power set of U and $A\subseteq E$. Then a pair (F; A) is called a soft set over U, where $F:A\rightarrow P(U)$ is a mapping.

That is, the soft set is a parameterized family of subsets of the set U. Every set $F(e)$, $\forall e\in E$, from this family may be considered as the set of e-elements of the soft set (F; E), or considered as the set of e-approximate elements of the soft set.

According to this manner, we can view a soft set (F, E) as consisting of collection of approximations: $(F,E)=\{(F(e),e)\mid e\in M\}$.

Definition 2.4 [29]: Let (G,M,R, F) be a rough soft formal context over the objects set G, and attributes set M. $B_1,B_2\subseteq M$, (F_1,B_1) and (F_2,B_2) are two soft sets over G on the rough soft formal context (G;M;R; F). F_1,F_2: $B\rightarrow\wp(G)$ is a set- value mapping over G on (G,M,R,F).
(i) If $B_1\subseteq B_2\subseteq M$, and $F_1(x)\subseteq F_2(x)$; $\forall x\in B_1\subseteq B_2$,then the soft sets (F_1,B_1) is a soft subset of the soft set $(\mathrm{F}_2,\mathrm{B}_2)$, denoted as $(F_1,B_1)\widetilde{\subset}(F_2,B_2)$.
(ii) Two soft sets (F_1,B_1) and (F_2,B_2) on the rough soft formal context (G; M; R; F) are said soft equal, if $(F_1,B_1)\widetilde{\subset}(F_2,B_2)$, and $(F_2,B_2)\widetilde{\subset}(F_1,B_1)$. We simply denote by $(F_1,B_1)=(F_2,B_2)$.

Definition 2.5 [29]: Let (F, B) is a rough soft formal set over the rough soft formal context (G;M;R; F) ,

(1) The relative complement of *(F, B)* is denoted by $(F,B)^c$ and is defined by $(F,B)^c$= (F^c,B), where F^c: $B \rightarrow \wp(G)$ and $F^c(x) = G - F(x)$; $\forall x \in B$. Clearly, $((F,B)^c)^c = (F; B)$.
(2) *(F, B)* is said to be a relative null rough soft formal set denoted by $\mathcal{N}$, if $\forall x \in B$; $F(x) = \emptyset$, if $B = M$, then is called absolute null rough soft formal set.
(3) *(F; B)* is said to be a relative whole rough soft formal set denoted by $\mathcal{W}$, if $\forall x \in B$; $F(x) = G$.

Example 1: Let (*G,M,R, F*). be a rough soft formal context, where $G = \{h_1, h_2, h_3, h_4, h_5\}$; $M = \{ e_1, e_2, e_3, e_4, e_5 \}$, in which e_1 :"expensive", e_2 : "beautiful", e_3: "wooden", e_4: "cheap", e_5: "in the green surrounding".
Let F_1, F_2: $M \rightarrow P(G)$, and suppose that:
$F_1(e_1) = \{ h_2, h_4\}$, $F_1(e_2) = \{h_1, h_3,\}$, $F_1(e_3) = \emptyset$; $F_1(e_4) = \{h_1, h_3, h_5\}$,$F_1(e_5) =\{h_1\}$.
Let $B = \{e_1, e_2\}$, then ${F_1}^c(e_1) = G - F_1(e_1) = \{h_1, h_3, h_5\}$, ${F_1}^c(e_2) = G - F_1(e_2) = \{h_2, h_4, h_5\}$, Hence, $(F_1; B)^c = ({F_1}^c,B) = \{(\{h_1, h_3, h_5\}, e_1), (\{h_2, h_4, h_5\}, e_2)\}$.
If let $B_1 = \{ e_1, e_3 \}; B_2 = \{ e_2 , e_4, e_5 \}$, then $(F_1, B_1) \widetilde{\subset} (F_2, B_2)$, and $(F_1, B_1) = (F_2, B_2)$.

Proposition 2.1[29]: Let (*G, M, R, F*) be a rough soft formal context over the objects set G, and attributes set *M*. $B_1, B_2 \subseteq M$, (F_1, B_1) and (F_2, B_2) are two soft sets over *G* on the rough soft formal context *(G; M; R; F)*. F_1, F_2: $B \rightarrow \wp(G)$ is a set- value mapping over *G* on *(G,M,R,F)* , *R*− upper approximation and *R*− lower approximation of (F_1, B_1) and (F_2, B_2) have many properties:
(1) $\underline{R}(F_1, B_1) \subseteq (F_1, B_1) \subseteq \overline{R}(F_1, B_1)$;
(2) $\overline{R}((F_1, B_1) \cup (F_2, B_2)) = \overline{R}(F_1, B_1) \cup \overline{R}(F_2, B_2)$ $\underline{R}((F_1, B_1) \cup (F_2, B_2)) \supseteq \underline{R}(F_1, B_1) \cup \underline{R}(F_2, B_2)$;
(3) $\overline{R}((F_1, B_1) \cap (F_2, B_2)) \subseteq \overline{R}(F_1, B_1) \cap \overline{R}(F_2, B_2)$ $\underline{R}((F_1, B_1) \cap (F_2, B_2)) = \underline{R}(F_1, B_1) \cap \underline{R}(F_2, B_2)$;
(4) if $(F_1, B_1) \subseteq (F_2, B_2)$, then $\underline{R}(F_1, B_1) \subseteq \underline{R}(F_2, B_2)$, $\overline{R}(F_1, B_1) \subseteq \overline{R}(F_2, B_2)$

Definition 2.6 [30] : Let (*G,M,R,F*) be a soft rough topological space over $\mathcal{T}$ which $\mathcal{T}$= (*G,M,R,F*) is a rough soft formal context, *G* is an initial universe set, *M* is a set of attributes, and (F_i, B_i),$B_i \subseteq M$; $i = 1,2$ is a soft set over *G*.
(i) The intersection of (F_1, B_1) and (F_2, B_2) is the soft rough formal set (*H;C*) is denoted as$(F_1, B_1) \cap (F_2, B_2)$ and is defined as $(F_1, B_1) \cap (F_2, B_2) = (H;C)$, where $C = B_1 \cap B_2$, and $\forall e \in C$, $H(e) = F_1(e) \cap F_2(e)$.
(ii) The extended union of (F_1, B_1) and (F_2, B_2) is the rough soft formal set (*H,C*), where $C = B_1 \cup B_2$, and $\forall e \in C$, denoted as $(F_1, B_1) \widetilde{\cup} (F_2, B_2) = (H; C) = (H, B_1 \cup B_2)$, where

$$H(e) = \begin{cases} F_1(e) & \text{if } e \in B_1 - B_2 \\ F_2(e) & \text{if } e \in B_2 - B_1 \\ F_1(e) \cup F_2(e) & \text{if } e \in B_1 \cap B_2 \end{cases}$$

(iii) The restricted intersection of (F_1, B_1) and (F_2, B_2) is the soft rough formal set (*H,C*) is denoted as $(F_1, B_1) \Cap (F_2, B_2)$ and is defined as $(F_1, B_1) \Cap (F_2, B_2) = (H,C)$, where $C = B_1 \cap B_2$, and $\forall e \in C$;$H(e) = F_1(e) \cap F_2(e)$.
(iv) The extended intersection of (F_1, B_1) and (F_2, B_2) is the rough soft formal set (*H;C*) , where $C = B_1 \cup B_2, \forall e \in C$, denoted as $(F_1, B_1) \sqcap_{\varepsilon} (F_2, B_2) = (H;C) = (H, B_1 \cup B_2)$, where

$$H(e) = \begin{cases} F_1(e) & \text{if } e \in B_1 - B_2 \\ F_2(e) & \text{if } e \in B_2 - B_1 \\ F_1(e) \cap F_2(e) & \text{if } e \in B_1 \cap B_2 \end{cases}$$

(v) The restricted union of (F_1, B_1) and (F_2, B_2) is defined as $(F_1, B_1) \cup_{\mathrm{R}} (F_2, B_2) = (H;C)$, where $C = B_1 \cap B_2$, and $\forall_e \in C$, $H(e) = F_1(e) \cup F_2(e)$.

Definition 2.7 [30]: Let $\mathcal{T}$ = (*G;M;R; F*) be a rough soft formal context over the object set *G* and attributes set *M*, $B_i \subseteq M$, $\tau = \{(F_i, B_i) \mid (F_i, B_i) \text{ is a soft set over } G\}$ which is the collection of soft sets on the rough soft formal context (*G;M;R; F*), if
(1) ϕ ,$\tilde{G}$ belong to τ;
(2) The union of any number of soft sets in τ belongs to τ, that is, τ is closed for the any union of soft sets over $\mathcal{T}$.
(3) The intersection of any two soft sets in τ belongs to τ, that is, τ is closed for the finite intersection of soft sets over $\mathcal{T}$.

Then the collection τ is called a soft topology over the rough soft formal context $\mathcal{T}$ (simply called soft rough topology).

The triplet (*G*, τ, *M*) is called a soft topological space over the rough soft formal context $\mathcal{T}$ (simply called soft rough topological space), and the members of τ are soft open sets in $\mathcal{T}$, the relative complement $(F,B)^c = (F^c, B)$ is said to be a soft closed set in $\mathcal{T}$ if $(F,B)^c \in \tau$.

III. THE ROUGH PROPERTIES OF THE SOFT TOPOLOGY

Definition 3.1 Let (*G*, τ, *M*) be a soft rough topological space over $\mathcal{T}$, in which $\mathcal{T}$ = (*G, M, R, F*) is a rough soft formal context, *G* is an initial universe set, *M* is a set of attributes. A pair (*F, B*) is a soft set over *G*, where $B \subseteq M$, and $F : B \rightarrow \wp(G)$ is a set-value mapping over *G*, furthermore, the lower and upper rough approximations of pair (*F; B*) are denoted by $\underline{R}(F,B) = (\underline{F},B)$, $\overline{R}(F;B) = (\overline{F};B)$, which are soft sets over *G* with the set-valued mappings given by $\underline{F}(x) = \underline{B}(F(x))$ and $\overline{F}(x) = \overline{B}(F(x))$, where $x \in B$. e can define the upper and the lower approximation of *A* about *R*:

$$\overline{\mathrm{A}_{\mathrm{R}}} = \cup\{Y \in G/R \mid Y \cap A \neq \emptyset\}$$
$$= \{x \in G \mid [\mathrm{x}]_{\mathrm{R}} \cap A \neq \emptyset\};$$
$$\underline{\mathrm{A}_{\mathrm{R}}} = \cup\{Y \in G/R \mid Y \subseteq A\} = \{x \in G \mid [\mathrm{x}]_{\mathrm{R}} \subseteq A\}.$$

$\overline{\mathrm{A}_{\mathrm{R}}}$, $\underline{\mathrm{A}_{\mathrm{R}}}$ are called *R*− upper approximation and *R*− lower approximation of *A*. $\overline{R}$,$\underline{R}$ are called the upper and lower rough approximation operators on soft set (*F;B*).

If $\overline{R} = \underline{R}$, we say that the soft set (*F, B*) is definable, otherwise, (*F,B*) is rough.

Proposition 3.1: Let (*G*, τ, *M*) be a soft rough topological space over $\mathcal{T}$ which $\mathcal{T}$ = (*G,M,R,F*) is a rough soft formal context, *G* is an initial universe set, *M* is a set of attributes, and (F_i, B_i), $B_i \subseteq M$, $i = 1, 2$ is a soft set over G. We have the following properties:

(1) $\underline{R}(F,B) \subseteq (F,B) \subseteq \overline{R}(F,B)$;
(2) $\underline{R}(\mathcal{N}_{(F,B)}) \subseteq \mathcal{N}_{(F,B)} \subseteq \overline{R}(\mathcal{N}_{(F,B)})$;
(3) $\underline{R}(\mathcal{W}_{(F,B)}) \subseteq \mathcal{W}_{(F,B)} \subseteq \overline{R}(\mathcal{W}_{(F,B)})$;
(4) $\underline{R}\ (\underline{R}(F,B)) = \underline{R}(F,B)$;
(5) $\overline{R}(\overline{R}(F,B) = \overline{R}(F,B)$;
(6) $\overline{R}(\underline{R}(F,B))= \underline{R}(F,B)$;
(7) $\underline{R}(\overline{R}(F,B)) = \overline{R}(F,B)$;
(8) $\underline{R}(F,B) = (\overline{R}(F,B)^c)^c$;
(9) $R(F,B) = (\underline{R}(F,B)^c)^c$;
(10) $\underline{R}((F_1,B_1) \cap (F_2,B_2)) = \underline{R}(F_1,B_1) \cap \underline{R}(F_2,B_2)$;
(11) $\overline{R}((F_1,B_1)\cap(F_2,B_2)) \subseteq \overline{R}(F_1,B_1)\cap \overline{R}(F_2,B_2)$;
(12)$\underline{R}((F_1,B_1)\cup(F_2,B_2)) \subseteq \underline{R}(F_1,B_1) \cup \underline{R}(F_2,B_2)$;
(13) $\overline{R}((F_1,B_1) \cup (F_2,B_2))= \overline{R}(F_1,B_1) \cup \overline{R}(F_2,B_2)$.

Proof: The proof is directed from the definition and the properties.

Proposition 3.2: Let (G, τ, M) be a soft rough topological space over $\mathcal{T}$ which $\mathcal{T}$ = (G, M, R, F) is a rough soft formal context, G is an initial universe set, M is a set of attributes, and (F_i, B_i), $B_i \subseteq M$, $i = 1, 2$ is a soft set over G, and $B_1 \cap B_2 \neq \emptyset$⍰, then

(1) $((F_1,B_1) \cup_{\mathrm{R}} (F_2,B_2))^c = (F_1,B_1)^c \Cap (F_2,B_2)^c$;
(2) $((F_1,B_1) \Cap (F_2,B_2))^c = (F_1,B_1)^c \cup_{\mathrm{R}} (F_2,B_2)^c$;
(3) $(F_1,B_1) \cup_{\mathrm{R}} ((F_2,B_2) \Cap (F_3,B_3)) = ((F_1,B_1) \cup_{\mathrm{R}} (F_2,B_2)) \Cap ((F_1,B_1) \cup_{\mathrm{R}} (F_3,B_3))$
(4) $(F_1,B_1) \Cap ((F_2,B_2) \cup_{\mathrm{R}} (F_3,B_3)) = ((F_1,B_1)$ e $(F_2,B_2)) \cup_{\mathrm{R}} ((F_1,B_1) \Cap (F_3,B_3))$.

Proof: They are easy using the definitions, in here, we omit them.

Theorem 1: Let (G, τ, M) be a soft rough topological space over $\mathcal{T}$ which $\mathcal{T}$ = (G, M, R, F) is a rough soft formal context, G is an initial universe set, M is a set of attributes, and (F_i, B_i), $B_i \subseteq M$, $i = 1, 2$ is a soft set over G. Then we have:

(1) $\underline{R}((F_1,B_1) \Cap (F_2,B_2)) =\underline{R}(F_1,B_1)\Cap \underline{R}(F_2,B_2)$;
(2) $\underline{R}((F_1,B_1) \sqcap_{\varepsilon} (F_2,B_2)) =\underline{R}(F_1,B_1) \sqcap_{\varepsilon} \underline{R}(F_2,B_2)$;
(3) $\overline{R}((F_1,B_1)\Cap(F_2,B_2)) \subseteq \overline{R}(F_1,B_1) \Cap \overline{R}(F_2,B_2)$;
(4)$\overline{R}((F_1,B_1)\sqcap_{\varepsilon} (F_2,B_2)) \subseteq \overline{R}(F_1,B_1) \sqcap_{\varepsilon} \overline{R}(F_2,B_2)$;
(5) $\underline{R}((F_1,B_1)\cup_{\mathrm{R}} (F_2,B_2)) \supseteq \underline{R}(F_1,B_1) \cup_{\mathrm{R}} \underline{R}(F_2,B_2)$;
(6) $\underline{R}((F_1,B_1)\tilde{\cup} (F_2,B_2)) \supseteq \underline{R}(F_1,B_1)\tilde{\cup}\ \underline{R}(F_2,B_2)$;
(7) $\overline{R}((F_1,B_1)\cup_{\mathrm{R}} (F_2,B_2))=\overline{R}(F_1,B_1) \cup_{\mathrm{R}} \overline{R}(F_2,B_2)$;
(8)) $\overline{R}((F_1,B_1)\tilde{\cup} (F_2,B_2)) = \overline{R}(F_1,B_1) \tilde{\cup}\ \overline{R}(F_2,B_2)$;
(9) $(F_1,B_1) \subseteq (F_2,B_2) \Rightarrow$
$\underline{R}(F_1,B_1) \subseteq \underline{R}(F_2,B_2); \overline{R}\,((FF_1,BB_1)\subseteq \overline{R}((FF_2,BB_2))$.

Proof: Let (G, τ, M) be a soft rough topological space over $\mathcal{T}$ which $\mathcal{T}$ = (G, M, R, F) is a rough soft formal context, G is an initial universe set, M is a set of attributes, and $(FF_i,BB_i), B_iBB_i \subseteq M$, $i = 1, 2$ is a soft set over.

(1) Let $(FF_1,BB_1) \Cap \overline{R}(FF_2,BB_2) =$ (H; C), then $C = BB_1 \cap BB_2$ and $H(x) = F_1(x)\cap F_2(x)$, $\forall\ x \in C$, using the definition, $\underline{R}\ ((FF_1,BB_1) \Cap (FF_2,BB_2)) =$ ($\underline{H}$, C), where $\underline{H}\ (x) = \underline{R}\ (H(x)) =\underline{R}\ \underline{R}(FF_1(x)\Cap FF_2(x))$, so we deduce that $\underline{H}\ (x) = \underline{RR}\ \underline{R}(FF_1(x)) \Cap \underline{R}(FF_2(x))$ for all $x \in C = B_1 \cap B_2$.

Hence, $\underline{R}((F_1,B_1) \Cap (F_2,B_2)) =\underline{R}(F_1,B_1)\Cap \underline{R}(F_2,B_2)$.

(2) Let $(F_1,B_1) \sqcap_{\varepsilon} (F_2,B_2)=$ (H,C), then $C = B_1 \cap B_2$, $\forall x \in C$, where

$$H(x) = \begin{cases} F_1(x) & \text{if } x \in B_1 - B_2 \\ F_2(x) & \text{if } x \in B_2 - B_1 \\ F_1(x) \cap F_2(x) & \text{if } x \in B_1 \cap B_2 \end{cases}$$

using the definition, $\underline{R}((F_1,B_1) \sqcap_{\varepsilon} (F_2,B_2))=$ ($\underline{H}$,C), where

$$\underline{H}(x) = \begin{cases} \underline{R}(F_1(x)) & \text{if } x \in B_1 - B_2 \\ \underline{R}(F_2(x)) & \text{if } x \in B_2 - B_1 \\ \underline{R}(F_1(x) \cap F_2(x)) & \text{if } x \in B_1 \cap B_2 \end{cases}$$

By proposition 3.1,
$\underline{R}(F_1(x) \cap F_2(x)) \supseteq \underline{R}(F_1(x)) \cap \underline{R}(F_2(x))$, so, for all $x \in C$,

$$\underline{H}(x) = \begin{cases} \underline{F_1}(x) & \text{if } x \in B_1 - B_2 \\ \underline{F_2}(x) & \text{if } x \in B_2 - B_1 \\ \underline{F_1}(x) \cap \underline{F_2}(x) & \text{if } x \in B_1 \cap B_2 \end{cases}$$

Hence,
$\underline{R}((F_1,B_1) \sqcap_{\varepsilon} (F_2,B_2)) = \underline{R}(F_1,B_1) \sqcap_{\varepsilon} \underline{R}(F_2,B_2)$.

(3) This is similar to the proof of (1).

(4) This is similar to the proof of (2).

(5) Let $(F_1,B_1)\cup_{\mathrm{R}} (F_2,B_2)=$ (H,C), then $C = B_1 \cap B_2; \forall x \in C$ and $H(x) =F_1(x)\cup F_2(x)$, using the definition,
$\underline{H}(x) =\underline{R}(F_1(x)\cup F_2(x)) =\underline{R}(F_1(x)\cup\underline{R}(F_2(x))$.

By proposition 3.1,
$\underline{R}((F_1,B_1)\cup(F_2,B_2)) \supseteq \underline{R}(F_1,B_1)\cup\underline{R}(F_2,B_2)$ Hence,
$\underline{R}((F_1,B_1)\cup_{\mathrm{R}} (F_2,B_2)) \supseteq\underline{R}(F_1,B_1)\cup_{\mathrm{R}} \underline{R}(F_2,B_2)$.

(6) Let $(F_1,B_1)\tilde{\cup} (F_2,B_2)=$ (H,C), then $C= B_1 \cup B_2$), $\forall x \in C$, where

$$H(x) = \begin{cases} F_1(x) & \text{if } x \in B_1 - B_2 \\ F_2(x) & \text{if } x \in B_2 - B_1 \\ F_1(x) \cup F_2(x) & \text{if } x \in B_1 \cap B_2 \end{cases}$$

using the definition, $\underline{R}((F_1,B_1)\tilde{\cup} (F_2,B_2))=$ (H;C), where

$$\underline{H}(x) = \begin{cases} \underline{R}(F_1(x)) & \text{if } x \in B_1 - B_2 \\ \underline{R}(F_2(x)) & \text{if } x \in B_2 - B_1 \\ \underline{R}(F_1(x) \cup F_2(x)) & \text{if } x \in B_1 \cap B_2 \end{cases}$$

By proposition 3.1,
$\underline{R}(F_1(x) \cup \underline{F_2}(x)) \supseteq \underline{R}(F_1(x)) \cup \underline{R}(F_2(x))$, so, for all $x \in C$,

$$\underline{H}(x) = \begin{cases} \underline{F_1}(x) & \text{if } x \in B_1 - B_2 \\ \underline{F_2}(x) & \text{if } x \in B_2 - B_1 \\ \underline{F_1}(x) \cup \underline{F_2}(x) & \text{if } x \in B_1 \cap B_2 \end{cases}$$

Hence,
$\underline{R}((F_1,B_1) \tilde{\cup} (F_2,B_2)) \supseteq \underline{R}(F_1,B_1) \tilde{\cup}\ \underline{R}(F_2,B_2)$.

(7) This is similar to the proof of (5).

(8) This is similar to the proof of (6).

(9) Suppose $(F_1,B_1) \subseteq (F_2,B_2)$, by definition 3.2, $F_1(x) \subseteq F_2(x), \forall x \in B_1$, using proposition 2.1,

$\underline{F_1}(x) \subseteq \underline{F_2}(x)$, and $\overline{F_1}(x) \subseteq \overline{F_2}(x)$, for all $\forall x \in B_1$.

Hence, we can conclude that

$\underline{R}(F_1, B_1) \subseteq \underline{R}(F_2, B_2); \overline{R}((F_1, B_1) \subseteq \overline{R}((F_2, B_2))$.

IV. CONCLUSION

In this paper, we discuss the rough properties of topological space based on soft formal context. Using the definition of rough set and soft set over the rough soft topology based on the rough formal context, we define the rough approximation operations over the soft topological space and discuss rough properties of the rough lower (supper) approximations based on the soft topology over the rough soft formal context. That is, we investigate the rough properties combing soft sets with rough sets, formal context and topology space, some different types of hybrid models are presented, which is rough properties with soft topology based on the rough soft formal context. That offers a new method and tool in data analysis. Therefore, we can deal with lots of data more easily and do more efficient decision in data mining, information system, human reasoning and so on.

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