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Super Resolution of an Image using Projection onto Convex Set Algorithm

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***Abstract* – Recently single image super resolution is important research area to generate high-resolution images from given low-resolution images. The term Super resolution is a process to increase the resolution of an image. This improvement quality of image is due to sub-pixel shift of low-resolution images from each other between images. In this paper, we concentrate on the problem of producing a high-resolution image from a single or multiple low-resolution images. We propose a POCS algorithm for the canonical problem of super-resolution (SR) image, which will obtain from low-resolution images. Single image super resolution algorithms are mainly based on frequency domain and spatial domain. Here image super resolution algorithm is presented which based on both spatial and frequency domain and consider the advantage of both. Algorithm is flexible and use back projection to minimize reconstruction error. Compared with traditional algorithm, experiment results show that the POCS algorithm eliminates the halo effect, thus the reconstructed image can achieve a good visual effect.**

***Keywords* – Projections Onto Convex Sets, Super Resolution, Low Resolution.**

I. INTRODUCTION

Super resolution (SR) is the term used to extract a high-resolution (HR) image from single or multiple low-resolution (LR) images [1]. The frequency domain methods are easy and computationally less complex. Recently, the spatial domain methods have attracted increased attention and having a major importance for SR image reconstruction. Interpolation techniques like pixel replication and bilinear interpolation up sample an image neglecting details of original image. These methods works better for smooth region by blurring edges and some texture [2]. The POCS algorithm for SR image construction was first suggested by Stark and Oskoui, and then extended by Tekalp to include observation noise. The major disadvantage of POCS algorithm is that Gibbs artifacts are existed on the edges of the HR reconstructed image. To overcome this shortcoming, we introduce an improved POCS algorithm. The paper is organized as follows: Section A introduces basics of POCS, Section B includes POCS algorithm, section C shows expected results on real and simulated images and the comparison to different algorithms based on its PSNR values. Section D gives expected results.

A. Basics of POCS

The term POCS is derived from two basic terms namely convex sets and projections. POCS is having large number of applications in Papoulis-Gerchberg Algorithm, Neural Network Associative Memory, Resolution at sub-pixel levels, Radiation Oncology / Tomography, JPG / MPEG repair, Missing Sensors etc.

In a vector space, a set C is convex $\forall$

$\vec{x} \in C$ and $\vec{Y} \in C$

$$\Rightarrow \lambda\vec{x} + (1-\lambda)\vec{y} \in C \quad \forall \quad 0 \le \lambda \le 1$$

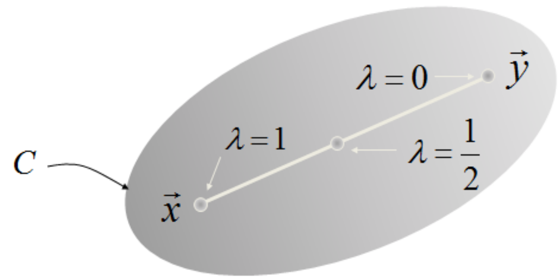

Fig.1. Given set C

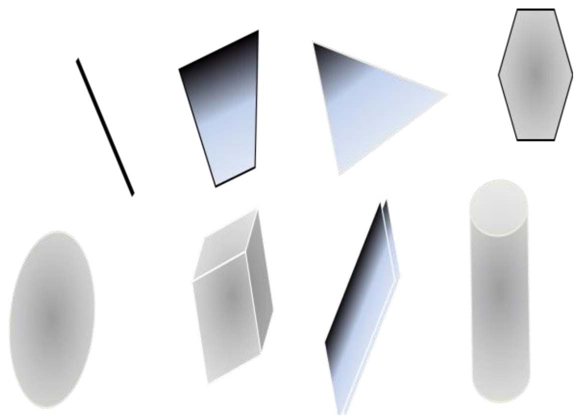

Fig.2. Example convex sets

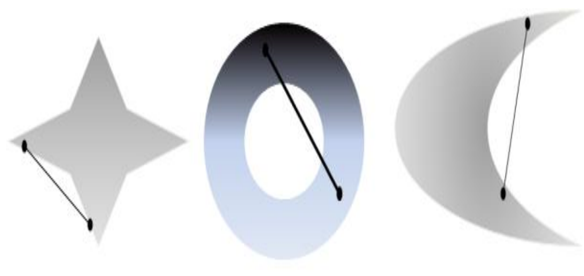

Fig.3. Example of non-convex sets

The concept of projection is given as, for every (closed) convex set, C, and every vector, x, in a Hilbert space, there is a unique vector in C, closest to x. This vector denoted P_C x, is the projection of x onto C:

The basic POCS algorithm to the SR reconstruction problem has been proposed in [4, 5]. The convex sets in any LR image are

$$c_{n1,n2,K,I} = \{x_K(m_1,m_2): |p_I^{x_j}(n_1,n_2)| \le \alpha_0\}$$

$0 \le n_1, n_2 \le N\text{-}1$, I = 1, 2………L (1)

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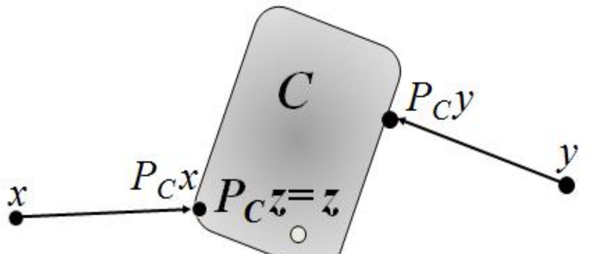

Fig.4. Projection of x on C

Where the value at each pixel is constrained such that it is associated residual

$$p_I^{x_j}(n_1,n_2) = o_K(n_1,n_2) - \sum_{m_1=0}^{M1}\sum_{m_2=0}^{M-1} x_K(m_1,m_2)\, f_K(m_1,m_2;n_1,n_2) \qquad (2)$$

is bounded in magnitude by α_0 for the set. Since α_0 is determined from the statistics of noise, the ideal image solution is a member of the set satisfying a certain statistical confidence. The projection of an arbitrary $x_K(m_1,m_2)$ onto $c_{n1,n2,K,I}$ is defined by

$$R_{n_1,n_2;j,K}[x_K(m_1,m_2)] = [x_I(m_1,m_2)] + \frac{(p_I^{x_K}(n_1,n_2) - \alpha_0) f_K(m_1,m_2;n_1,n_2)}{\sum_{m_1=0}^{M1}\sum_{m_2=0}^{M-1} f_K{}^2(m_1,m_2;n_1,n_2)}$$

$$p_I^{x_K}(n_1,n_2) > \alpha_0$$

$$0, -\alpha_0 < p_I^{x_K}(n_1,n_2) < \alpha_0 \qquad (3)$$

$$[x_I(m_1,m_2)] + \frac{(p_I^{x_K}(n_1,n_2) - \alpha_0) f_K(m_1,m_2;n_1,n_2)}{\sum_{m_1=0}^{M1}\sum_{m_2=0}^{M-1} f_K{}^2(m_1,m_2;n_1,n_2)}$$

$$p_I^{x_K}(n_1,n_2) > -\alpha_0$$

Additional constraints such as bounded energy, positivity, and limited support utilized to improve the results. Utilized altitude constraint set is

$$R_A[x_K(m_1,m_2)] = \begin{cases} 0, x_K(m_1,m_2) < 0 \\ x_K(m_1,m_2), 0, \le x_K(m_1,m_2) \le 255 \\ 255, x_K(m_1,m_2) > 255 \end{cases}$$

II. Proposed POCS Algorithm

A. Image Registration

To accurately find the motion parameters between different LR images, precise sub-pixel registration is necessary. In this paper, we used a method, which is based on the Taylor series expansion for a given function [5]. Using this method, the two-dimensional image function can be described as (4) if we only use the first three terms of the Taylor series

$$I(m_0,n_0) \cong I(m_1,n_1) + (m_0 - m_1)\frac{\partial I(m_1,n_1)}{\partial m} + (n_0 - n_1)\frac{\partial I(m_1,n_1)}{\partial n} \qquad (4)$$

Where $I(m_0,n_0)$ and $I(m_1,n_1)$ denote the reference image and other LR images respectively. (m_0-m_1) and (n_0-n_1) represent global translation between a pair of observed LR images. For simplicity, these will be written as E_x and E_y. E_x and E_y are estimated by minimizing the square error.

$$\varepsilon = \sum_m^M \sum_n^N \left[I(m_0,n_0) - I(m_1,n_1) - E_x \frac{\partial I(m_1,n_1)}{\partial m} - E_y \frac{\partial I(m_1,n_1)}{\partial n} \right]$$

Minimizing E is achieved by differentiation with respect to E_x and E_y, and setting the result to zero. The obtained equation can be expressed as

$$\begin{bmatrix} \sum_m^M \sum_n^N \left(\frac{\partial I(m_1,n_1)}{\partial m}\right)^2 & \sum_m^M \sum_n^N \frac{\partial I(m_1,n_1)}{\partial m}\frac{\partial I(m_1,n_1)}{\partial n} \\ \sum_m^M \sum_n^N \frac{\partial I(m_1,n_1)}{\partial m}\frac{\partial I(m_1,n_1)}{\partial n} & \sum_m^M \sum_n^N \left(\frac{\partial I(m_1,n_1)}{\partial n}\right)^2 \end{bmatrix} \cdot \begin{bmatrix} E_x \\ E_y \end{bmatrix} = \begin{bmatrix} \sum_m^M \sum_n^N \left(I(m_0,n_0) - I(m_1,n_1) \times \frac{\partial I(m_1,n_1)}{\partial m}\right) \\ \sum_m^M \sum_n^N \left(I(m_0,n_0) - I(m_1,n_1) \times \frac{\partial I(m_1,n_1)}{\partial n}\right) \end{bmatrix} \qquad (5)$$

The gradients in (5) were obtained from the Prewitt operator. In this LR, images were created by down sampling the original image. Because some aliasing was introduced in the process of down sampling, it is very important to pre-filter the images with a low pass filter to smooth the images. The motion parameters can be obtain for a low-resolution image in the pyramid, and then interpolated into a large image.

B. Bilinear interpolation

We assume the I of low-resolution image with size M×N is enlarged to the H of high-resolution image with size 2M×2N. The $H_{2i,2j}$ is zoomed from $I_{i,j}$. We determine the homogenous pixels at $H_{2i+1,2j}$, $H_{2i,2j+1}$ and $H_{2i+1,2j+1}$ using pixel difference criterion. Bilinear interpolation algorithm is used to interpolate when these pixels are homogenous [6]. The procedure for determining pixel difference is as follows:

$\Delta H_1 = |H_{2i,2j} - H_{2i+2p,2j+2q}|$

$\Delta H_2 = |H_{2i+2,2j} - H_{2i,2j+2}|$

$\Delta H_3 = |H_{2i,2j} - H_{2i+2,2j+2}|$

If $\Delta H_1 <$ threshold then
$H_{2i+p,2j+q} = (H_{2i,2j} + H_{2i+2p,2j+2p})/2$

Else

$H_{2i+p,2j+q}$ is edge pixel

Where p, q $\in$ {(0, 1), (1 , 0)}

If $\Delta H_2 <$ threshold and $\Delta H_3 <$ threshold then

$\Delta H_{min} = \min\{\Delta H_2\ \Delta H_3\}$

If $\Delta H_{min} = \Delta H_2$

$H_{2i+1,2j+1} = (H_{2i+2,2j} + H_{2i,2j+2})/2$ else

$H_{2i+1,2j+1} = (H_{2i,2j} + H_{2i+2,2j+2})/2$ else if

$\Delta H_2 <$ threshold then
$H_{2i+1,2j+1} = (H_{2i+2,2j} + H_{2i,2j+2})/2$ else if

$\Delta H_3 <$ threshold
$H_{2i+1,2j+1} = (H_{2i,2j} + H_{2i+2,2j+2})/2$ else

$H_{2i+1,2j+1}$ is edge pixel.

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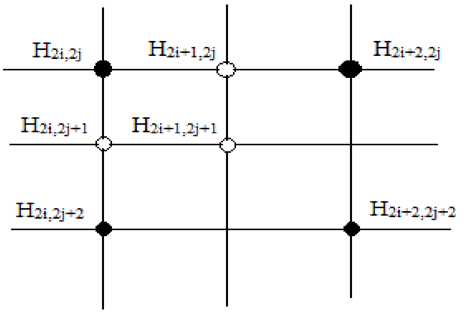


Fig.5. Interpolation of homogenous pixels

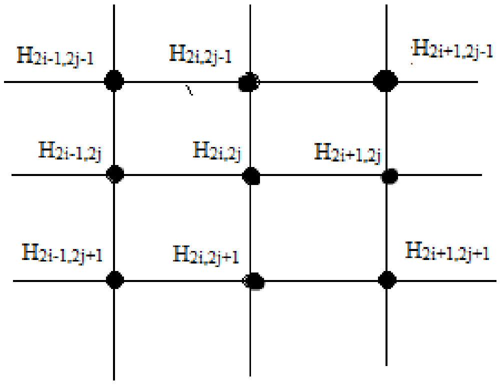


Fig.6. Interpolation of edge pixels.

C. *Image warping*

Image warping is a transition, which is applied to the source domain, which modifies the geometrical properties of the image. Consider an image I. Image warping produces a new image I_1 that is as follows:

$I_1(T_\theta(x)) = I(x)$ for each $x \in D$ Where,

T_θ is 2D mapping function where θ vector of parameters of transformation .x is is point to be mapped on domain D. The fundamental steps of a warping algorithm are the following:

1) Computation of the bounding box of the warped image (forward mapping).
2) Backward mapping of lattice points that sample the bounding box of the warped image (avoid “holes”).
3) Validation of the backward mapped points (must belong to the domain of the source image).
4) Intensity transfer via re-sampling.

D. *POCS super resolution*

I low-resolution (LR) images can be seen after geometric distorting, fuzzy linear space, down sample and additional noise as a high resolution (HR). Therefore, the imaging process can be written as [4]

$$O_I = D_I B_I M_I H_I + \Psi_I \qquad (6)$$

O_I is the observed image of size M X M

D_I is down sample matrix of size M X L

M_I is geometric distortion matrix of size LX L

B_I is fuzzy linear space matrix of size L X L

H_I is the original high-resolution image of size L X M

Ψ_I is Gaussian white noise of size M ×M

Different observed images are obtained from different geometric distortion, down sampling and additional noise to ideal images. The model above can be expressed as follows:

$$\begin{pmatrix} O_1 \\ . \\ . \\ . \\ O_N \end{pmatrix} - \begin{pmatrix} D_1B_1M_1 \\ . \\ . \\ . \\ D_1B_1M \end{pmatrix} * X + \begin{pmatrix} \psi_1 \\ . \\ . \\ . \\ \psi_N \end{pmatrix} = \begin{pmatrix} F_1 \\ . \\ . \\ . \\ F_N \end{pmatrix} * X + \psi_I \qquad (7)$$

III. Results

Here we loaded low-resolution images of resolution 480×640 with 0.3 mega pixel camera and registered them. The process of registering the image is shown below

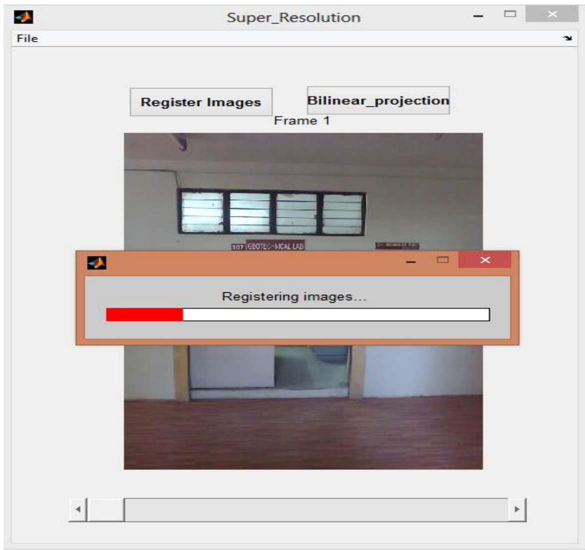


Fig.7 Image registration

The portion to be super-resolved is first selected and then cropped. The selection of portion is shown below

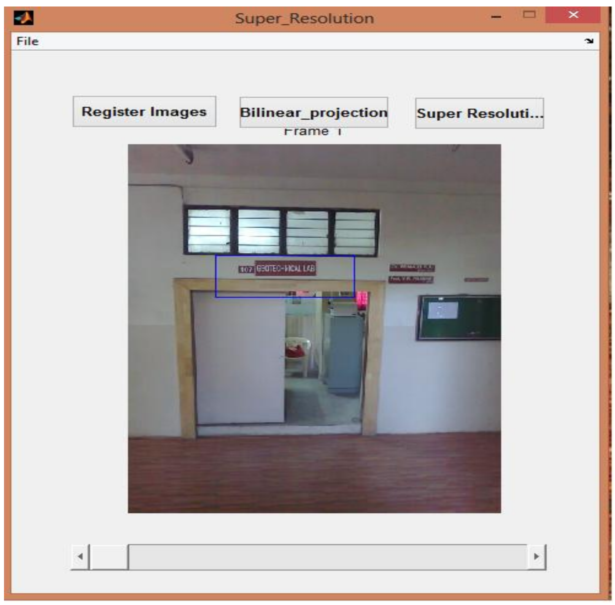


Fig.8 Selection of image

The cropped portion of the image is shown

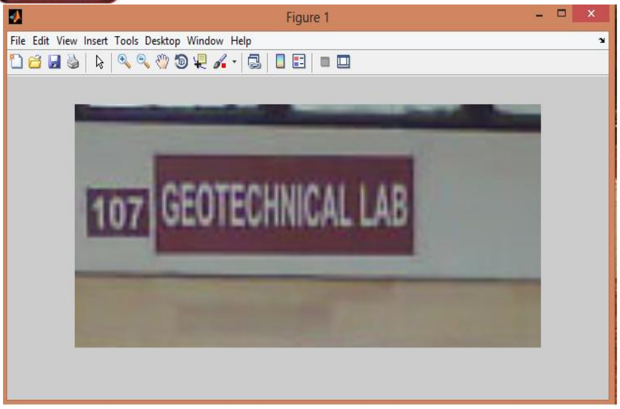


Fig.9. Cropped image

The values of image pixels before and after interpolation is as follows

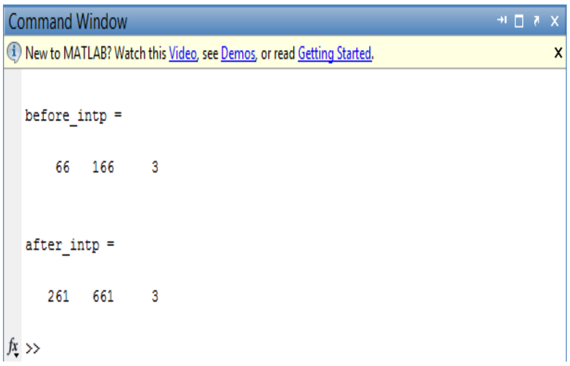
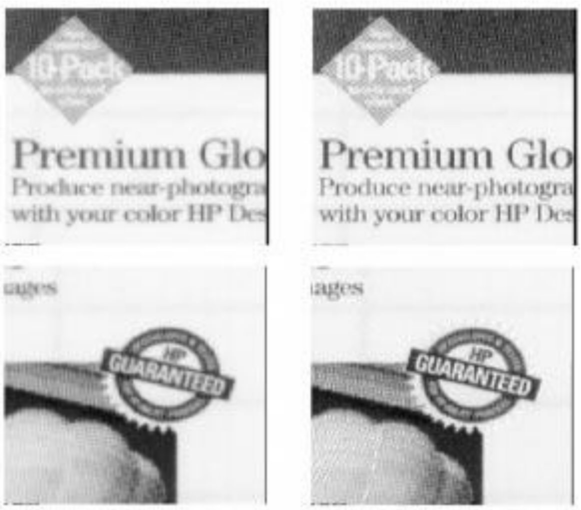


Fig.10 Interpolation values



(a) (b)

Fig.11. Expected results (a) Original image and (b) Expected output image

For performance evaluation of algorithm, visual quality and PNSR ratio are considered as parameters [4]. The PSNR is defined as:

$$PSNR = 10\, log \left[\frac{255^2}{\frac{1}{N} \left\| \hat{O} - O \right\|} \right]$$

Where N is the total number of pixels, O is the original image, and $\hat{O}$ is the reconstructed image. It can be seen that the proposed algorithm highly improves the quality of the reconstructed high-resolution images for all processed images, which are compared with POCS algorithm.

IV. Conclusion

A new method to estimate the unknown parameters in a high-resolution image reconstruction problem has been proposed using POCS method performed both in frequency and in spatial domain. In addition, it shows effectively results when applied to a practical application such as super-resolution imaging. The performance of proposed algorithm shows improvement in MSE and PSNR, which is good and computational complexity is low.

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