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Multi-Personal Communications in Bass Diffusion Model: Application to Forecasting of Development of Internet

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***Abstract* – Possibility of application of Bass diffusion model to forecasting of development of Internet is analyzed. It is shown that a quantitative description of the dynamics of sales based on the Bass diffusion model, in certain cases (Internet, particularly) is expedient to consider the influence of communications of orders, higher than the second (multi-personal communications). In this case, the differential equation describing the dynamics of sales includes terms with the degree higher than the second. Communications of higher orders can be interpreted from the standpoint of the cultural coercion phenomenon, i.e. a combination of factors forcing members of society to consume goods or services**

***Keywords* – Bass Diffusion Model, Internet, Multi-Personal Communications, Cultural Coercion.**

I. Introduction

Bass diffusion model [1] is well-known method of prediction and analysis of sales dynamics of new products or services (innovations). There is ample evidence [2-5] that the model adequately describes the dynamics of the marketing of innovations, but at the same time it is noted that the actual curves may differ from the predictions obtained on the base of the model. It is necessary to emphasize that theories similar to [1] mainly are used as some tools for market investigations; however, as shown in this paper, their significance is actually much greater. In particular, the analysis of the marketing of innovations (notably, this applies to new telecommunication systems) makes it simple to obtain important information about the structure of the concrete society.

In numerous studies, particularly, [2-5], there were attempts to make adjustments to the Bass's model. However, main assumptions that formed the basis of the model were left almost unchanged.

Namely, it was assumed that the promotion of innovation in the market is determined by two types of informational impact on the consumer. One of them is determined by the influence of advertising, and the other - by the transmission of information directly from one consumer to another («Word-in-mouth»). Accordingly, the Bass's equation using in the classical form consists of two terms, that reflect the mentioned informational impacts, and the equation's solution (logistic curve) is symmetrical in respect to the inflection point.

However, a more detailed analysis of the spread of information in society shows that the classical view does not reflect the features of the information transmission completely. Actual society has a rather complex structure of communication channels, also at least in some types of innovations the phenomenon of cultural coercion [6] significantly affects the dynamics of the process.

The phenomenon of cultural coercion may be explained by an example of innovation in telecommunications. Here it is seen more clearly that the need for a new product or service depends not only on the preferences of the individual, but also on how well the product is distributed in society. A typical example is mobile communications or Internet; to be more precise, with the expansion of their market even the conservative part of consumers, who initially refused to purchase cell phones or Internet, were forced to use their services.

The same factors of cultural coercion (including an idea of the "prestige", "modern", etc.) are also applicable to other types of goods and services, though in a less visible manner.

The classical model of Bass and its modifications do not consider such factors. In this paper it is shown that their consideration can be achieved by comparatively simple tools. Moreover, the quantitative analysis of this phenomenon provides relevant data on the informational structure of society and gives possibility to give more precise predictions of market of Internet services.

II. Observed Results

Figure 1 shows the curve of the dynamics of the Russian market of mobile phones in the 1997-2009 years. Such a curve is typical of any market for an innovative product; generally such dependencies are well described by the solution resulting from the Bass's model, i.e. logistic curve. As shown below, in the case where the curve described by the Bass equation, its phase portrait should be symmetrical. The phase portrait represents the dependence of a function's derivative from the value of the function. Applying to a logistic curve it should be a parabola.

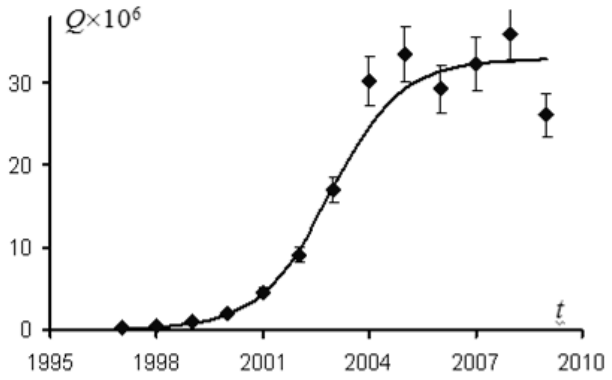

Fig.1. The dynamics of mobile phone sales in the Russian market by years, according to "Euroset", 2010.

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The curve in Figure 2 shows the phase portrait of Figure 1. It is obtained by numerical differentiation of the data from Figure 1 with subsequent numerical filtering. Approximation of the phase portrait by a parabolic dependence is shown in the same figure, curve 1.

It is seen that the phase portrait of the curve is really close to the parabolic. However, the considered curve is accurately approximated by a cubic polynomial, and for the initial part of the curve (where the market fluctuations do not appear) the accuracy of the approximation is very high (Fig. 2, curve 2). As shown in the next section, such polynominal dependence allows to take into account the above mentioned factors (the existence of rather complex informational structure of the society and factor of cultural coercion).

III. Cultural Compulsion and the Informational Structure Of Society

Model [1] can be formulated in terms of the differential equation, which takes into account the informational impact of the above mentioned types on the dynamics of sales.

$$\frac{dN}{dt} = \alpha\left(N_0 - N\right)N + \beta\left(N_0 - N\right) \qquad (1)$$

where coefficients α and β characterize the intensity of informational influences that determine the dynamics of the marketing of an innovation, N_0 - market potential for this product or service, $N(t)$ - the number of consumers at a moment *t*. Traditionally, the term proportional to α is associated with interpersonal communication, the term proportional to β - with the impact of the mass media and advertisement.

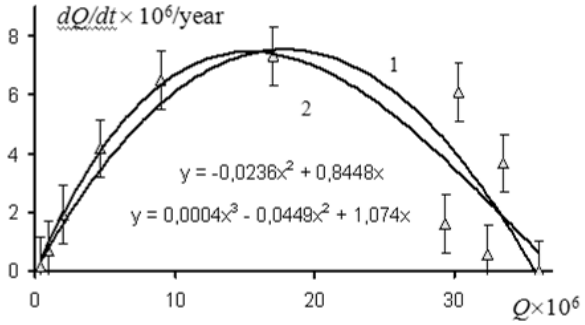

Fig.2. Phase portrait of the sales curve shown in Fig. 1

The solution of this equation can be written as:

$$N(t) = \frac{N_0 + N_1}{1 + c\exp\left(-t/\tau\right)} - N_1 \qquad (2)$$

,

where $N_1 = \beta/\alpha$, $\frac{1}{\tau} = \alpha\left(N_0 + N_1\right) = \alpha N_0 + \beta$,

a constant c is determined from the initial conditions.

Equation (1) can be considered as an equation for the phase portrait of the empirical curves; derivatives can be found with the numerical differentiation of experimental curve (Fig.2.).

For the purpose of this work it is significant that the equation (1) considers two types of informational influence (mass influence of advertisement, and mass influence of information, transferred in individual communications). Such a classification is approximate: firstly, society is characterized as a complicated informational structure [7], secondly, there is such phenomenon as cultural coercion, considered, for example, in [6], as it was mentioned above.

Existence of the cultural coercion is evident in case of next example connecter with cellular phones. It is well known that a sufficiently large group of people (especially in CIS countries), mostly of adult age, who for a long time refused to use mobile phones. However, with the spread of the cellular, they were forced to adopt the technology.

Mathematical description of the phenomenon of the cultural coercion can be easily developed based on consideration of two competing services with the long-term nature. Examples of such services could be - services of mobile connections, as well as virtual services, provided by different web-sites (search services, etc.).

Starting from the formal analogy with the Bass model, next two equations similar to (1), that describe the dynamics of sales of the service (or the dynamics of visits to virtual services) should be supplemented by members that reflect competition in the following form.

$$\frac{dN_1}{dt} = \alpha_{01}\left(N_0 - N_1 - N_2\right)N_1 + \beta_{01}\left(N_0 - N_1 - N_2\right) + \beta_{21}N_2 - \beta_{12}N_1 + \alpha_{21}N_2N_1 - \alpha_{12}N_2N_1 \qquad (3)$$

$$\frac{dN_2}{dt} = \alpha_{02}\left(N_0 - N_1 - N_2\right)N_2 + \beta_{02}\left(N_0 - N_1 - N_2\right) - \beta_{21}N_2 + \beta_{12}N_1 - \alpha_{21}N_2N_1 + \alpha_{12}N_2N_1 \qquad (4)$$

where $N_{1,2}$ - sales (or the number of site visitors) of the first and second product, respectively, α_{ij} - coefficients characterizing the informational influence of consumers at each other, β_{ij} - coefficients that characterize the effect of mass media.

In particular, in (3) and (4) based on the formal analogy with the equation of Bass, appears a member $\alpha_{21}N_2N_1$, which describes the transition of users from project 2 to project 2 by information that users 1 report to users 2.

If we consider the most extreme case of the competition that meets the condition of market saturation, and if, for the purpose of clarity, we exclude from consideration the impact of advertising and other sources of information.

$$N_0 = N_1 + N_2 \ . \qquad (5)$$

Equation (5) expresses that in the case of saturation of the market the average total number of users that consider services (users of both sites) remains constant. Increasing the number of users of one of them can take place only by reducing the number of users of the other. This corresponds to the most extreme case in which a change of the parameters under consideration can take place only through competition between the services.

Then, equations (3) and (4) take the following form.

$$\frac{dN_1}{dt} = \alpha_{21}N_2N_1 - \alpha_{12}N_2N_1 \qquad (6)$$

$$\frac{dN_2}{dt} = -\alpha_{21} N_2 N_1 + \alpha_{12} N_2 N_1 \quad (7)$$

In the case when actual consumer properties of products 1 and 2 are similar, the coefficients α_{12} and α_{21} must be equal with the high accuracy. An additional argument in favor of this conclusion is the fact that the average communication skills of consumers in both projects are obviously the same. (They are members of the same society.)

In these circumstances equations (6) and (7) degenerate.

$$\frac{dN_1}{dt} = 0 \quad (8)$$

$$\frac{dN_2}{dt} = 0 \quad (9)$$

In other words, the formal analogy with the Bass's model gives obviously inadequate result, contrary to observations. Usually one of the two original products completely displaces the other. It can be seen, in particular, in examples of the spread of Web search engines in the Internet.

The way to overcome the observed difficulty is to take into account that it is unnecessary more than two people may be involved in communication. (The communication with n participants will be called the n-degree communication.)

Considering the 3-rd degree communications equations (8) and (9) take the next form.

$$\frac{dN_1}{dt} = \gamma_{21} N_1^2 N_2 - \gamma_{12} N_2^2 N_1 \quad (10)$$

$$\frac{dN_2}{dt} = -\gamma_{21} N_1^2 N_2 + \gamma_{12} N_2^2 N_1 \quad (11)$$

where coefficients γ_{ij} characterizes the informational fusion of consumers to each other in the 3-rd degree communications .

In this case term $\gamma_{21} N_1^2 N_2$ defines the change of 2nd service user's opinion influenced by the information reported to him by two 1st service users and member $\gamma_{12} N_2^2 N$ - the reverse effect. It is easy to see that any other communications of third degree does not really affect the sales dynamics.

Considering this result, the equations describing the dynamics of competing services should be written as follows .

$$\frac{dN_1}{dt} = \gamma N_1 N_2 (N_1 - N_2) + \beta_{21} N_2 - \beta_{12} N_1 \quad (12)$$

$$\frac{dN_2}{dt} = -\gamma N_1 N_2 (N_1 - N_2) - \beta_{21} N_2 + \beta_{12} N_1 \quad (13)$$

Note that in this study the competition between the services is primarily considered for best clarity. Specifically, this example shows the importance of communication of a degree higher than second.

It is also clear that the communications of higher degree may be important not only in the case of two competing services. In particular, the analog of Bass's equation should be written in the next form.

$$\frac{dN}{dt} = \alpha (N_0 - N) N + \beta (N_0 - N) + \gamma (N_0 - N) N^2 \quad (14)$$

This form meets the cubic approximation of the phase portrait, considered in section 2.

It should be emphasized that the probability of the communication of a higher-than-second degree in most cases is relatively low (the simplest example is the conversation of three or more people at a time.) There is even lower probability that in such communication two consumers of a particular service (goods and services) will enter simultaneously.

This probability becomes significant only if the particular service (product, service) really gets the mass distribution (the evident example is Internet as whole). That is why it can be argued that the terms of higher degrees describe the phenomenon of cultural coercion. In a certain sense, they reflect the influence of the environment on the dynamics of consumption.

Further, equation (14) can be formally generalized, because along with the third-degree communications there is a probability that communications of 4th and higher orders will appear. In the case of such communication, efficiency of informational influence increases. In particular, if a certain person hears positive feedback on the product or service from three or more interlocutors at once, he will more likely develop a positive opinion on it. So we have.

$$\frac{dN}{dt} = \sum_{n=0} \alpha_n N^n (N_0 - N) \quad (15)$$

.

At first glance, such a generalization has a purely formal nature; however, it can be used to predict the dynamics of sales or, more broadly, the dynamics of promoting innovation.

IV. Possibilities of Using the Mathematical Description of the Phenomenon of Cultural Coercion to Solve Forecasting Problems

In order to demonstrate the use of the decomposition (15) to solve prognostic problems, we will use next example having evident interest for Internet researches. Figure 3 shows the number of users of Facebook in time (curve 1), site data [7].

The same figure shows the smoothed curve (curve 2) obtained on the basis of observations by the method of splines.

By the numerical differentiation of the smoothed curve we obtained the phase portrait of studying dependence, presented in Figure 4. The second and fourth degree polynomial approximation of the phase portrait is shown in the same figure (curves 1 and 2, respectively). You can see that a polynomial of degree 4 describes the phase portrait built on the basis of the field observations data, with higher accuracy.

The choice of the fourth degree of the polynomial is determined by the following considerations. Qualitatively, the shape of the curve describing the phase portrait should

be close to the parabola. More precisely, the phase portrait should reflect the existence of only two stable states, i.e. cross the x-axis at two points. Cubic approximation doesn't always meet this condition, in which case you should go to the approximation of the 4th degree.

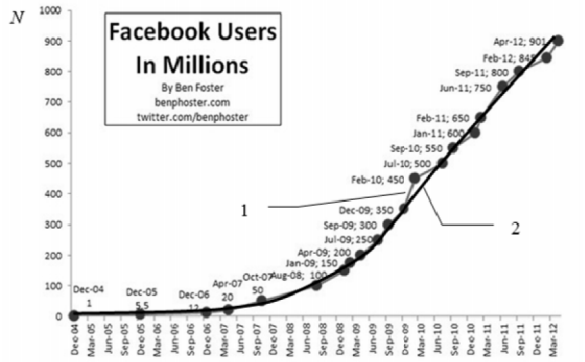

Fig.3. The growing number of users of Facebook.
1-raw data, http://www.justsocial.com/facebook-updated-stats.html/, 2 - smoothed approximation

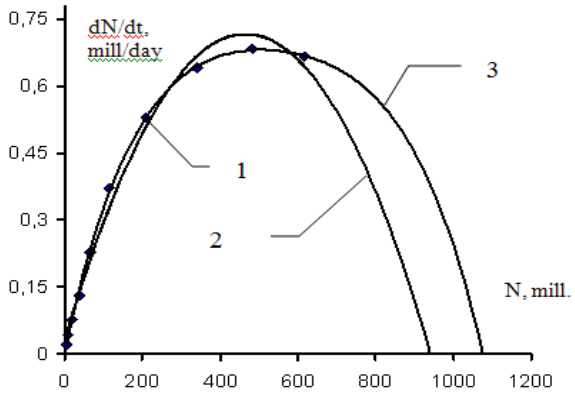

Fig.4. Phase portrait of the curve 2, Figure 3
1-point on the basis of experimental data, 2-quadratic polynomial approximation, 3 - by the 4th degree polynomial.

It is seen that polynomial of the fourth degree, in contrast to the second-degree polynomial, adequately describes the phase portrait obtained on the basis of experimental data.

These differences significantly affect the forecasts obtained on the basis of the Bass's diffusion model.

Figure 5 presents extrapolations of the number of Facebook users, derived from the phase portraits, figure 4, curves 2 and 3, respectively. It can be seen that the forecast obtained directly on the Bass's model, gives essentially different results than the similar forecast, which takes into account the higher-degree communications, interpreted through the cultural factor of coercion.

Equations were used with following numeric coefficients.

$$\frac{dN}{dt} = -3\cdot 10^{-06}x^2 + 0.0029x + 0.0306$$

$$\frac{dN}{dt} = -4.5\cdot 10^{-12}x^4 + 0.99\cdot 10^{-08}x^3 - 9.2\cdot 10^{-06}x^2 + 0.0041x + 0.0034$$

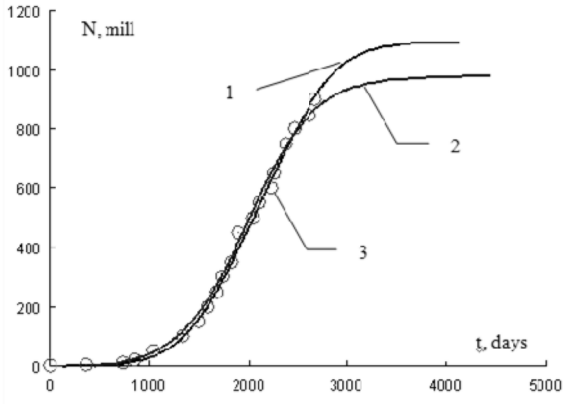

Fig.5. Extrapolation of the number of Facebook users by the Bass Model (2) and improved model (1), (3) – initial data

V. Conclusion

Thus, the existence of the phenomenon of cultural coercion makes it reasonable to take into account higher-degree communications when considering the Bass diffusion model and its analogs. Communications of a degree higher than the second (i.e., those that involve more than 2 people at the same time) can be accounted for by additional terms in the differential equation derived from the Bass,s Model.

Taking into account this type of communication is essential when the commodity (service) goes to mass distribution as it is evident for Internet. In this case the forecasting based on the classical Bass's model and its modification proposed in this study may significantly differ from each other.

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