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Convergence Newton – Codes Quadrature of Difference Sampling Period to Bilinear Transformation

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Abstract **— There are two types of numerical integration rules implemented by Newton– Codes quadrature rule(NCQ) and other is Gauss legendry integration rule[9]. We Proposed convergence of NCQ of different sampling periods for various order m to bilinear transformation of S to Z by introducing Lagrange interpolation appropriate fractional Z Values.**

Keywords **— Newton – Codes, Gauss – Legendre, Lagrange Interpolation.**

I. Introduction

THE DIGITAL integrator is an important device in the areas of control, biomedical engineering, and radar [1]–[8]. The design methods of digital integrator generally can be classified into two categories. One is the linear phase finite-impulse response (FIR) filter approach in which the filter coefficients are determined by using maximal flatness constraints [1], [2], the other is the infinite-impulse response (IIR) filter method in which the filter coefficients are obtained directly from well-known rectangular, trapezoidal and Simpson methods of numerical integration [3]–[8]. In the literature, some typical transfer functions of IIR integrators are given by

$$H_1(Z) = T\frac{Z^{-1}}{1-Z^{-1}} \quad (1)$$

$$H_2(Z) = \frac{T}{2}\frac{1+Z^{-1}}{1-Z^{-1}} \quad (2)$$

$$H_3(Z) = \frac{T}{3}\frac{1+4Z^{-1}+Z^{-2}}{1-Z^{-2}} \quad (3)$$

There are two types of numerical integration rules to be studied. One is Newton–Cotes quadrature (NCQ) rule, the other is Gauss–Legendre integration rule.

II. Design Based on Different Sampling Period for Various Orders M

Now, let us study how to use the composite Newton–Cotes rules $Q_{NC}(m)$ with h = (T) /(m-1) to design digital IIR integrators. Given signal x(n), the output y(n) of the integrator is given by and explained in [9].

$$y(n) = \sum_{l=0}^{\infty} \beta(m)\frac{T}{m-1}$$

$$y(n) = \sum_{t=0}^{\infty} \beta(m)\frac{T}{m-1} \times \Big[\sum_{K=0}^{M-1} \gamma_m (m-1-$$

$$kx\, n{-}km{-}1{-}l \quad (4)$$

Taking Z-transform on both sides of (4) and using the sym-metric condition in (14) in [9]we have

$$Y(z) = \beta(m)\frac{T}{m-1}\left(\sum_{l=0}^{\infty}\sum_{K=0}^{M-1}\gamma_m(k)Z^{-\frac{k}{m-1}-l}\right)X(z)$$

$$\beta(m)\frac{T}{m-1}\left(\sum_{K=0}^{M-1}\gamma_m(k)Z^{-\frac{k}{m-1}-l}\right)\times\left(\sum_{l=0}^{\infty}Z^{-l}\right)X(z)$$

$$= \beta(m)\frac{T}{m-1}\frac{\sum_{K=0}^{M-1}\gamma_m(k)Z^{-\frac{k}{m-1}}}{1-Z^{-1}}X(z) \quad (5)$$

Thus, the transfer functions for quadrature rules $Q_{NC}(m)$ are given by

$$F_m(Z) = \frac{Y(z)}{X(z)} = \beta(m)\frac{T}{m-1}\frac{\sum_{K=0}^{M-1}\gamma_m(k)Z^{-\frac{k}{m-1}}}{1-Z^{-1}} \quad (6)$$

Based on (12) in [9] and (6), some typical transfer functions with $m = 2,3,4,5$ are given by

$$E_2(Z) = \frac{T}{2}\frac{1\pm Z^{-1}}{1-Z^{-1}} \quad (7)$$

$$E_3(Z) = \frac{T}{6}\frac{1\pm 4Z^{-1/2}\pm Z^{-1}}{1-Z^{-1}} \quad (8)$$

$$E_4(Z) = \frac{T}{8}\frac{1\pm 3Z^{-1/3}\pm 3Z^{-2/3}\pm Z^{-1}}{1-Z^{-1}} \quad (9)$$

$$E_5(Z) = \frac{T}{90}\frac{7\pm 32Z^{-1/4}\pm 12Z^{-1/2}\pm 32Z^{-3/4}\pm 7Z^{-1}}{1-Z^{-1}} \quad (10)$$

III. Lagrange Interpolation

Lagrange interpolation method for approximating a fractional delay $z^{-(I \square d)}$ [10], is used as the following

$$Z^{-(I+d)} = \sum_{n=0} h(n)Z^{-n} \quad (11)$$

d = 1/2

$$Z^{-(I+d)} = h(0) + h(1)Z^{-1}$$

For a first order if I = 0 and d= ½ m=1, n=0

$$h(n) = \prod_{m=0\, m\neq n}^{1}\frac{0+\frac{1}{2}-m}{n-m}$$

$$h(0) = \frac{0+\frac{1}{2}-1}{0-1} = \frac{-1/2}{-1} = \frac{1}{2}$$

$$h(1) = \frac{0+\frac{1}{2}-0}{1-0} = \frac{1/2}{1} = \frac{1}{2}$$

$$Z^{-(I+d)} = h(0) + h(1)Z^{-1}$$

$$Z^{-(0+\frac{1}{2})} = \frac{1}{2} + \frac{1}{2}Z^{-1}$$

$$Z^{-(1/2)} = \frac{1}{2} + \frac{1}{2}Z^{-1} \quad (12)$$

d = 1/3

$Z^{-(I+d)} = \sum_{n=0}^{1} h(n)Z^{-n}$

For a first order if I = 0 and d= 1\3 m=1, n=0

$Z^{-(0+1/3)} = \sum_{n=0}^{1} h(n)Z^{-n}$

$Z^{-(I+d)} = h(0) + h(1)Z^{-1}$

$h(n) = \prod_{m=0\ m\neq n}^{1} \frac{I+d-m}{n-m}$

$h(0) = \frac{0+\frac{1}{3}-1}{0-1} = 1 - \frac{2/3}{-1} = 2/3$

$h(1) = \frac{0+\frac{1}{3}-0}{1-0} = \frac{1/3}{1} = \frac{1}{3}$

$Z^{-(I+d)} = h(0) + h(1)Z^{-1}$

$$Z^{-(1/3)} = \frac{1}{3} + \frac{1}{3}Z^{-1} \qquad (13)$$

d = 2/3

$Z^{-(I+d)} = \sum_{n=0}^{1} h(n)Z^{-n}$

$Z^{-(I+d)} = h(0) + h(1)Z^{-1}$

$h(n) = \prod_{m=0\ m\neq n}^{1} \frac{I+d-m}{n-m}$

For a first order if I = 0 and d= 2\3 m=1, n=0

$h(0) = \frac{0+\frac{2}{3}-1}{0-1} = \frac{-1/3}{-1} = 1/3$

$h(1) = \frac{0+\frac{2}{3}-0}{1-0} = \frac{2/3}{1} = \frac{2}{3}$

$Z^{-(0+2/3)} = h(0) + h(1)Z^{-1}$

$$Z^{-(2/3)} = \frac{1}{3} + \frac{2}{3}Z^{-1} \qquad (14)$$

d = 1/4

$Z^{-(I+d)} = \sum_{n=0}^{N} h(n)Z^{-n}$

$h(n) = \prod_{m=0\ m\neq n}^{N} \frac{I+d-m}{n-m}$

$Z^{-(I+d)} = \sum_{n=0}^{1} h(n)Z^{-n}$

$Z^{-(I+d)} = h(0) + h(1)Z^{-1}$

For a first order if I = 0 and d= 1\4 m=1, n=0

$h(0) = \prod_{m=0\ m\neq n}^{1} \frac{0+1/4-1}{0-1} = \frac{-3/4}{-1} = 3/4$

$h(0) = 3/4$

$h(1) = \frac{0+\frac{1}{4}-0}{1-0} = \frac{1/4}{1} = \frac{1}{4}$

$Z^{-(0+1/4)} = h(0) + h(1)Z^{-1}$

$$Z^{-(1/4)} = \frac{3}{4} + \frac{1}{4}Z^{-1} \qquad (15)$$

d = ¾

$Z^{-(I+d)} = \sum_{n=0}^{N} h(n)Z^{-n}$

$h(n) = \prod_{m=0\ m\neq n}^{N} \frac{I+d-m}{n-m}$

$Z^{-(I+d)} = \sum_{n=0}^{1} h(n)Z^{-n}$

$Z^{-(I+d)} = h(0) + h(1)Z^{-1}$

For a first order if I = 0 and d= 3\4 m=1, n=0

$h(0) = \prod_{m=0\ m\neq n}^{1} \frac{0+3/4-1}{0-1} = \frac{-1/4}{-1} = 1/4$

$h(0) = 1/4$

$h(1) = \frac{0+\frac{3}{4}-0}{1-0} = \frac{3/4}{1} = \frac{3}{4}$

$Z^{-(0+3/4)} = h(0) + h(1)Z^{-1}$

$$Z^{-(3/4)} = \frac{1}{4} + \frac{3}{4}Z^{-1} \quad (\qquad 16)$$

IV. Proposals

1. Substituting equ.(12) in equ, (8) leads to equ. (7) i.e., bilinear transformation of S to Z
2. Substituting equ.(13) ,equ (14) in (9) leads to equ. (7) i.e., bilinear transformation of S to Z
3. Substituting equ.(12) , equ, (15), equ. (16) in equ. (10) leads to equ. (7) i.e., bilinear transformation of S to Z

From our proposals based on legrange interpolation any higher order of different sampling period of a discrete signal leads to bilinear.

V. Conclusion

We conclude as different sampling period signals of a transfer function leads to bilinear transfer function by substituting corresponding fractional Z values based on legrange interpolation.

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