Design and Simulation of High ‘g’ and Low Cross Axis Sensitivity Single Axis Piezoresistive MEMS Accelerometer

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Abstract – In this work the design of a single axis micro electro mechanical system (MEMS) accelerometer based on piezoresistive sensing has been presented. The accelerometer design is based on cantilever consisting of a proof mass supported by four thin flexural beams which are fixed at the supporting frame ends along with piezoresistors employed over these fixed ends of the flexures for maximum sensitivity. The sensor has been designed for optimum performance and characterized using finite element method (FEM) CAD tools achieving low cross axis sensitivity, acceleration range of 500g and showing linear performance. Its sensitivity was observed to be 0.225mV/g along the sensing axis (z axis) and the cross axis sensitivities along x and y axes were found to be as low as 0.525 V/g and 0.546 V/g respectively that are merely 0.234 % and 0.243 % of the z axis sensitivity. Also, the bandwidth of the sensor was found to be 5.868 kHz giving high figure of merit of 61171.96 m-1. The proposed sensor structure is of compact size and is observed to be of superior performance in comparison to the reported sensors. Hence the present design can possibly be used for high acceleration measurements with low cross axis sensitivity and higher bandwidth.

Keywords – Accelerometer, Acceleration, MEMS, Piezoresistive.

I. INTRODUCTION

Transducers are fundamental part of every sensor which converts the quantity to be measured into an intelligible output signal. One of the most prominent transducers in the microrealm is the piezoresistive cantilever which translates information from the mechanical into the electrical domain. In the present work also a Micro Electro-Mechanical System (MEMS) accelerometer based on cantilever has been designed which finds wide application in various fields such as automotive, aeronautical and military, robotic systems, biomedical instruments etc. [1], [2] because of inherent advantages like reduced size, cost and power, high precision and resolution as compared to the conventional devices. The various detection methods of sensing based on cantilever are optical, piezoresistive [3], [4], [5], piezoelectric, electrostatic, tunneling [6] and thermal. The basic principle of optical cantilever operation lies in optical reflection from the back surface of the beam while the front surface interacts with the measurand [7]. The detection mechanism in Piezoelectric cantilever is based on the generation of an electric field resulting from the introduction of stress to single or multiple layers of piezoelectric materials such as ZnO and PZT [8], and the electrostatic detection is done by forming a capacitor between the cantilever surface and a fixed plate in which the deflection of the cantilever can be measured as a change in capacitance [9]. And, in thermal type the cantilever beam acts as a heat conduction path for a thermal probe [10]. Each of the above sensing mechanism has its own merits and demerits but piezoresistive sensing is preferred due to its structural simplicity, simple fabrication process and easy transduction mechanism as compared to the other types of accelerometers. Also, these are less prone to parasitic capacitance and electromagnetic interferences. Further, most of the accelerometers are considered as linear spring mass system [11] and are designed to have various desired properties like high sensitivity, high bandwidth, wide operation range, good linearity, low cross axis sensitivity and high shock survivability. Gaining high bandwidth and high sensitivity simultaneously is a non-trivial task since both of them are interdependent. Hence the interdependency is described by a dimensionless constant ‘φsensor’ called geometrical constant [12, 13, 14, 15, 16]. The two properties are related to each other by the following equation [11, 17]

\[ S\omega^2 = \phi_{sensor} \]  

(1)

where \( S \) denotes sensitivity and \( \omega \) is the lowest resonant frequency of spring mass system and increasing the sensitivity and bandwidth simultaneously is only possible by maximizing ‘\( \phi_{sensor} \)’ hence this constant is characterized as a figure of merit (FOM). Another important point of interest is cross axis sensitivity which has to be reduced to the lowest for enhancing the performance of sensor. Though the other parameters of design have shown constant improvement but cross axis sensitivity and sensitivity-bandwidth product have not been improved significantly and have scope for further research work. Therefore in the present work the design of a single axis MEMS accelerometer having high sensitivity-bandwidth product and very low cross axis sensitivity has been proposed and optimized and characterized through FEM simulations using Intellisuite MEMS CAD tools.

The following sections describe piezoresistive theory, cantilever principle, sensor design parameters and FEM analysis based performance characterization of the proposed sensor design. Also, a comparison of the performance of the proposed design with recently reported designs has been carried out and the present accelerometer design has been found to be superior in performance and size.
II. PIEZORESISTIVE THEORY

The piezoresistive effect is a widely used sensing principle. An electrical resistor changes its resistance when it experiences a strain and deformation. This effect provides an easy and direct signal transduction mechanism between the mechanical and the electrical domain. Today, it is used in the MEMS field for a wide variety of sensing applications, including accelerometers, pressure sensors, gyro rotation rate sensors, tactile sensors, flow sensors, sensors for monitoring structural integrity of mechanical elements and chemical / biological sensors. The resistance value of a resistor with the length \( l \) and the cross-sectional area \( A \) is given by

\[
R = \rho \frac{l}{A}
\]

(2)

The resistance value is determined by bulk resistivity \( \rho \) and the dimensions. Accordingly, there are two important ways by which the resistance value can change with applied strain. First, the dimensions, including the length and cross section, will change with strain. Secondly, the resistivity of certain materials may change as a function of strain [18]. Piezoresistors signify the resistors whose resistivity changes with applied strain. The resistivity of semiconductor silicon changes as a function of strain therefore silicon is an ideal material to be used as piezoresistor due to its excellent mechanical properties. The change in resistance is linearly related to the applied strain according to the relation

\[
\frac{\Delta R}{R} = G \frac{\Delta l}{l}
\]

(3)

The proportionality constant \( G \) in the above equation is called the gauge factor of a piezoresistor. By rearranging the terms in this equation we arrive at an explicit expression for \( G \),

\[
G = \frac{\Delta R}{R} \Delta \frac{l}{l}
\]

(4)

Where \( \varepsilon \) is strain, \( R/R \) and \( ll/l \) are respectively the fractional changes in resistance and dimension. The change of resistance under the longitudinal stress component measured along its longitudinal axis is called longitudinal piezoresistivity. The relative change of measured resistance to the longitudinal strain is called the longitudinal gauge factor. On the other hand, the change of resistance under transverse strain components is called transverse piezoresistivity and the relative change of measured resistance to the transverse strain is called the transverse gauge factor.

The longitudinal and transverse strains both are generally present simultaneously however one of them may play a dominating role. The total resistance change is the summation of individual changes taking place under the influence of longitudinal and transverse stress components namely

\[
\frac{\Delta R}{R} = \left( \frac{\Delta R}{R} \right)_{\text{Longitudinal}} + \left( \frac{\Delta R}{R} \right)_{\text{Transverse}}
\]

(5)

The fractional change in the resistance of a resistor \( R \) that is subjected to longitudinal \( (\sigma_l) \) and transverse \( (\sigma_t) \) stresses is given by

\[
\frac{\Delta R}{R} = \pi_1 \sigma_l + \pi_t \sigma_t
\]

(6)

\( \pi_1 \) and \( \pi_t \) respectively are the longitudinal and transverse piezoresistive coefficients.

The three independent piezoresistive constants namely longitudinal, transverse and shear, which need to be considered in the case of a cubic silicon crystal are \( \pi_{11}, \pi_{12} \) and \( \pi_{44} \) and can be used to derive the longitudinal and transverse piezoresistive coefficients for any crystal direction [19], [20].

\[
\pi_l = \pi_{11} \cdot 2 (\pi_{11} + \pi_{12} + \pi_{44}) (l_1^2 m_1^2 + l_2^2 n_1^2 + m_1^2 n_2^2),
\]

and

\[
\pi_t = \pi_{11} \cdot 2 (\pi_{11} - \pi_{12} + \pi_{44}) (l_1^2 m_2^2 + m_1^2 n_2^2 + n_1^2 l_2^2)
\]

where \( (l_1, m_1, n_1) \) and \( (l_2, m_2, n_2) \) are respectively the sets of direction cosines between the longitudinal resistor directions (subscript 1) and the crystal axis, and between the transverse resistor directions (subscript 2) and the crystal axis. Further, the longitudinal piezoresistive coefficients are maximum in <110> direction in (100) plane wafers [21] and <110> is also the direction of primary flat of wafers which is used to align them. Therefore in the fabrication of piezoresistive sensors, the piezoresistors should be perpendicular or parallel to the primary flat of the wafers. Also, the longitudinal and transverse cosines for the <110> directions oriented piezoresistors are (1/√2, 1/√2, 0) and (-1/√2, 1/√2, 0) respectively. These result in

\[
\pi_{110} = \frac{1}{2} (\pi_{11} + \pi_{12} + \pi_{44})
\]

(7)

and

\[
\pi_{110} = \frac{1}{2} (\pi_{11} - \pi_{12} + \pi_{44})
\]

(8)

These equations yields

\[
\pi_{110} = \frac{1}{2} \pi_{44}
\]

(9)

\[
\pi_{110} = - \frac{1}{2} \pi_{44}
\]

(10)

Since \( \pi_{11} \) and \( \pi_{12} \) are negligible as compared to \( \pi_{44} \) for p-type diffused resistors.

Combining "(6)", "(9)" and "(10)" we get the general expression for fractional change in piezoresistance given by

\[
\frac{\Delta R}{R} = \pi_{44} (\sigma_l - \sigma_t)
\]

(11)

Figure 1 shows the top view of the designed single axis accelerometer having its z axis as sensing axis. The sensor consists of a proof mass with four thin flexures along with the piezoresistors to be employed at the fixed ends of the flexures and connected in a Wheatstone bridge fashion with their axes aligned to <110> direction. The longitudinal and transverse stresses both acts on each resistor. If a resistor experiences a stress \( \sigma_l \) then it must also be subjected to a stress \( \sigma_t \) along its width and vice versa, where \( v \) is the Poisson’s ratio. Also, the stresses on the resistors \( R_1 \) and \( R_2 \) and on resistors \( R_3 \) and \( R_4 \) are equal but act in direction 90° to each other, that is, the transverse stress on \( R_1 \) and \( R_2 \) is the longitudinal stress on \( R_3 \) and \( R_4 \), and vice versa[22].

The four piezoresistors are connected in Wheatstone bridge (figure 2) and the output voltage \( V_{out} \) of the bridge is calculated by obtaining their node voltages \( V_n \) and \( V_p \)

\[
V_n = E \frac{R_1 + \Delta R_1}{R_1 + \Delta R_1 + R_3 + \Delta R_3}
\]

(12)

\[
V_p = E \frac{R_3 + \Delta R_3}{R_3 + \Delta R_3 + R_4 + \Delta R_4}
\]

(13)

\[
V_{out} = V_n - V_p
\]

(14)

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Cantilever is a singly supported beam forming a basic mechanical structure having well described strain and bending characteristics [23] which makes it useful in many of the static sensing applications. Furthermore, a cantilever can be described as a second-order system with distinct resonance characteristics determined by the cantilever mass and spring constant as well as by the ambient conditions [24].

Therefore any changes in these parameters will be reflected as a variation in its resonance characteristics and thus making the cantilever a prominent choice for dynamic sensing applications. The motion of a cantilever resonator as a lumped mass-spring system can be described by the differential equation of a second-order system with constant coefficients [24]

\[ m \frac{d^2 y}{dt^2} + B \frac{dy}{dt} + ky = F(t) \Omega t \]  

(15)

Where \( y(t) \) is the displacement of the lumped mass \( m \), \( k \) is the spring constant, \( B \) is the velocity-related damping coefficient, and \( F \) is the excitation force applied on the lumped mass with an angular velocity of \( \Omega \). From the properties of the second-order system the natural resonance frequency \( f_0 \) is calculated [24]:

\[ f_0 = \frac{\omega_0}{2\pi} = \frac{1}{2} \sqrt{\frac{k}{m}} \]  

(16)

where \( \omega_0 \) is the natural angular velocity (radial resonance frequency) of the system. In case of cantilever beam the stiffness \( K \) is given by

\[ K = 3 \frac{E I}{L^3} \]  

(17)

where \( E \) is Young’s modulus, \( I \) is moment of inertia of cantilever beam of width \( b \) and height \( h \) which is given by

\[ bh^3/12 \]  

(18)

For calculation of sensitivity of an accelerometer based on cantilever the stress developed at the flexure need to be calculated. For a point force \( F \) applied at the point \( x_f \) (figure 3) on a single-layer rectangular cantilever, the stress is [23]

\[ \sigma_x = \begin{cases} \frac{12x(x_f-x)}{h^3b}, & x \leq x_f \\ \frac{12x(x_f-x)}{h^3b}, & x_f \leq x \leq L \end{cases} \]  

(19)

The stress magnitude is highest at the cantilever clamped end and linearly decreases toward the tip being suggestive of the piezoresistors to be employed at the fixed ends of the flexures.

### Table I: Design Parameters of Sensor

<table>
<thead>
<tr>
<th>Element</th>
<th>Length (µm)</th>
<th>Width (µm)</th>
<th>Thickness (µm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proof Mass</td>
<td>1000</td>
<td>1000</td>
<td>400</td>
</tr>
<tr>
<td>Flexures</td>
<td>1000</td>
<td>70</td>
<td>30</td>
</tr>
<tr>
<td>Support Frame (inner square dimension)</td>
<td>3000</td>
<td>3000</td>
<td>1000</td>
</tr>
<tr>
<td>Support Frame (outer square dimension)</td>
<td>5000</td>
<td>5000</td>
<td>1000</td>
</tr>
</tbody>
</table>
In the present work a single axis MEMS accelerometer with piezoresistive sensing and z axis as sensing axis has been designed. The designed sensor consists of a proof mass suspended by four thin flexures along x and y directions that are fixed to an outer frame which in turn is fixed to the system whose acceleration has to be measured. The frame moves with the whole structure when it accelerates whereas the proof mass tries to remain in its previous position due to inherent inertia and in the process gets deflected up / down in accordance to the direction of the motion of the system. These results in to a stress developed between the frame and proof mass ends on each of the flexures. To measure these stress piezoresistors are implanted at maximum stress points towards the fixed end sides on each flexure.

The proposed accelerometer is designed using Intellisuite® FEM CAD software which is equipped with the virtual prototyping tool to make the structure as per design. The material for the design in the software is fixed to be silicon of Young Modulus 130 GPa and poisson’s ratio 0.226. The developed dimensions of the designed device are given in table 1 and its FEM prototype model is shown in figure 4.

V. **FINITE ELEMENT METHOD SIMULATION**

The designed sensor was analyzed using FEM analysis through Intellisuite® FEM simulation software by carrying out static analysis of the accelerometer for an acceleration range of 500g. The natural frequency of the sensor was determined using frequency analysis tool for finding its operational bandwidth and was observed to be 5.868 kHz. Also, the stress locations of maximum stress necessary to achieve high sensitivity were determined through simulation and this is shown in red colour region on the flexures (figures 5 and 6), and the sensor sensitivity was computed using stress value that was found to be 0.225 mV/g. Further, a graph plotted between output and acceleration shows that the designed sensor performance is linear in its operating range (figure 7). Also, a performance comparison was made with the previously reported designs [25, 26] and the proposed design of the sensor was found to be superior on account of various parameters viz., figure of merit (FOM), and range and cross axis sensitivity etc. as detailed in table 2.

VI. **RESULTS AND DISCUSSIONS**

In the present work a single axis piezoresistive MEMS accelerometer based on cantilever has been designed for a range of 500g acceleration. The performance of the designed sensor has been characterized using FEM CAD tools of Intellisuite® for making and analyzing the virtual prototype of the sensor. The sensitivity of the sensor was calculated using expression for piezoresistive sensing in which the piezoresistors are employed at fixed ends of the flexures to come across maximum stress and hence achieve maximum sensitivity that was found to be 0.225 mV/g along sensing axis (z axis). Also, the cross axis sensitivities along x and y axes were observed to be 0.525 V/g and 0.546 V/g which are 0.234 % and 0.243 % of z axis sensitivity respectively. The bandwidth of the sensor...
has been calculated by using frequency analysis tool of FEM CAD software and was found to be 5.868 kHz. A graph between output and acceleration has been plotted and it shows a linear relationship in its operating range.

Also, a performance comparison of the sensor has been done with previously reported designs and is found to be superior in performance than the reported design with respect to the range of operation, figure of merit and lower cross axis sensitivity that are required for a high performance accelerometer. In addition to these the designed sensor is more compact in size as compared with the reported designs.

VII. CONCLUSION

A single axis MEMS accelerometer based on cantilever principle has been designed and its performance is evaluated using MEMS CAD tools. The sensor is designed to achieve high sensitivity along with low cross axis sensitivity and operating range as high as 500g based on piezoresistive sensing employing high stress generating architecture and its calculated sensitivity was found to be 0.225 mV/g along sensing axis, while its cross axis sensitivity along x and y axes have been observed to be as low as 0.525 V/g and 0.546 V/g which are 0.234 % and 0.243 % of z axis sensitivity respectively. The bandwidth of the designed sensor has been found to be 5.868 kHz yielding figure of merit as high as 61171.96 m-1. Hence the present work may be an attractive choice for design of low cost MEMS accelerometers having high operating range, high bandwidth and very low cross axis sensitivity.

REFERENCE