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Some Fundamental Properties of Petri Nets

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Abstract – **The aims of this work are to find ways to represent and remodel Petri nets. The importance of matrix representation for Petri net structures is highlighted and used to represent Petri nets. Depending on the structure of the Petri nets certain properties can be clearly identified in the corresponding input, output flow and incidence matrices. These properties are clearly explained. It is shown how inverting or swapping places and transitions results in consistent changes in the incidence matrix. Relationships, results and findings using matrix representation are presented using examples and discussed**

Keywords – **Petri Nets, Representation, Matrices, Matrix Algebra.**

I. Introduction

Petri nets are powerful graphical and mathematical formalisms, useful for modeling many types of system behaviour. They are useful for modeling concurrency, asynchronous, distributed, parallel, deterministic, real-time, and many other types of system behaviour [1],[2]-[5],[8],[9]. Usually Petri nets share properties with flow charts, block diagrams, network diagrams, UML 2 activity models and graphs [9],[10]. The mathematical counterpart of Petri nets are founded on the basic static properties and dynamic behaviour of Petri nets. Normally the outline structure of the net is fixed and does not change with time. I.e. places and transitions remain unchanged throughout the execution of the net. This is indicated in [1], [2].

The static properties relate mainly to the structural representation of the net, while the dynamic properties focus on the conditions required for transition enabling and firing, whilst another important aspect concerns changes that can occur in the global token representation when a transition or multiple transitions fire at a specific point, provided, there are enabling conditions for token firing present as is shown in [2].

Petri nets can be represented using equations, state equations, or structurally by constructing a special type of matrix that captures the inflows and outflows from places to transitions and vice-versa [1].

The normal properties of Petri nets, that are explored, are mainly related to the static and behavioural properties. The latter is more difficult to analyze, if there is no proper solution, or for undefined behaviour [3]. Given the idea of representing well formed and well behaved basic Petri nets, it is possible to understand some of the basic analysis methods, that rely on the principle that the Petri net has a given solution and is restricted, i.e. it has a limited state space. The properties are normally derived from the transition or place invariants that can cover parts or all of the net. Boundedness, safeness, liveness, reversibility, home states, coverability, reachability are main properties that allow for simple verification [2]. These properties are obtainable from the incidence matrix in conjunction with the marking vector. These can be used to generate the place and transition invariants along with the mark graph.

The incidence matrix has other interesting fundamental properties that allow for different experimentation and analysis for Petri nets representation.

II. Motivation

The main reason for this work is to find useful properties for the examination and remodeling of Petri nets. The idea is to use a base model to generate new models. This is a positive idea allowing for experimentation with different configurations.

Petri nets even of the simplest type, offer non-trivial modeling solutions for diverse problem classes. Unfortunately the more complex the net the more difficult it is to examine its dynamic behaviour properties [11]. However, as long as the net is of finite size, some very basic properties of the Petri net are still examinable.

One property of the Petri net, that is simple to construct and represents the inflows and outflows between places and transitions, is the incidence matrix [1]. The idea of having an incidence matrix for the Petri net is similar to the adjacency matrix or lists used for compact graph representation. The incidence matrix can be constructed for large nets independently of the type of net. Obviously, the more complex the net the larger the size of the incidence matrix. On the other hand certain types of invariants and other modeling methods are not possible and do not exist for every type of net.

Just by examining this matrix, the outline structure of a Petri net, some basic properties can be inferred and examined, using simple methods and important conclusions can be made.

III. Problem Formulation

The incidence matrix has some interesting fundamental properties that allow for the analysis and experimentation with the structural representation of Petri nets.

IV. Some Basic Petri Net Properties

A. Basic Definition of Petri nets

A basic or simple Petri net is defined as bi-partite digraph having two types of vertices as in[1],[2]. This can is represented as a four tuple set, PN = (P,T,F,W). P is a finite non empty set of places P= $\{p_1,p_2,p_3,\ldots,p_n\}$ and T is a finite non empty net of transitions T= $\{t_1,t_2,t_3,\ldots,t_n\}$. F is a finite non empty set of flows from a place to a transition and vice-versa, given as $F \subseteq \{(PxT) \cup (TxP)$. Normally (PxT) represents the input arcs also denoted as I and (TxP) represent the output arcs denoted as O.

W is a weight function or marking value for the tokens

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at a place p, given as $W : P \rightarrow \{1,2,3,...,n\}$. Places and transitions are disjoint i.e. $P \cup T = \phi$ and $T \cup P = \phi$. Nodes are not isolated. The Petri net can have an initial marking normally denoted as M_o.

B. Ordinary Vs Non-Ordinary Petri Nets

A Petri net is ordinary iff, $\forall p \in P, t \in T, I(p,t) \leq 1$ and $O(p,t) \leq 1$.

This implies that all arcs have a multiplicity of 1. This property can be directly observed from the input flow matrix and the output flow matrix. If there are no entries > 1 in both matrices then the Petri net is ordinary. If $\exists_{(p \in P)}; I(p,t) > 1$ or $\exists_{(p \in P)}; O(p,t) > 1$. If exists an entry > 1 then it is a non-ordinary net.

The concept of non-ordinary nets can be extended to different classes of Petri nets. Here by non-ordinary it is implied mainly that the arc multiplicity is > 1.

C. The Concept of Nodes

A node in a Petri net refers to either a place or a transition, y is a node, **iff** $y \in P \cup T$. The input set or pre set of a transition t implies the set of all input places to t. This can be written as ${}^{\bullet}t = \{p : p \in P \cap I(p,t) \neq 0\}$. The output set or post set of t is the set of all output places from t. This can be written as $t^{\bullet} = \{p : p \in P \cap O(t,p) \neq 0\}$. An *elementary path* in the Petri net is identified as a sequence of nodes: $a_1, a_2, ..., a_n$; where $n \geq 1$ and $\exists$ arc(a_i, a_{i+1}) for i $\in N_{n-1}$ if n>1 and $a_i = a_j$ implies that i=j where $N_n = \{1,2,...,n\}$ possibly defining a self-loop, elementary loop or a circuit.

D. Incidence Matrix-Concise Form

A concise way to represent Petri nets structurally for analysis is to represent Petri nets using matrices. The incidence matrix is fixed, i.e. it does not change for the Petri net unless the structure of the Petri net is altered. The incidence matrix captures all the structural information about a Petri net, i.e. the connections from places to transitions and vice-versa, along with all the number of transitions are represented in the incidence matrix.

The incidence matrix can be constructed for a net independently if the net is live or not or there are some other issues. I.e. the incidence matrix is just a generic structural way of representing a Petri net.

E. Incidence Matrix Representation

The incidence matrix for a Petri net is composed of $C_{ij} = O_{ij} - I_{ij}$. I.e. the incidence matrix C_{ij} also denoted simply as C is composed of the difference between the output flow matrix O_{ij} and the input flow matrix I_{ij}. The incidence matrix representation can also be written as $C_{ij} = C_{ij}^{+} - C_{ij}^{-}$, where $C_{ij}^{+} = W(i,j)$ if $t_j \in {}^{\bullet}P_i$ else it is zero and $C_{ij}^{-} = W(i,j)$ if $t_j \in P_i^{\bullet}$ else it is zero and W(i,j)= weight of an arc from $i \rightarrow j$ or $j \rightarrow i$. Simply C_{ij}^{+} represents the output of transitions to places and C_{ij}^{-} represents the input of places to transitions. The O_{ij} matrix is the complete set of output flows from transitions to places. I.e. ${}^{\bullet}p = \{t : (t,p) \in F\}$. The I_{ij} matrix is the complete set of input flows from place to transitions. I.e. $p^{\bullet} = \{t : (p,t) \in F\}$.

To see the full picture of the Petri net, the input flow matrix and the output flow matrix have to be considered together. This is because in the final incidence matrix C_{ij} it is possible to end up with flows that cancel each other out.

F. Transition Firing Using Vectors

Following the above explanation in section IV. d) about the incidence matrix, transition firing simply follows. Transition firing is simply written as $M_1 = M_0 + Cf$. M_0 is the initial marking vector and f is the firing vector, i.e. which transition is to fire. M_1 is the resultant marking. The initial marking vector represents the basic state of the net, or the number of tokens in their respective places. Place and transition invariants are easily derived from the incidence matrix.

V. Incidence Matrix Properties in Relation to Petri Net Representation

This section contains some terminology, definitions and basic facts concerning Petri nets and the properties of Petri nets as viewed through the incidence matrix. Properties of matrices are explained in [6]-[7].

A. Elements of the Incidence Matrix

The address of the element in the matrix are determined by the values i,j. where i represents the row number, i.e. the place row and j represents the column, i.e. the transition column.

B. Direct Sum of Subnet Matrices

In certain cases and for Petri nets having equal number of places and transitions only, i.e. square incidence matrices for the subnets, it is possible to have square matrices A_1, A_2, A_3, …., A_n of respective orders m_1,m_2,m_3, ….m_n. A generalized ordered matrix A can be created from these matrices, where the direct sum of A_1, A_2, A_3, …., A_n is the diagonal (A_1, A_2, A_3, …., A_n). This property only holds if the subnets represented by A_1, A_2, A_3, …., A_n are not at all connected. I.e. if A_1, A_2, A_3, …., A_n are completely separate nets.

C. Dimensions of the Incidence Matrix

The incidence matrix dimensions rows by columns represent the number of places and transitions in the Petri net. i.e. if a Petri net has 22 places and 30 transitions, then the incidence matrix C_{ij} would be a 22x30 matrix.

The larger the Petri net the larger the size of the incidence matrix. But this is still very useful and can be represented with some difficulty.

If the number of places and transitions in the Petri net are identical, this is significant because it implies that the incidence matrix C_{ij} is a square matrix and square matrices do have special properties. These special properties do not exist for non-square matrices.

D. Square Incidence Matrix

If a normal Petri net has the same number of transitions

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and places then by definition, there must exist a square incidence matrix for the Petri net. This is because the columns represent transitions and the rows represent places in the incidence matrix. If the number of places and transitions are equal, then the incidence matrix is definitely a square one. Similarly the input and output matrices must be square.

E. Petri Net Place and Transition Inversion

By Petri net place and transition inversion or simply called inversion in this case, it is implied that transition and places are swapped, but no other alterations are carried out. Inversion is defined simply by replacing the original nodes of the Petri net. Transitions are replaced as places and vice-versa.

From a perspective of viewing the Petri net as a digraph, if nodes are treated as being of the same type, the outline structure remains unchanged. I.e. the number of edges remains completely unaltered.

If for a given Petri net the places and transitions are inverted an interesting observation can be made. This is that the original incidence matrix denoted as C_{org} is identical to $-C^T_{inv}$, where $-C^T_{inv}$ is the negative of transposed incidence matrix for the place and transition inverted net. i.e. the negative of the transposition of the incidence matrix for the Petri net with places and transitions inverted.

Thus the Petri net place and transition inversion can be a) carried out directly from the incidence matrix, i.e. in two steps, i) transposing the matrix and ii) multiplying the matrix by – sign. or b) simply redrawing the Petri net with places and transitions swapped and generating the incidence matrix for this new net.

F. Transpose of the Incidence Matrix

The transpose of a matrix is given as turning the rows of a given matrix into columns and vice-versa. The transpose of matrix C_{ij} is given as C^T_{ij}. I.e. the matrix of order R x C is transposed into C x R. Rows and columns are interchanged. i.e. $C_{ij} = C_{ji}\ \forall_{i,j};(i \in I),(j \in J)$ unless $C_{ij} = 0$.

For a Petri net, the transpose of its incidence matrix can result in a matrix having different dimensions if the number of places and transitions in the net are not equal. If the number of places and transitions are equal, the resultant matrix being square, has similar order and dimensions.

G. Transpose of the Input/Output Flow Matrices

In a similar pattern, to the transpose of the incidence matrix, the input and output flow matrices are also transposed, and the input matrix becomes the output matrix for the new net and vice-versa. I.e. if the net places and transitions are inverted, then the new input flow and output flow matrices that result are simply based on the transposition of the original matrices, the inflow matrix becomes the output matrix and the outflow matrix becomes the input matrix. I.e. the new output flow matrix is $O_{inv} = I^T_{org}$ and the new input flow matrix is $I_{inv} = O^T_{org}$.

H. Equal Matrices

For Petri net place and transition inversion it is observed that $C_{org,}$ $-C^T_{inv}$ are equal.

I.e. the negative of the transposed matrix for the inverted net is always equal to incidence matrix of the original net.

Both matrices have the same order and each element of one is equal to the corresponding element of the other.

This can be written as $C_{org} = -C^T_{inv}$ and $C_{org} + C^T_{inv}$ = 0. Both matrices have commutative properties.

VI. Some Examples

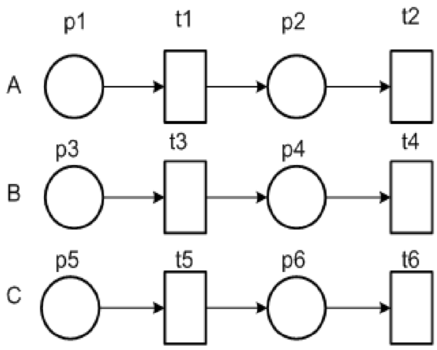

Fig.1. Three Simple Petri Nets A, B, C

Some trivial examples are shown to demonstrate these findings. For constructing the matrices, the Petri net places and transitions must follow an ordering sequence labeling and the correct place and transition labeling sequence has to be maintained.

A. Simple Unconnected Petri Nets

In fig.1 three unconnected Petri nets A,B,C are shown. The structure of each net is identical. Thus the input flow, output flow and incidence matrices for these nets are identical.

The input and output flow matrix for subnet A are written as:

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \text{ and } O = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} \quad C = \begin{bmatrix} -1 & 0 \\ 1 & -1 \end{bmatrix}$$

This is identical for B and C. The generalization of the individual incidence matrices for A,B,C is a square 6x6 matrix which can be called D. Where diag(D) = diag(A,B,C).

$$D = \begin{bmatrix} -1 & 0 & 0 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 \end{bmatrix}$$

These properties could be useful for creating duplicates using a repeated pattern. I.e. subnets A, B, C are identical and have a repeated pattern.

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B. Simple Sequential Cyclical Petri Net

In fig.2 (a) a simple sequential Petri net having a repetitive cycle is shown. Fig. 2 (b) shows the same net but the places and transitions have been swapped, i.e. inverted. For the Petri net in fig. 2 (a), the input flow, output flow and incidence matrix C are all 4x4 square matrices given as:

$$I=\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix}\quad O=\begin{bmatrix}0&0&0&1\\1&0&0&0\\0&1&0&0\\0&0&1&0\end{bmatrix}$$

$$C=\begin{bmatrix}-1&0&0&1\\1&-1&0&0\\0&1&-1&0\\1&0&1&-1\end{bmatrix}$$

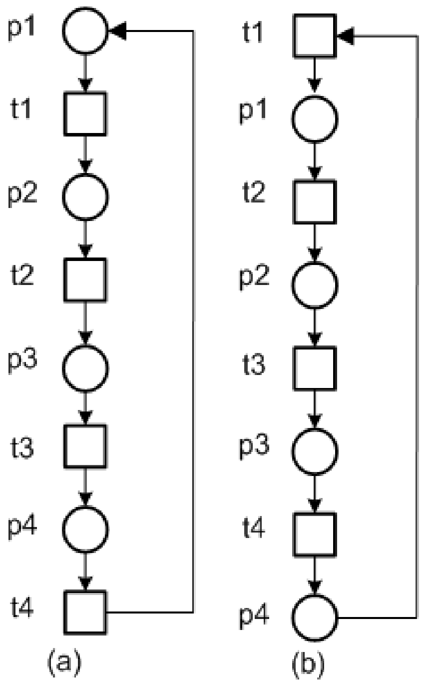


Fig.2. (a) Cyclical Sequential Petri Net and (b) Same net: Places and Transitions Inverted

For fig.2 (b) the matrices are given as follows:

$$I=\begin{bmatrix}0&1&0&0\\0&0&1&0\\0&0&0&1\\1&0&0&0\end{bmatrix}\quad O=\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix}$$

$$C=\begin{bmatrix}1&-1&0&0\\0&1&-1&0\\0&0&1&-1\\-1&0&0&1\end{bmatrix}$$

C. Petri Net with Choice/Conflict

In fig. 3 (a) the Petri net structure is slightly more complex than the previous ones. If there were tokens in places p1 and p2, then transitions t1, t2 and t3 can be enabled simultaneously. However, from a structural point of view and the isolated view of the incidence matrix, these details do not affect the underlying structure.

Given that the number of places and transitions are equal, for both fig. 3 (a) and 3 (b) the matrices are all square matrices.

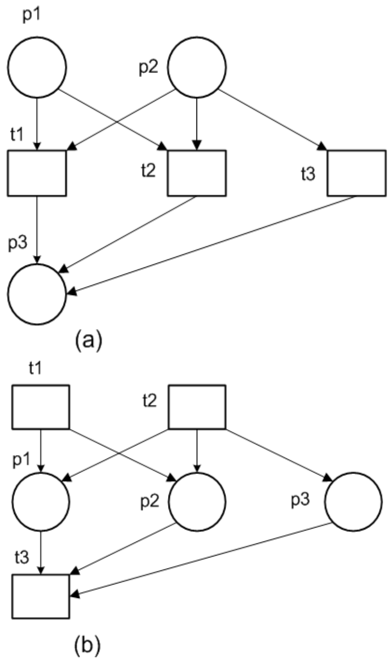
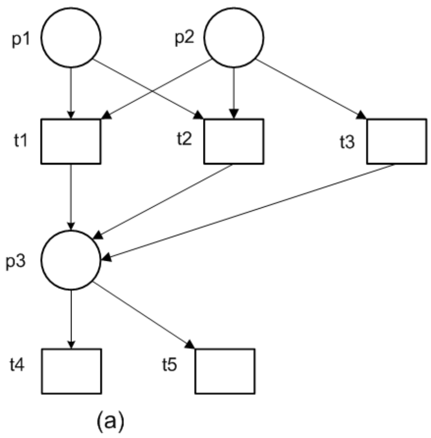


Fig.3. (a) Petri Net with Choice/Conflict and (b) Same net: Places and Transitions Inverted

The matrices for fig 3 (a) are:

$$I=\begin{bmatrix}1&1&0\\1&1&1\\0&0&0\end{bmatrix}\quad O=\begin{bmatrix}0&0&0\\0&0&0\\1&1&1\end{bmatrix}$$

$$C=\begin{bmatrix}-1&-1&0\\-1&-1&-1\\1&1&1\end{bmatrix}$$



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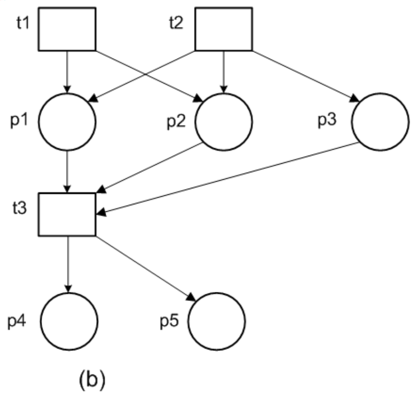

Fig.4. (a) Petri Net with Non Equal No. of Places and Transitions and (b) Same net: Places and Transitions Inverted

The matrices for fig 3 (b) are:

$$I = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} \quad O = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 1 & -1 \\ 1 & 1 & -1 \\ 0 & 1 & -1 \end{bmatrix}$$

D. Petri Net with a Non Equal Amount of Places and Transitions

For the Petri net in fig. 4 (a), the input, output flow and incidence matrices are definitely non-square ones given as:

$$I = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix} \quad O = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \end{bmatrix}$$

$$C = \begin{bmatrix} -1 & -1 & 0 & 0 & 0 \\ -1 & -1 & -1 & 0 & 0 \\ 0 & 0 & 0 & -1 & -1 \end{bmatrix}$$

For the Petri net with inverted places and transitions in fig. 4 (b), the following matrices are given:

$$I = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad O = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 1 & -1 \\ 1 & 1 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

VII. Results and Findings

The results and findings explain some aspects of the Petri net examples using the matrices for simple structural analysis.

i) For the Petri net in fig.1 it is obvious that there are three disconnected nets or subnets labeled A,B,C. The input flow matrix is symmetric both for the individual subnets and the nets grouped together in a generalized matrix. The generalized incidence matrix D represents all the three nets input and output flows.

The input flow matrix, is a diagonal and identity matrix, having properties of both where $I = I^T$. Adding another net would be a simple matter of repetition, i.e. a fixed pattern is visible in the input, output and incidence matrices. The concept of putting separate nets together can be extended to different Petri net structures, subject to the condition that the number of places and transitions are equal. I.e the matrices for the nets are square, and also that the places of the nets have proper labeling sequences. These must be kept.

ii) Fig. 2 (a) represents a simple sequential Petri net that has a loop or repeated behaviour. In this instance inverting the places and transitions as shown in fig. 2 (b) does not really alter the main properties of the net, except for the transition and place ordering. Again, it is evident that as there are an equal number of places and transitions the input, output and incidence matrices are square. The input matrix for the net in fig. 2 (a) has symmetrical matrix properties and it is a diagonal and identity matrix. Transposing this matrix does not change it. It is possible to deduce that if the input flow matrix is symmetric then if the net places and transitions are inverted then the output flow matrix will be symmetric and vice-versa. This is evident in example in section VI) b. which are fig. 2(a) and fig. 2(b) I.e. the output flow matrix of the inverted net = transpose of the input flow matrix of the initial net and vice-versa. The property for the incidence matrix where $C_{org,} = -C_{inv}^T$ holds.

iii) The example in section VI) c. shows an initial net fig. 3 (a) that has choice or conflicting transitions. The input and output flow matrices are still square as there is an equal number of places and transitions; however the number of arcs from a place to transitions are more than one and this is reflected here. The matrices are definitely not symmetrical ones. Inverting the places with transitions and vice-versa yields the net in fig. 3 (b). Again the property $C_{org,} = -C_{inv}^T$ holds.

iv) The example in section VI) d. shows a net, fig. 4(a) that does not have an equal number of places and transitions. This implies that the input, output flows and incidence matrices are non-square ones, as shown. The incidence matrix for the inverted place/transition net in fig. 4(b) has different dimensions of the previous one. The property $C_{org,} = -C_{inv}^T$ still holds. Table 1 summarizes the results.

Table 1 Summary of Properties for Example Nets

Ex.	Square Matrices	Symmetric Matrix	Property $C_{org,} = -C_{inv}^T$
VI a)	Yes	Yes	Yes
VI b)	Yes	Yes	Yes
VI c)	Yes	No	Yes
VI d)	No	No	Yes

More properties of the matrices exist and can be found, i.e. for some cases of Petri nets, the square matrices would definitely have determinants. The image of a matrix,

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depicting the columns as independent column vectors can be found. I.e. In the case of the incidence matrix the independent column vector would depict the inflows and outflows for a particular transition. For the example given in section VI) b. or fig. 2 (a) the given the initial incidence matrix C, the column vectors would be:

$$t_1\begin{bmatrix}-1\\1\\0\\1\end{bmatrix}+t_2\begin{bmatrix}0\\-1\\1\\0\end{bmatrix}+t_3\begin{bmatrix}0\\0\\-1\\1\end{bmatrix}+t_4\begin{bmatrix}1\\0\\0\\-1\end{bmatrix}$$

From the column vectors for the incidence matrix it is possible to examine the properties of a transition in order to see if the inputs and outputs to a transition are balanced or not.

VIII. Some Practical Uses and Ideas

Other ways of representing the Petri nets could be by using augmented matrices. Reduced form matrices could be found for certain Petri nets where obviously arc multiplicity is considered.

The idea of inverting places and transitions, normally implies transforming actions or steps into results or inputs and transforming inputs or outputs into actions or steps.

A real life example is, a place representing a customer and a transition representing an action like order ticket. The place transition inversion is the customer into customer action and order ticket into ticket.

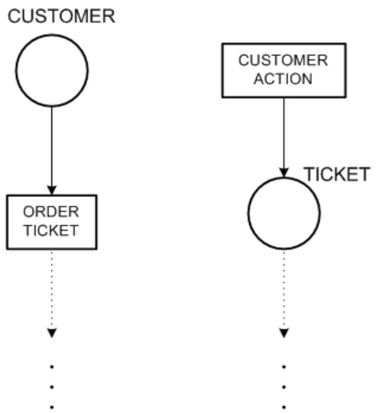

Fig.5. Petri Net Fragment Showing Action/Input Inversion

Fig.5 illustrates the idea of inverting places and transitions using a simple trivial example. This is depicted as a Petri net fragment or incomplete net. Obviously the concepts of liveness, reachability, etc. are ignored here.

There is the concept of having a Petri net within a net. i.e. the transitions can decompose into an entire system or network, similar ideas can be used for places which can be viewed as communication channels or storage points. Hence a transition of a place in a higher order net structure is a sort of black box. This work can help for sorting or restructuring an entire network whilst retaining fundamental properties from the original network. Hence it could be useful for representing high level analogies in certain types of systems. Certain problems can be considered e.g. when constructing a system, if it is of the correct form.

It is possible to identify sequences or patterns from the matrices. These would have to be interpreted in conjunction with the system being described for their relevance.

More practical uses have still to be sought out.

II. Conclusions

It is obvious that for different Petri net structures the matrix properties can be i) significantly different. I.e. it is not straightforward to compare the incidence matrices of two different nets. This implies that each case requires special attention and individual analysis

The matrices could be ii) used for reduction and the result would be a completely different Petri net.

iii) Different methods of verification and matrix properties are identifiable. However these will change depending on the Petri net structure and are not general properties. I.e. Both General properties and unique properties can be identified using matrix theory for different nets.

iv) Other forms of inversion for the Petri net can be considered. E.g. it is possible to invert the flows or flow direction of the net, retaining the same nodes, i.e. places and transitions. However, this requires detailed treatment elsewhere.

Petri nets offer the advantage of having a vast amount of support and use in the domain of computing and information systems. They have over three decades of coverage. They are important for understanding and representing system specification and behaviour from an informal and formal point of view. The properties explained in this work are useful for finding new methods of representing Petri nets structurally. The properties pointed out are useful for finding repeatable patterns within existing structures.

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