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Bound Accuracies for Localization Methods in Wireless Sensor Networks

B. Srilatha
Assistant Professor
KUCET, Warangal, India
Email: srilathabasani@gmail.com

Asiya Sulthana
Assistant Professor
BIT, Narsampet, Warangal, India
Email: asiya806@yahoo.co.in

E. Ramakrishna
Associate Professor
VITS, Karimnagar, India
Email: erroju.rk@gmail.com

***Abstract* – Node localization is an important issue for wireless sensor networks to provide context for collected sensory data. Sensor network designers need to determine if the desired level of localization accuracy is achievable from their network configuration and available measurements. The Chapman–Robbins bound or Hammersley–Chapman–Robbins bound is a lower bound on the variance of estimators of a deterministic parameter. It is a generalization of the Cramér–Rao bound, compared to the Cramér–Rao bound, it is both tighter and applicable to a wider range of problems.**

***Keywords* – Wireless Sensor Networks, Localization, Sensor Networks.**

I. Introduction

A Wireless Sensor network of very sensitive instruments is presently being assembled across the globe with a view to directly detecting cosmic gravitational radiation on Earth. Although detection is the initial goal of the network, it must ultimately function as an astrophysical observatory. In this capacity it will study a range of sources of gravitational waves, such as neutron stars, black holes, and the early universe. The network will also provide data through which to learn new and fundamental physics about the gravitational field. Arguably, broadband instruments based upon the method of laser interferometry offer the best long-term potential for making astrophysical observations. Several such instruments have been funded and are being constructed at the present time.

Evaluation methods for localization systems serve two purposes. First, they allow a network designer, prior to the creation of the senor network, to obtain a quantitative bound on how well the localization of sensor nodes can be performed with given types of localization measurements and with different geometric arrangements of the measuring base stations. A network designer can then determine which of a set of possible network designs will achieve the required localization accuracy for their application.

Second, these tools can be used to evaluate the performance of an existing localization system to see if all the potential location accuracy is being achieved or if further improvements are possible. The tools help to quantify the cost and accuracy tradeoffs of different component choices in a localization system's design.

To provide accuracy information for localization in the presence of multipath and NLoS propagation, figures of merit have been derived in the localization literature such as the Cramér-Rao lower bound on the mean square error of the location estimates. In other words, the Cramér-Rao lower bound assumes that the node location is a deterministic value and the localization system has no other information about the node location prior to the measurements. Other sources of information, such as the sensor selection procedure or the measurements taken in the past, are not considered, so the Cramér-Rao lower bound is no longer a valid lower bound for localization systems where this information is available.

II. Chapman–Robbins Bound or Hammersley-Chapman-Robbins Bound Statement

Let $\theta \in R^n$ be an unknown, deterministic parameter, and let $X \in R^k$ be a random variable, interpreted as a measurement of θ. Suppose the probability density function of X is given by p(x; θ). It is assumed that p(x; θ) is well-defined and that p(x; θ) > 0 for all values of x and θ.

Suppose δ(X) is an unbiased estimate of an arbitrary scalar function g: $R^n \rightarrow R$ of θ, i.e., $E\{\delta(X)\} = g(\theta)$ for all θ.

The Chapman–Robbins bound then states that

$$\mathrm{Var}(\delta(X)) \geq \sup_{\Delta} \frac{[g(\theta+\Delta)-g(\theta)]^2}{E\left[\left[\frac{p(x;\theta+\Delta)}{p(x;\theta)}-1\right]^2\right]}.$$

III. Lower Bounds on Location Estimation Error with Prior Information

If information on the node location is available before the localization procedure begins, then a Bayesian localization procedure with localization MSE that is lower than the Cramér-Rao bound described above is available. The most common Bayesian bound is the Bayesian Cramér-Rao Bound (BCRB) given by

$$\mathrm{E}\{[\boldsymbol{\theta}(k)-\hat{\boldsymbol{\theta}}(k)][\boldsymbol{\theta}(k)-\hat{\boldsymbol{\theta}}(k)]^T\} \geq \{\tilde{\mathbf{J}}[\boldsymbol{\theta}(k)]\}^{-1}$$

where the matrix $J[\theta(k)]$ is defined as

$$\tilde{\mathbf{J}}[\boldsymbol{\theta}(k)] = -\mathrm{E}\{\nabla_{\boldsymbol{\theta}(k)}[\nabla_{\boldsymbol{\theta}(k)} \log f(\mathbf{z}(k),\boldsymbol{\theta}(k))]^T\}$$

When $f(\mathbf{z}(k), (k))$ is the joint probability density function of $\mathbf{z}(k)$ and $\theta(k)$ with no restrictions on the bias of the estimator for bound validity.

The inequality of A≥B indicates that the matrix given by D=A−B is non-negative definite. Using standard Bayesian theory, the joint probability density function is given by

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$f(\mathbf{z}(k), \theta(k)) = f((k)|\theta(k)) f(\theta(k))$ where $f(\mathbf{z}(k) \mid \theta(k))$ is the conditional probability density function used to calculate the standard Cramér-Rao bound.

Algorithm1. Monte Carlo integration calculation of Bayesian Cramér-Rao bound.

1. Generate N independent samples of

   $\theta(k): t^1, t^2, ..., t^N$

   t^k has probability density function of $f(\theta(k))$

2. Calculate $\tilde{J}_D(\theta(k))$:

   $$\tilde{J}_D(\theta(^k)) = \frac{1}{N}\sum_{k=1}^{N}(D^k)^T C^{-1}(D^k)$$

   Where $D^k = \nabla_{\theta(k)} E[z(^k) \mid \theta(k)]\big|_{\theta(k)=t^k}$

3. Calculate $\tilde{J}_P(\theta(k))$:

   $$\tilde{J}_P(\theta(_k)) = -\frac{1}{N}\sum_{k=1}^{N} B^k$$

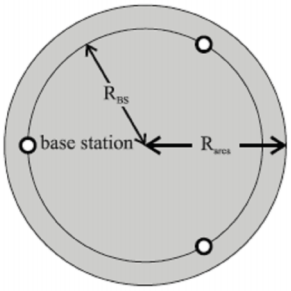
   Where $B^k = \nabla_{\theta(k)}[\nabla_{\theta(k)} \log_f(\theta(k)]^T\big|_{\theta(k)=t^k}$

4. Calculate bound on localization MSE from (10).

IV. Bounds for a Simple Localization Scenario

The simulated network environment for the first three sets of localization for these simulations, the node locations are uniformly distributed within a disk of radius *Rarea*. The node localization is performed from measurements made by *m=3* base stations uniformly located on a circle of radius *RBS* with the same center as the disc for the terminal locations. The local Cramér-Rao bound is not valid for this localization because of the existence of a prior probability density function for node location. The Cramér-Rao bound would give a bound on localization error only for localization performed without the use of prior information. The BCRB is not valid for this localization either because the finite support for the prior probability density function of the node location causes the regularity conditions of the bound to be violated.

Localization is performed based on RSS, ToA, or AoA measurements made by the base stations with the LoS propagation measurement model. The first sets of simulations are ToA measurements performed with R_{area} = R_{BS} = 15m, which results in localization with low GDOP. The RMSE for the ToA MMSE localization is compared with bounds calculated. The BCRB MSE 'bound' exceeds the optimal MSE as the mean noise power increases. The BCRB calculations for each $\boldsymbol{\theta}(k)$ value use only the local curvature of $f(\mathbf{z}(k), \theta(k))$ and do not include the boundary conditions. For the same reason, local Cramér-Rao bounds calculated near the boundaries of the region of support for $f(\theta(k))$ are also invalid as they do not exclude estimated locations where $f(\theta(k)) = 0$. The BCRB in this case is a lower bound on localization performed without use of the prior probability density function $f(\theta(k))$ also known as Maximum Likelihood Estimation



Node bound range estimation Model

V. Estimation Theory

Estimation theory is a branch of statistics and signal processing that deals with estimating the values of parameters based on measured/empirical data that has a random component. The parameters describe an underlying physical setting in such a way that the value of the parameters affects the distribution of the measured data. An estimator attempts to approximate the unknown parameters using the measurements.

For example, it is desired to estimate the proportion of a population of voters who will vote for a particular candidate. That proportion is the unobservable parameter; the estimate is based on a small random sample of voters. Or, for example, in radar the goal is to estimate the range of objects (airplanes, boats, etc.) by analyzing the two-way transit timing of received echoes of transmitted pulses. Since the reflected pulses are unavoidably embedded in electrical noise, their measured values are randomly distributed, so that the transit time must be estimated.

In estimation theory, it is assumed the measured data is random with probability distribution dependent on the parameters of interest. For example, in electrical communication theory, the measurements which contain information regarding the parameters of interest are often associated with a noisy signal. Without randomness, or noise, the problem would be deterministic and estimation would not be needed

VI. Estimation Process

The entire purpose of estimation theory is to arrive at an estimator, and preferably an implementable one that could actually be used. The estimator takes the measured data as input and produces an estimate of the parameters.

It is also preferable to derive an estimator that exhibits optimality. Estimator optimality usually refers to achieving minimum average error over some class of estimators, for example, a minimum variance unbiased estimator. In this case, the class is the set of unbiased estimators, and the average error measure is variance (average squared error between the value of the estimate and the parameter). However, optimal estimators do not always exist.

The general steps to arrive at an estimator:

In order to arrive at a desired estimator, it is first necessary to determine a probability distribution for the measured data, and the distribution's dependence on the

unknown parameters of interest. Often, the probability distribution may be derived from physical models that explicitly show how the measured data depends on the parameters to be estimated, and how the data is corrupted by random errors or noise. In other cases, the probability distribution for the measured data is simply "assumed", for example, based on familiarity with the measured data and/or for analytical convenience.

After deciding upon a probabilistic model, it is helpful to find the limitations placed upon an estimator. This limitation, for example, can be found through the Cramér-Rao bound.

Next, an estimator needs to be developed or applied if an already known estimator is valid for the model. The estimator needs to be tested against the limitations to determine if it is an optimal estimator (if so, then no other estimator will perform better).

Finally, experiments or simulations can be run using the estimator to test its performance.

After arriving at an estimator, real data might show that the model used to derive the estimator is incorrect, which may require repeating these steps to find a new estimator. A non-implementable or infeasible estimator may need to be scrapped and the process started anew.

In estimator estimates the parameters of a physical model based on measured data.

To build a model, several statistical "ingredients" need to be known. These are needed to ensure the estimator has some mathematical tractability instead of being based on "good feel".

The first is a set of statistical samples taken from a random vector (RV) of size N. Put into a vector,

$$\mathbf{x} = \begin{bmatrix} x[0] \\ x[1] \\ \vdots \\ x[N-1] \end{bmatrix}$$

Secondly, we have the corresponding M parameters

$$\theta = \begin{bmatrix} \theta_1 \\ \theta_2 \\ \vdots \\ \theta_M \end{bmatrix},$$

which need to be established with their probability density function (pdf) or probability mass function (pmf) $p(x \mid \theta)$.

It is also possible for the parameters themselves to have a probability distribution (e.g., Bayesian statistics). It is then necessary to define the Bayesian probability $\pi(\theta)$.

After the model is formed, the goal is to estimate the parameters, commonly denoted $\hat{\theta}$, where the "hat" indicates the estimate.

One common estimator is the minimum mean squared error (MMSE) estimator, which utilizes the error between the estimated parameters and the actual value of the parameters

$$e = \hat{\theta} - \theta$$

as the basis for optimality. This error term is then squared and minimized for the MMSE estimator

VII. Local Bounds

The formulation of local bounds is based on the premise that the unknown parameters one seeks to measure are deterministic quantities. The bounds are local in the sense that they are placed on the MSE's for each different possible value of the intrinsic parameter vector. Local bounds have two serious limitations. First, they are restricted in application to estimators that are *unbiased*. In practice, biased estimation is often unavoidable. If the space of a parameter is finite, for example, then an unbiased estimator of it does not exist. Secondly, local bounds are unable to incorporate any prior information that one might have about the parameters.

The Cramer-Rao bound (CRB) is a familiar example of a local bound and it therefore has only limited utility. This bound states that for any unbiased estimator of a parameter vector θ, based upon noisy observations x, the error covariance matrix must be larger than or equal to the inverse of the Fisher information at θ. Thus

$$\mathcal{R}_{ij} = \langle (\hat{\theta}_i - \theta_i)(\hat{\theta}_j - \theta_j) \rangle \geq \mathcal{J}_{ij}^{-1},$$

where $\mathcal{J}$ is the Fisher information matrix whose elements are given by

$$\mathcal{J}_{ij} = \left\langle \frac{\partial^2}{\partial\theta_i \partial\theta_j} \ln\Lambda(\mathbf{x};\boldsymbol{\theta}) \right\rangle$$

and $\Lambda(x;\theta)$ is the likelihood ratio

$$\Lambda(\mathbf{x};\boldsymbol{\theta}) = \frac{p(\mathbf{x}|\boldsymbol{\theta})}{p(\mathbf{x}|0)}.$$

VIII. Conclusion

Several methods for calculating performance bounds for node localization in wireless sensor networks, the bounds account for information available on the node location from the sensor selection process and consider complicating effects such as radio propagation including both LoS and NLoS propagation paths within the network environment which works efficiently Localization bound accuracies which helps in estimation of bounds for node localization in wireless Sensor Networks

The *Cramér–Rao bound* with estimation theory helps in tight and accurate calculation of localization bounds in wireless sensor network for better performance of data and other services for WSN's

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