Traffic Modelling for Capacity Analysis of CDMA Networks using Gaussian Approximation Method

B. O. Omijeh
Department of Electronic and Computer Engineering, University of Port Harcourt
Email: omijehb@yahoo.com

R. E. Okonigene
Department of Electrical and Electronic Engineering, Ambrose Alli University Nigeria

R. Ochi
Department of Electronic and Computer Engineering, University of Port Harcourt

Abstract – This paper presents, modelling telephone traffic in cellular networks operating with Code Division Multiple Access (CDMA) for the purpose of capacity analysis of such systems. With the current use of CDMA as the preferred multiple access technique due to its capacity advantage, there is the need for tools that will assist in ensuring quality of service and proper network dimensioning. This work produces a model useful for capacity analysis. Focusing on the reverse link, this is achieved by modelling telephone traffic using Gaussian assumptions to generate a CDMA blocking probability that is adapted into Erlang B formula for capacity calculations. A program written in MATLAB is used to realise the blocking probability formula with graphical outputs that is a tool for dimensioning. Results show that variations in network parameters affect CDMA capacity and that CDMA has a huge capacity advantage over TDMA and FDMA.

Keywords – Modelling, CDMA, Gaussian, MATLAB, Erlang, Telephone, Traffic.

I. INTRODUCTION

Code Division Multiple Access (CDMA) is the preferred access technique for mobile communications. Its development is mainly for capacity reasons [1]. Its advantage over other multiple access schemes (like FDMA and TDMA) include higher spectral reuse efficiency, greater immunity to multipath fading, more robust handoff procedures, gradual overload capability and voice activity effects.

In CDMA however, the separation between traffic and transmission issues is not clear with capacity being determined by interference caused by all the transmitters in the network [2]. This calls for more researches to understand its traffic behaviour.

Cooper in his work [3] carried out analysis of cell interference in spread spectrum. An extension of this which includes the effects of shadowing and voice activity monitoring, is found in the work of Gilhousen. In the paper[4] by Fapojuwo in 1993, a computationally intensive procedure is presented for the evaluation of the teletraffic capacity of both forward and reverse links in a CDMA cellular system. Kim in 1993[5], did a very similar analysis with the exception that the fixing of the PLE at 4 leads to analytic expressions for the interference from the circular cells. This is extended to an analytic result for variance [6]. A standard hexagonal cellular layout is assumed with the propagation model in the paper[7] by Kohno et al in 1995, which includes lognormal shadowing taken to be independent on distinct paths. An extension of the reverse link analysis of the work by Gilhousen et al is discussed in a paper [8] by Viterbi et al(1992). Robert, in 1996[9][10] carried out an investigation of the effect of using the actual distance of users when calculating the capacity of a CDMA network. Simulations were carried out for twenty-seven cell CDMA network. The simulation results show that for a uniform user distribution, the difference in capacity determined using relative actual interference and relative average interference is too small to warrant the incursion of heavy computational load involved in the former case. This discovered advantage is explored in the capacity analysis in this paper. Blocking of subscribers trying to make calls occurs when the reverse link multiple access interference power reaches a predetermined level that is set to maintain acceptable signal quality.

II. CDMA CAPACITY FUNDAMENTALS

Consider a CDMA (IS-95) system with band width W MHz with each user employing an uncoded bit rate \( R_b = 1/T_b \), where \( T_b \) is the bit duration. If the system employs orthogonal signalling waveform, then the maximum number of users is given by

\[
M = \frac{W}{R_b} = WT_b
\]

(1)

Assuming that the received power of each user is \( S_r \), then the total received power is

\[
P_r = MS_r
\]

(2)

The required signal-to-noise power ratio (SNR) or \( E_b/N_0 \) (bit energy-to-noise spectral density ratio) is given to be

\[
\left( \frac{E_b}{N_0} \right)_{\text{req}} = \frac{S_r/R_b}{N_0} \cdot \frac{P_r/M}{N_0 R_b}
\]

(3)

From which we obtain

\[
M = \frac{P_r}{\left( \frac{E_b}{N_0} \right)_{\text{req}} R \cdot \left( \frac{E_b}{N_0} \right)_{\text{req}}}
\]

(4)

The capacity of a CDMA system is proportional to the processing gain of the system, which is the ratio of the spread bandwidth to the data rate. This fact may be shown as follows: assuming first that the system is isolated from all other forms of interference (i.e., a single cell), the carrier power

\[
C = S_r E_b/T_b = RE_b
\]

(5)

The interference power at the base station may be defined as

\[
I = W \cdot N_0
\]

(5)

Where \( W \) is the transmission bandwidth and \( N_0 \) is the interference power spectral density.
Thus a general expression for the carrier-to-interference power ratio for a particular mobile user at the base station is given by

$$ C = \frac{R.E_b}{W/N_0} = \frac{E_b/N_0}{W/R} $$

(6)

The quantity $E_b/N_0$ is bit energy to noise power spectral density and $W/R$ is the processing gain of the system.

If $M$ denote the number mobile users and power control is used to ensure that every mobile has the same received power at the base station, then (neglecting thermal noise) the interference power caused by the $M - 1$ interferers is

$$ I = C(M - 1) $$

(7)

Substituting for $I$, the carrier-to-interference ratio can now be expressed as

$$ C = \frac{C}{C(M-1)} = \frac{M}{M-1} $$

(8)

Substituting $C/I$ from equation (6) into (8) the capacity of CDMA is found to be

$$ M \approx M - 1 = \frac{W}{R} \cdot \frac{1}{E_b/N_0} $$

(9)

Thus, the capacity of CDMA is proportional to the processing gain.

A. CDMA Capacity and Duty Cycle

Studies have shown that the average duty cycle of a full-duplex voice conversation is approximately 35%. Because transmission is not eliminated entirely but reduced during such pauses, the effective duty cycle of the digital waveform is closer to 40% or 50%.

If the duty cycle of the speech traffic channels in the CDMA system is denoted by the variable $\alpha$, then the capacity equation becomes

$$ M = \frac{W}{\alpha R} \cdot \frac{1}{E_b/N_0} \cdot \frac{1}{\alpha} $$

(10)

B. Capacity and Directivity

If the base station employs directional antennas that divide the cells into sectors, each antenna will only receive a fraction of the interference from within the cell. In practice, the receiving antennas have overlapping coverage areas of approximately 15%. Standard implementation divide the cell into three sectors, which provides an effective capacity increase of $G=3$: 0.85 = 2.55, and the corresponding capacity equation is

$$ M = \frac{W}{R} \cdot \frac{1}{E_b/N_0} \cdot \frac{1}{\alpha} \cdot G $$

(11)

C. Capacity in a Multicellular CDMA Network Subject to Interference

The above capacity equations assumed a single, isolated CDMA cell. M in equation (9) is comparable with the number of frequencies available to users in a sector of a single-cell FDMA deployment. In a multicell system, the interference from signals originating in other cells must be taken into account when determining the capacity of a particular “home” cell. Such interference is diminished by the attenuation incurred by the interferers in propagating to the home cells.

The equation for CDMA capacity may be modified to include a reuse efficiency (treated more clearly in subsequent sections of this thesis) for other cell interference, $F_c$:

$$ M = \frac{W}{R} \cdot \frac{1}{E_b/N_0} \cdot \frac{1}{\alpha} \cdot G \cdot F_c $$

(12)

III. CDMA BLOCKING PROBABILITY MODELING

In this section, a novel CDMA blocking model is developed as a tool for the capacity analysis of CDMA cellular networks. This blocking model is also modelled by software to allow for more flexibility in the analysis in this thesis.

The number of users for which the CDMA blocking probability, denoted $B_{CDMA}$, equals a certain value (usually 1 or 2%) is defined to be the Erlang capacity of the system and is related to an equivalent number of channels in an FDMA or TDMA cellular system. Thus, the calculation of the CDMA blocking probability is based on an analysis of the other-user interference.

A. Formulation of the Blocking Probability

First we consider a single, isolated CDMA cell with $M$ active users. The total reverse link signal-plus-noise power received at the base station can be written as

$$ \alpha G P_1 + \alpha G P_2 + \ldots + \alpha M P_M + (N_0 W) $$

(13)

Where

- The $\{\alpha_i\}$ are random variables representing the reverse link voice activity, which have the experimental values given as $E[\alpha_i] = \alpha _r = 0.4$ and $E[\alpha_i^2] = \alpha _t = 0.31$
- The $\{P_i\}$ are the random signal powers for the $M$ active users.
- The number of signals $M$ is itself an RV, assumed to have a Poisson distribution, so that $E[M]=\bar{M} = Var[M]$

To a potential $(M + 1)$st reverse link user, the total power for the $M$ active users and the thermal noise is interference power. Thus, we may write

$$ I = I_0 + \sum_{i=1}^{M} \alpha_1 P_1 + \alpha_1 \alpha_2 P_2 + \ldots + \alpha_1 \ldots \alpha_M P_M + (N_0 W) $$

(14)

$$ I_0 = \frac{I}{W} = \frac{\alpha_1 P_1 + \alpha_1 \alpha_2 P_2 + \ldots + \alpha_1 \ldots \alpha_M P_M + N_0 W}{W} $$

(15)

Normalized by $I_0 R_b$, where $R_b$ is the data bit rate, the total interference is characterized by the quantity

$$ \frac{I}{I_0 R_b} = \frac{W}{R_b} \cdot \frac{\alpha_1}{I_0} \cdot \frac{E_b}{I_0} + \frac{\alpha_2}{I_0} \cdot \frac{E_b}{I_0} + \ldots + \frac{\alpha_M}{I_0} \cdot \frac{E_b}{I_0} + \frac{N_0 W}{I_0 R_b} $$

(16)

Where $P$ is $E_b R_b$,

$$ Z = \sum_{i=1}^{M} \alpha_i \rho_i = \frac{W}{R_b} (1 - \eta) $$

(17)

$$ \rho_i = \frac{E_b}{I_0} $$

(18)

and

$$ \eta = \frac{N_0}{I_0} \text{ (thermal noise) } $$

(19)

is a parameter indicating the loading of the CDMA system and $W/R_b$ is the spread-spectrum processing gain.

Given the value of $\eta$, the quality of the channel that is available to the $(M + 1)$st mobile user is characterized by the value of the random variable $Z$; if $Z$ exceeds some threshold value, then the channel is effectively unavailable (blocked) to the $(M + 1)$st user. In terms of the distribution
of the random variable $Z$, the probability that the $(M + 1)_{th}$ mobile CDMA user will be blocked is the probability that $Z$ exceeds some threshold value $Z_0$ as a function of a threshold value of the interference parameter $\eta_0$.

$$B_{\text{CDMA}} = \Pr \{ Z > Z_0 = \frac{W}{R_0} (1 - \eta_0) \} \quad (20)$$

$$= \Pr \{ \sum_{i=1}^{M} \alpha_r \rho_i > \frac{W}{R_0} (1 - \eta_0) \} \quad (21)$$

If a probability density function $p_r(x)$ is known or assumed for $Z$, then the evaluation of $B_{\text{CDMA}}$ is simply a matter of integrating that pdf over the region defined by $Z > Z_0$.

$$B_{\text{CDMA}} = \int_{Z_0}^{\infty} d \rho \cdot p_r(x) \quad (22)$$

The exact pdf of $Z$ is not known, however, so an approximation is needed to compute $B_{\text{CDMA}}$.

The CDMA blocking probability can be manipulated to the form

$$B_{\text{CDMA}} = \Pr \{ Z > Z_0 \} = \Pr \left( \frac{Z - E[Z]}{\sqrt{\text{Var}[Z]}} > Z_0 - E[Z] \right) \quad (23)$$

$$= Q \left( \frac{Z_0 - E[Z]}{\sqrt{\text{Var}[Z]}} \right) \quad (24)$$

Where $Q(z)$ is notation for the complementary cumulative distribution function of the standardized (zero-mean, unit variance) version of the RV $Z$.

The approximation method to be considered in what follows is Gaussian approximation, based on the fact that $Z$ is a sum (Central Limit Theorem) and so we can write

$$Q(\rho) \approx Q \left( \frac{\rho}{\sqrt{\text{Var}[\rho]}} \right) \quad (25)$$

That is, the blocking probability can be calculated using

$$B_{\text{CDMA}} = Q \left( \frac{Z_0 - E[Z]}{\sqrt{\text{Var}[Z]}} \right) \quad (26)$$

The approximation method is based on identifying the actual mean and variance of $Z$ with the mean and variance of a Gaussian RV. Next, the mean and variance of $Z$ and its specification in relation to Gaussian RV can be determined.

### B. Mean and Variance of $Z$

The form of the interference statistics $Z$ is the weighted sum of the $M$ RVs $\{\rho_i, i = 1, 2, \ldots, M\}$:

$$Z = \alpha_{r_1} \frac{E_{\text{tot}}}{I_0} + \alpha_{r_2} \frac{E_{\text{tot}}}{I_0} + \ldots + \alpha_{r_M} \frac{E_{\text{tot}}}{I_0}$$

$$= \alpha_{r_1} \rho_1 + \alpha_{r_2} \rho_2 + \ldots + \alpha_{r_M} \rho_M \quad (27)$$

Propagation measurements in general indicate that received signal powers, when expressed in dB units, are nearly Gaussian. Therefore, the $\{\rho_i\}$ above, when measured in dB units, are close to having a Gaussian probability distribution with median $m_{\text{db}}$ and standard deviation $\sigma_{\text{db}}$:

$$\rho_i \ (\text{dB}) = 10 \log_{10} \rho_i = m_{\text{db}} + \sigma_{\text{db}} G_i \quad (28)$$

Therefore, the RV $\rho_i$ is lognormal and can be written

$$\rho_i = 10^{(m_{\text{db}} + \tau G_i)/10} = (e^{\ln(10)^{m_{\text{db}} + \tau G_i}}) / 10$$

$$= e^{(\ln(10))m_{\text{db}} + \tau G_i} \quad \text{using } \beta = (\ln 10)/10 \quad (29)$$

The median, mean, and mean square of $\rho_i$ are assumed to be same for all $i$ and are obtained as follows:

- Median
  $$\frac{1}{2} \pm \Pr \{ \rho_i \leq \rho_{med} \} = \Pr \left\{ e^{\beta (mdB + \rho dB)} \leq \rho_{med} \right\}$$

  $$\Pr \left\{ G \leq \frac{\tau \ln \rho_{med} - m_{\text{db}}}{\sigma_{\text{db}}} \right\} = G_{\text{med}} = 0 \quad (30)$$

  Thus $\rho_{med} = e^{\beta m_{\text{db}}}$

- Mean
  $$E(\rho_i) = E \left\{ e^{\beta (mdB + \rho dB)} \right\} = e^{\beta m_{\text{db}}} E \left\{ e^{\beta \rho dB} \right\}$$

  $$= \rho_{med} E \left\{ e^{\beta \rho dB} \right\} = \beta \sigma dB \quad (31)$$

  Where $E \left\{ e^{\beta \rho dB} \right\} = M_G(u) = u^{2/2} \quad (MGF)$

  Thus, $E \{ \rho_i \} = \rho_{med} M_G(\beta \sigma dB)$

- Means Square:
  $$E \{ \rho_i^2 \} = E \left\{ e^{2\beta (mdB + \rho dB)} \right\}$$

  $$= e^{2\beta m_{\text{db}}} E \left\{ e^{2\beta \rho dB} \right\} = \rho_{med}^2 M_G(2\beta \sigma dB)$$

  $$= \rho_{med}^2 e^{1/2(2\beta \sigma dB)^2} = \rho_{med}^2 e^{2(\beta \sigma dB)^2} \quad (32)$$

Because $M$ is analogous to the number of calls in progress through a switch, which has a Poisson distribution, it is reasonable to postulate that the mean and variance of $M$ are equal, as in the case of Poisson RV. The mean, mean square and variance of $Z$ therefore are

$$E[Z] = E[mdE[Z|M]] = E[M \sum_{i=1}^{M} E[\alpha_{r_i} \rho_i]]$$

$$= E[M E[\alpha_{r_i} \rho_i]] = E[M] E[\alpha_{r_i} \rho_i]$$

$$= M \bar{\alpha}_{r_i} e^{\beta m_{\text{db}}} + \frac{1}{2} \beta^2 \sigma^2 dB \quad (33)$$

$$E[Z^2] = E[M \sum_{i=1}^{M} E[\alpha_{r_i} \rho_i]^2]$$

$$= E[M \sum_{i=1}^{M} E[\alpha_{r_i}^2 \rho_i^2] + E[\alpha_{r_i} \rho_i]^2]$$

$$= E[M \sum_{i=1}^{M} E[\alpha_{r_i}^2 \rho_i^2]]$$

$$= E[M E[\alpha_{r_i}^2 \rho_i^2]] + E[\alpha_{r_i} \rho_i]^2$$

$$= E[\alpha_{r_i} \rho_i^2] + E[\alpha_{r_i} \rho_i]^2$$

$$= E[\alpha_{r_i} \rho_i]^2$$

$$= E[\alpha_{r_i} \rho_i]^2 = \frac{M^2 - M}{E[\alpha_{r_i} \rho_i]^2}$$

$$= \frac{M^2 - M}{E[\alpha_{r_i} \rho_i]^2} [E[\alpha_{r_i} \rho_i]^2]$$

$$= \bar{\alpha}_{r_i} e^{2\beta m_{\text{db}} + \beta^2 \sigma^2 dB} \quad (34)$$

The interference due to mobiles in other cells can be accounted for by using first- and second-order frequency reuse factors $F = 1 + \xi$ and $F^1 = 1 + \xi$, respectively, where

$$\xi = \frac{\text{Total other cell received (median)power}}{\text{Total same cell received (median)power}}$$

$$\xi' = \frac{\text{Total other cell mean square received power}}{\text{Total other cell mean square received power}} \quad (38)$$
A typical analytical value of $\xi = 0.086$ and $\xi' = 0.55$. With this method of accounting for interference from other cells, the mean and variance for $Z$ become and

$$E[Z] = \bar{M} \rho_m \rho_{med} e^{\frac{1}{2} \beta^2 \sigma^2} (1 + \xi)'$$

$$\text{Var}[Z] = \bar{M} \rho_m^2 \rho_{med}^2 e^{2 \beta^2 \sigma^2} (1 + \xi')$$

(39) and (40)

C. Approximations for the Probability Distribution of $Z$

Because the $M$ RVs $\{p_i, i=1, 2, \ldots, M\}$ are lognormal RVs, the interference statistic $Z$ is the weighted sum of lognormal RVs. One approximation for the distribution of $Z$ is based on assuming that the summing of variables to produce $Z$ causes its distribution to converge to a Gaussian distribution according to the CLT. Another approach is to assume that the lognormal character of the $\{p_i\}$ makes $Z$ have an approximately lognormal distribution.

$$Z = \sum_{i=1}^{M} \rho_{med} e^{\beta \Delta dB + \beta dB} + \sum_{i=1}^{M} \rho_{med} e^{\beta dB}$$

$$\approx \sum_{i=1}^{M} \rho_{med} e^{\beta (\Delta dB + dB)} + \sum_{i=1}^{M} \rho_{med} e^{\beta dB}$$

$$\approx m + \sigma_M G$$

D. CDMA Blocking Probability Formula for Gaussian Assumptions

Under the Gaussian assumption, the mean and variance of $Z$ are identified as the mean and variance of a Gaussian RV, $m_M + \sigma_M G$:

$$B_{CDMA} = Pr[Z > Z_0]$$

$$\approx Pr[m_M + \sigma_M G > Z_0] = Pr\left( G > \frac{Z_0 - m_M}{\sigma_M} \right)$$

Substituting the expressions for the mean and variance of $Z$, we obtain a general expression for the CDMA blocking probability under the Gaussian approximation for the interference statistic, given by

$$B_{CDMA} = Q\left( \frac{w_{b}}{\bar{M} \rho \rho_{med} e^{\frac{1}{2} \beta^2 \sigma^2} (1 + \xi')} \right)$$

(41)

The interference parameter $\eta$ is related to cell loading by the relation

$$\eta = 1 - X$$

(42)

Thus the threshold $\eta_0$ can be converted to loading threshold as $X_0 = 1 - \eta_0$ giving

$$B_{CDMA} = Q\left( \frac{w_{b}}{\bar{M} \rho \rho_{med} e^{\frac{1}{2} \beta^2 \sigma^2} (1 + \xi')} \right)$$

(43)

Which is of the form $B_{CDMA} = Q\left( \frac{w_{b} - \frac{\bar{M}}{\rho \rho_{med}}}{\frac{\sigma_M}{\rho \rho_{med}}} \right)$

Using typical numerical values for the parameters with the following set of values [Jong S. L. et al, 1998]

$$\sigma_{db} = 2.5dB, m_{db} = 7dB, W = 1.2288MH, R_0 = 9.6kbps, X_0 = 0.9, \overline{\alpha} = 0.4, \overline{\alpha}' = 0.31, \beta = (ln 10)/10, \rho_{med} = e^{0.05dB}$$

The $B_{CDMA}$ can be reduced to

$$B_{CDMA} = \left( \frac{115.2 - 2.37 (1+\xi') \bar{M}}{3.89 (1+\xi')} \right)$$

(44)

This is the blocking probability of CDMA network as a function of Erlang capacity $\bar{M}$. $\xi, \xi'$ can assume typical experimental values such as $\xi = \xi' = 0.55$.

IV. RESULTS AND DISCUSSION

A. Graphical Plots and Analysis of CDMA Blocking Probability Formula for Gaussian Assumptions

The blocking probability formula derived by Gaussian Assumptions in the previous session is as follows:

$$B_{CDMA} = Q\left( \frac{w_{b}(X_0) - \bar{M} \rho \rho_{med} e^{\frac{1}{2} \beta^2 \sigma^2} (1 + \xi')} \right)$$

(45)

Taking a plot of $B_{CDMA}$ versus $\bar{M}$ for a single cell ($\xi = \xi' = 0$) and for multiple cells ($\xi = \xi' = 0.55$) using the following typical parameter values: $\sigma_{db}=2.5dB, m_{db}=7dB, W=1.2288MH, R_0 = 9.6kbps, X_0 = 0.9, \overline{\alpha} = 0.4, \overline{\alpha}' = 0.31$.

It is observed that the plots are parametric in $m_{db}$, $N_0$, which takes the values 5.6 and 7dB. The Figures below show the results obtained.

$$\overline{\alpha} = 0.4, \overline{\alpha}' = 0.31, X_0 = 0.9, \frac{w}{R_b} = 128$$

Fig.1. CDMA blocking probability (Gaussian approximation) versus average number of mobile users, SNR requirement varied.

It is observed that the value of $E_b/N_0$ needed for link operations affects the average number of users that can be accommodated at a given level of blocking. Raising the $E_b/N_0$ requirement increases the blocking probability for the same value of $M$ or decreases the capacity for the same probability. For example, when the blocking probability is chosen to be $B_{CDMA} = 10^{-2} = 0.01 = 1\%$, Figure 1 shows for multiple cells that the corresponding value of the Erlang capacity $\bar{M}$ is 18 for $m_{db} = 7dB$, 24 for $m_{db} = 6dB$, and 33 for $m_{db} = 5dB$.

If the Erlang capacity for a single cell is denoted by $\bar{M}$ and the capacity for multiple cells by $\bar{M}$, for $B_{CDMA} = 1.0\%$ and $E_b/N_0 = 6dB$, reading from figure 1, $\bar{M} = 37.5$ and $\bar{M}' = 24.1$. The ratio of these values is $37.5/24.1 = 1.56$, a value that is approximately equal to the assumed value of 1.56.
the reuse factor, $F = 1 + \xi = 1.55$. This is consistent with
the definition of the reuse factor.

\[ \alpha_r = 0.4, \alpha_T^2 = 0.31, \frac{E_b}{N_0} = 6dB, \frac{W}{R_b} = 128 \]

Fig. 2. CDMA blocking probability (Gaussian approximation) versus average number of mobile users, loading threshold varied.

Assuming that $m_{dB} = E_b / N_0 = 6$ dB, the effect of varying the loading threshold $X_0$ on $B_{CDMA}$ and $M$ is illustrated in Figure 2, in which $X_0$ takes the values $X_0 = 0.66, 0.75$, and 0.9. These values correspond to the multiple access interference power being twice, three times, and nine times as strong as the thermal noise. Raising the loading threshold has the effect of relaxing the system requirements, and is seen in Figure 2 to result in either a decrease in the blocking probability for the same value of $M$, or an increase in $M$ for the same value of $B_{CDMA}$. If we substitute specific numerical parameter values into the general expression (45), such as $m_{dB} = 2.5$ dB, $m_{dB} = 7$dB, $W = 1.2268$ MHz, $R_b = 9.6kbps$, $X_0 = 0.9, \alpha_r = 0.4, \alpha_T^2 = 0.31$, and we obtain

\[ B_{CDMA} = Q\left(\frac{\sqrt{115.2 - 2.37(1+\xi)^3}}{2\alpha_T^3/\alpha_r^3}\right) \]

Which is of the form

\[ B_{CDMA} = Q\left(\frac{a-bM}{\sqrt{CM}}\right) \]

Because $Q(0) = 0.5$. It is inferred from (46) that the blocking probability is 50% when $M = a/b$. This high a blocking probability is of course numerical, so we know that an acceptable value of blocking probability is realized only when $M$ is much less than $a/b$. It is interesting therefore to note by comparing (46) and (45) that the upper limit on $M$ based on having a small blocking probability is

\[ M < \frac{a}{b} \approx \frac{w}{\alpha_T p_{med}} e^{2\sigma^2 a_{dB}(1+\xi)} = \frac{P_G}{P_{G}/N_0} \cdot \frac{1}{\alpha_T F} \cdot \frac{X_0}{\epsilon^2 a_{dB}} \]

The ideal CDMA capacity

\[ M = \frac{P_G}{E_b/N_0} \cdot \frac{1}{\alpha_T F} = \frac{P_G}{E_b/N_0} \cdot \frac{1}{\alpha_T} \cdot Fe \]

is valid under perfect power control and omni-directional cell antenna assumptions. Note that under the conditions of perfect power control ($m_{dB} = 0$ dB) and 100% cell loading in the ideal situation ($X_0 = 1$), then the Erlang capacity bound in (47) is equal to the ideal capacity in (48).

B. Sensitivity To $\xi'$

The sensitivity of the CDMA blocking probability to the value of the “second order” reuse fraction $\xi'$ is considered in Figure 3, in which $\xi = 0.55$ and $X_0 = 0.55$ are seen to be slightly to the left of those for $\xi = 0.086$ for small blocking probabilities. This indicates that the terms in the analysis that involve the mean square power are not critical and that use of assumption that $\xi = \xi'$ does not incur a significant loss in accuracy, provided that the blocking probability is taken to be greater than 1%.\[ \alpha_r = 0.4, \alpha_T^2 = 0.31, X_0 = 0.9, \frac{R_b}{W} = 128 \]

Fig. 3. CDMA blocking probability (Gaussian approximation) versus average number of mobile users, reuse fractions varied.

C. Erlang Capacity Comparisons of CDMA, FDMA, AND TDMA

Here we consider calculations for the Erlang capacity of a CDMA telephone system and compare it with that of FDMA (AMPS) and TDMA (IS-54) systems, using the blocking probability curves that we have developed thus far under the Gaussian approximation for the interference statistics.

Thus, Figure 3 can be used for Erlang capacity determination for all cases of interest with respect to the values of $E_b/N_0$. The comparison of Erlang capacities is based on first reading the CDMA Erlang capacity for a specified blocking probability from Figure 3 and then treating it as the offered load, so that we can use the Erlang B probability expression:

\[ B_{CDMA} = \frac{(M)^{X/N}}{\Sigma_{n=0}(M)^n/n!} \]

(49)

to find N, which is an “equivalent number of channels” to be compared with the numbers of channels in the FDMA and TDMA systems.
Having found the equivalent number of channels in a CDMA system that corresponds to a value of blocking probability, the capacities of CDMA, FDMA, and TDMA cellular systems can be compared. For this purpose numbers of channels (N) are given in Table 1. The Erlang capacities and equivalent values of $\bar{M}$ were read from the curves in the Figure 3, for the case of $(\xi, \xi^*) = (0.55, 0.086)$, and the values of N are based on the Erlang B table (in the appendix). For example, for $E_b/N_0 = 7$ dB and $B = B_{\text{CDMA}} = 1\%$, from Figure 3 we read the Erlang capacity $\bar{M} = 20$ Erlangs. Now, for $\bar{M} = A$, the offered load, we need to find the equivalent number of channels N that satisfies (49). For this N, we look the Erlang B table and read the corresponding number of channels as $N = 30$, as shown in Table 1.

The number of channels in Table 1 is for a CDMA cellular system in one 1.25-MHz band (one FA). Assuming a noncontiguous 12.5-MHz cellular band, the CDMA system can use as many as nine 1.25-MHz FAs.

Table 1: Erlang capacities and corresponding numbers of channels, based on Figure 3, for $\xi = 0.55$ and $\xi^* = 0.086$.

<table>
<thead>
<tr>
<th>$B_{\text{CDMA}}$</th>
<th>$E_b/N_0 = 5$ dB</th>
<th>$E_b/N_0 = 6$ dB</th>
<th>$E_b/N_0 = 7$ dB</th>
</tr>
</thead>
<tbody>
<tr>
<td>1%</td>
<td>$M = 35$</td>
<td>$M = 26$</td>
<td>$M = 20$</td>
</tr>
<tr>
<td></td>
<td>$N = 46$</td>
<td>$N = 37$</td>
<td>$N = 30$</td>
</tr>
<tr>
<td>2%</td>
<td>$M = 36$</td>
<td>$M = 28$</td>
<td>$M = 21$</td>
</tr>
<tr>
<td></td>
<td>$N = 46$</td>
<td>$N = 37$</td>
<td>$N = 30$</td>
</tr>
</tbody>
</table>

Thus, to compare the effective number of CDMA channels available in a particular sector with the number of channels available in the AMPS(FDMA) and TDMA (IS-54) cellular systems, the numbers for N in Table 1 must be multiplied by the number of FAs, which is nine. After this multiplication is done, the corresponding Erlang capacity can be obtained from an Erlang B Table or calculation by identifying the offered load with a multi-FA CDMA Erlang capacity denoted by $M_{\text{multi-FA}}$. As diagrammed in Figure 4, this procedure of finding the offered load corresponding the aggregate number of channels (nine times N in Table 1) assumes that all those N times N are available for assignment and that mobile is capable of accessing any of them.

**D. Number of Subscribers at the Busy Hour**

We are interested in computing not only the number of active users at a given time, but also the number of subscribers that may be supported by the CDMA system in any given cell.

In terms of traffic theory, consider a telephone switch and its trunk of N lines, for system-planning purposes, the traffic during the “busy hour” of the day is used. Typically, a user is likely to be on the telephone at any given time during the busy hour with the probability of 0.02 to 0.03. That is, each subscriber is considered to offer $A_0 = 0.02$ to 0.03 Erlangs of traffic. The number of subscribers ($M_0$) that can be supported by the trunk during the busy hour for a specified blocking probability that results in the total load $A$ then is given by the formula

$$M_0 = A/A_0$$

(52)

**V. CONCLUSION**

Erlang capacity “formula” for the CDMA cellular system under an approximation for the interference statistics has been derived using Gaussian approximation, by invoking the CLT. The stochastic nature of call arrivals and departures were characterized using statistical means. The interference contributed by each user was modeled as a Poison RV that summed up to a statistical RV with Gaussian characteristics.

The blocking probability formula so derived was programmed in MATLAB and Erlang capacities calculated and results graphically displayed. See Figure 5. Blocking occurred when the reverse link multiple access interference power reached a predetermined level that is set to maintain acceptable signal quality. When the total user interference at a base station receiver exceeded the set threshold, the system blocked the next user attempting to place a call.

The number of users for which the CDMA blocking probability equaled 1% as chosen was taken to be the Erlang capacity of the network. Thus, a new CDMA blocking probability model is developed that enabled the estimation and analysis of Erlang capacity of CDMA networks.

Graphic results for the blocking model generated showed the effect of variations in interference parameters on CDMA capacity. The Erlang capacity from the model was adapted into Erlang B formula to estimate capacity in...
terms of channels, and the number of subscribers a typical CDMA sector could accommodate.

Comparative capacity analysis showed that CDMA has a huge capacity advantage over TDMA and FDMA.

REFERENCES


