Performance Evaluation for Super-Orthogonal Concatenated Convolutional Coding Scheme for Multiple Access Applications over Nonlinear Satellite Channels

Hala M. A. Mansour, Labib Francis Gergis, Mostafa A. R. Eltokhy, Hoda Z. Said

Abstract — In digital satellite communications there are many earth stations and satellites sharing the transmission channel hence, a multiple access scheme is needed in order to increase the number of simultaneous users and make efficient utilization of power and bandwidth of the channel. Code-division multiple-access (CDMA) has been considered as one of the most efficient and reliable schemes for multiple access satellite communications. In CDMA system, ultimate capacity can be achieved using low rate error correcting codes to perform both coding and spreading. Orthogonal and super-orthogonal convolutional codes have been proposed for this purpose. The super-orthogonal convolutional code performs well in very low rates and requires less bandwidth expansion than the orthogonal convolutional code. Concatenating two or more super-orthogonal convolutional codes through interleavers achieves near Shannon limit performance. A major drawback in satellite communications is the presence of high power amplifier which is needed to provide the signal power required to overcome the large distance between transmitter and receiver. The high power amplifier introduces nonlinear distortion to the amplified signal when it operates in its maximum power efficiency region which results in degradation to the CDMA signal. In order to compensate for this nonlinear response a linearization technique is employed or the operating point of the amplifier is backed-off to its linear region. In this paper we present a new low rate concatenated convolutional coding scheme suitable for spread spectrum CDMA applications. Analysis for the bit error probability performance of the proposed coding scheme is derived in both additive white Gaussian noise (AWGN) channel, and nonlinear satellite channel. Results showed that, the proposed coding scheme achieves superior performance compared to the conventional low rate convolutional codes.

Keywords — CDMA, Code Concatenation, HPA, Low Rate Codes.

I. INTRODUCTION

Code-division multiple-access [1]-[3] is a widely employed multiple access technique in satellite systems. It has robustness in multipath fading channels and in addition, it has the ability to accommodate large number of users with lower interference levels.

In conventional CDMA system, the presence of the multiple-access interference (MAI) limits the number of simultaneous users supported by the channel. To minimize the effect of MAI, powerful error correcting codes are employed followed by the spreading process which is accomplished using pseudo random sequences [4][5]. It was proved by Viterbi [6][7] that, spread spectrum CDMA systems can reach their maximum theoretical performance by employing low-rate error correcting codes alone to perform both coding and bandwidth spreading which is known as code-spread. These low rate codes are suitable only for spread spectrum applications where the code rate can be as small as the inverse of the spreading factor of the spreading process.

The early introduced low rate error correcting codes are orthogonal [6], and super-orthogonal convolutional codes [8][9]. Later, super-orthogonal turbo codes [10] was presented which is a concatenation of two or more super-orthogonal convolutional codes. Super-orthogonal turbo code has superior performance to both orthogonal and super-orthogonal convolutional codes. A significant amount of work has been done in order to obtain good low rate codes for application in CDMA systems [11]-[15].

Powerful error correcting codes can be obtained by concatenating at least two codes through interleavers which is known as code concatenation [16]. Two concatenated coding schemes are commonly used: serial concatenation [17], and parallel concatenation [18][19].

In satellite communications, in order to provide the transmit signal levels needed to overcome the loss between the transmitter and receiver, an efficient high power amplifier (HPA) is employed. Power amplifier operates most efficiently at its saturation region. As driven near saturation, the power amplifier exhibits nonlinear distortion [20][21] which causes degradation to the signal-to-noise ratio (SNR), and interference to adjacent channels.

In this study, a new low rate concatenated convolutional coding scheme is presented called super-orthogonal parallel-serial concatenated convolutional code (SOPSCCC). The proposed concatenated coding scheme is suitable for spread spectrum applications. The bit error probability performance of the coding scheme is studied in both AWGN channel, and nonlinear satellite channel where a high power amplifier is employed. Results proved the superior performance of the proposed concatenated coding scheme to that of the classical low rate concatenated convolutional codes.

This paper is organized as follows, in section II a brief review on orthogonal and super-orthogonal convolutional codes is presented. Section III presents an overview for the conventional concatenated convolutional coding schemes. In section IV, a review on high power amplifier is introduced. Section V introduces the construction of the proposed SOPSCCC. Analysis of the SOPSCCC is illustrated in section VI and the results for the bit error probability performance is presented in section VII. In section VIII conclusion is discussed.
II. ORTHOGONAL AND SUPER-ORTHOGONAL
CONVOLUTIONAL CODE

Orthogonal convolutional codes are low rate codes whose codewords are orthogonal to each other. Encoder structure for anorthogonal convolutional code is shown in Fig. 1. The encoder consists of a k-stage shift register, the states of the shift register are connected to an orthogonal signal selector. One of the possible ways to obtain orthogonal signal set is by using Hadamard matrices [10].

Fig.1. Block diagram for an orthogonal convolutional encoder.

Every one bit input, all the k bits (contents of the shift register) are used to select a row from the orthogonal code. The selected codewords are orthogonal to each other since they are generated form an orthogonal signal set. Every bit interval the encoder outputs one of $2^k$ codewords each of length $2^k$ bits so, the code rate of the orthogonal encoder is $1/2^k$. By performing some modification on the orthogonal convolutional encoder, a new class of orthogonal convolutional codes is obtained called, super-orthogonal convolutional codes. Encoder structure of the super-orthogonal convolutional code is same as that of orthogonal convolutional code except that, only the inner k-2 stages of the shift register are used to select a codeword from the orthogonal code hence, the code rate of the super-orthogonal convolutional codes is $1/2^{k-2}$.

III. CODE CONCATENATION

A. Serial concatenated convolutional code

Serial concatenated convolutional code, as shown in Fig. 2, is derived by combining an outer code $C_o$ and an inner code $C_i$, through an interleaver with length $N$. The outer code has rate $R_o=h/p$ and the inner code has rate $R_i=p/h$, the resulting SCCC, $C$, has rate $R=R_o=R_i=h/n$.

Fig.2. Serial concatenated convolutional code.

For aseral concatenated convolutional code, the conditional weight enumerating function (CWEF), $A^{C_i}(w, X)$ which enumerates the weights of all codewords associated with particular information weights is expressed as [17]

$$A^{C_i}(w, X) = \sum_{z=0}^{N} A^{C_i}(z, X) \times A^{C_i}(z, X)$$

(1)

where $A^{C_i}(z, X)$ is the number of codewords of the outer code, $C_o$ that have weight $z$ associated with an information word of weight $w$, and $A^{C_i}(z, X)$ is the CWEF of the inner code $C_i$.

B. Parallel concatenated convolutional code

Parallel concatenated convolutional code (PCCC), that is shown in Fig. 3, consists of two codes, $C_1$ with rate $R_1=h/n_1$, and $C_2$ with rate $R_2=h/n_2$ joined by an interleaver with length $N$. The output code word length $n=n_1+n_2$ and the overall PCCC rate is $R_o=h/n$.

Fig.3. Parallel concatenated convolutional code.

CWEF of a parallel concatenated convolutional code is expressed as [18][19]

$$A^{C_i}(w, X) = A^{C_i}(w, X) \times A^{C_i}(w, X)$$

(2)

where $A^{C_i}(w, X)$ and $A^{C_i}(w, X)$ are the CWEFs of the two constituent codes $C_1$ and $C_2$ respectively.

IV. HIGH POWER AMPLIFIER

In satellite communication systems, because the large distances between transmitter and receiver, high efficiency power amplifiers are needed. There are two major types of power amplifiers, travelling wave tube amplifiers (TWTA) and solid state power amplifiers (SSPA). At high power levels, TWTA's offer the best performance in terms of size, cost and efficiency, but lag behind SSPAs in linearity. The non-linear behavior of TWTA is usually described according to Saleh model [22] using amplitude/amplitude (AM/AM), and amplitude/phase (AM/PM) conversions that are illustrated in Fig. 4. One method to avoid the nonlinear effect of the HPA is to utilize a linearization technique which generates a nonlinear inverse function to that introduced by the HPA. The most widely used linearization technique is the predistortion linearization [23][24]. Another simple method to compensate for the amplifier nonlinearity is to back-off the operating point of the HPA from saturation to the linear region. The output back-off (OBO) of the HPA is defined as [25]

$$OBO = 10 \log \frac{P_{out}}{P_{sat}} \text{ dB}$$

(3)
V. CONSTRUCTION OF THE PROPOSED LOW RATE CONCATENATED CODING SCHEME

The super-orthogonal parallel-serial concatenated coding scheme presented in this study combines the principles of low rate coding with that of both parallel and serial concatenation. Block diagram of the coding scheme is illustrated in Fig. 5. It constructed out of two serially concatenated codes, each of them is a serial concatenation of a classical convolutional code and a super-orthogonal convolutional code. The two serially concatenated codes are then further concatenated in parallel. The two outer codes, $C_{p \parallel}$ and $C_{o}$ of the two serially concatenated codes, $C_{p \parallel}$ and $C_{o}$ are two identical rate 1/1 convolutional codes whose generator matrix is 

$$G(D) = [1 \ (1+D) \ (1+D)] \ (4)$$

and whose encoder block diagram is shown in Fig. 6. The rate 1/1 convolutional encoder can be obtained from the rate 1/2 recursive systematic convolutional encoder of Fig. 7 by not sending the systematic bit of the encoder. The two inner codes, $C_{i1}$ and $C_{i2}$ are two identical super-orthogonal convolutional codes that described in section II. In the proposed concatenated coding scheme, we have $2^{k-1}$ output bits from $C_{p \parallel}$ and $2^{k-2}$ output bits from $C_{o}$ so, the overall SOPSCCC rate is $R=1/(2 \times 2^{k-2})$.

VI. PERFORMANCE ANALYSIS

The proposed coding scheme is originally a parallel concatenation of two convolutional codes, $C_{p \parallel}$ and $C_{o}$. For a parallel concatenated convolutional code, the bit error

where $P_{sat}$ is the saturation power of the HPA, and $P_{av}$ is the average output power of the transmitted signal. For large back-off values, the nonlinear distortion of the HPA can be greatly reduced but, on the other hand, the power efficiency of the amplifier decreases, which leads to an unacceptable performance. So, a trade-off must be made between minimizing the nonlinear effects and maximizing the power efficiency of the HPA.
of all nonzero codewords, that is, a codeword weight enumerating function (WEF). The modified state diagram of the outer rate 1/1 convolutional encoder of Fig. 6 is shown in Fig. 8.

![Modified state diagram of the outer rate 1/1 convolutional encoder.](image)

Fig. 8. Modified state diagram of the outer rate 1/1 convolutional encoder.

In the modified state diagram of Fig. 8, the power of Z in a branch is the weight of the n encoded bits on that branch.

There are two definitions related to the modified state diagram and are used in computing its transfer function. These definitions are:

- **Forward path**: Forward path is the path connecting the initial state to the final state and does not go through any state twice. The path gain is the product of the branch gains along the path. We will denote the gain of the forward path as $F_i$.

- **Cycle**: A cycle is a closed path starting at any state and returning to the same state without going through any other state twice. The gain of the cycle is denoted as $C_i$.

A set of cycles is non-touching if no state belongs to more than one cycle in the set. Let $\{i\}$ be the set of all cycles in the modified state diagram, $\{i, j\}$ be the set of all pairs of non-touching cycles, $\{i', j', l\}$ be the set of all triples of non-touching cycles, and so on. A quantity $\Delta$ is then defined as [26]

$$\Delta = 1 - \sum_i C_i + \sum_{i < j} C_i C_j - \sum_{i < j < l} C_i C_j C_l + \ldots$$  \hspace{1cm} (6)

where $C_i$ is the sum of the cycle gains, $\sum_{i < j} C_i C_j$ is the product of the cycle gains of two non-touching cycles summed over all pairs of non-touching cycles, $\sum_{i < j < l} C_i C_j C_l$ is the product of the cycle gains of three non-touching cycles summed over all triples of non-touching cycles, and so on. Finally, $\Delta$ is a quantity defined exactly like $\Delta$ but only for the portion of the graph not touching the $i$th forward path. Transfer function, $A(Z)$ of the modified state diagram can then be calculated as [27]

$$A(Z) = \frac{\sum F_i \Delta_i}{\Delta}$$  \hspace{1cm} (7)

If the modified state diagram is augmented by labeling each branch corresponding to the input information block with $W^w$, where the power of $W$ is the input information bits in that branch, and labeling each branch with $L$, then, the codeword input-output weight enumerating function (IOWEF) of the encoder, which is the transfer function of its augmented modified state diagram, is expressed as [26]

$$A(W, Z, L) = \sum_{w, z, l} A_{w, z, l} W^w Z^z L^l$$  \hspace{1cm} (8)

where $A_{w, z, l}$ denotes the number of codewords with weight $z$, whose associated information weight is $w$, and whose length is $l$ branches. The augmented modified state diagram for the outer encoder is shown in Fig. 9.

![Augmented modified state diagram of the outer rate 1/1 convolutional encoder.](image)

Fig. 9. Augmented modified state diagram of the outer rate 1/1 convolutional encoder.

In the augmented modified state diagram of Fig. 9:

- There are three cycles:
  - Cycle 1: $S_1S_2S_0S_1$ ($C_1 = WZ^L$)
  - Cycle 2: $S_0S_1S_1$ ($C_2 = WLZ^L$)
  - Cycle 3: $S_1S_0$ ($C_3 = WL$)

- There is one pair of non-touching cycles:
  - Cycle pair 1: Cycle 2, Cycle 3 ($C_4C_5 = W^2ZL^L$)

- There are no other sets of non-touching cycles. Therefore, according to (6), $\Delta$ can be expressed as

$$\Delta = 1 - (Z^L + WL + WL W^{2L} + W^2Z^L) \equiv 1 - W^2Z^L(W + L + W^L + W^2L)$$  \hspace{1cm} (9)

After computing $\Delta$ we need to calculate the quantity $\Delta_i$. In the augmented modified state diagram of Fig. 9 there are two forward paths, the paths and their gains are:

- Forward path 1: $S_0S_1S_0S_1$ ($F_1 = WZ^L$)
- Forward path 2: $S_0S_1S_0S_2$ ($F_2 = WZ^L$)

The subgraph not touching the forward path 1 is the single branch around state $S_1$ which is shown in Fig. 10.

![Subgraph of Fig. 9 not touching the forward path 1.](image)

Fig. 10. Subgraph of Fig. 9 not touching the forward path 1.

For the sub-graph illustrated in Fig. 10 we can find $\Delta_i$ as

$$\Delta_i = 1 - WL$$  \hspace{1cm} (11)

Forward path 2 touching all states in the graph so, there are no subgraphs not touching the forward path 2 then,

$$\Delta_2 = 1$$  \hspace{1cm} (12)

Now, we can find $F_i \Delta_i$ for the augmented modified state diagram as

$$F_i \Delta_i = F_1 \Delta_1 + F_2 \Delta_2 = W^2Z^L + WZ^LW^2Z^L = WZ^L + W^2Z^L$$  \hspace{1cm} (13)

Finally, substituting from (9) and (13) into (7), the IOWEF $A(W, Z, L)$ of the outer code can be calculated as...
\[
A^{C_{w2}}(W, z, L) = \frac{W^2ZL^4 + W^2ZL^8 + W^2ZL^4}{1 - ZL^4 \cdot (1 + L + \cdots + L^{w-2}) + W^2ZL^8}
\]  

CWEF of the encoder, which enumerates the weights of all information block generating a particular output weight \(z\), is expressed as \[26\]
\[
A(W, z) = \sum A_{n,z} W^n
\]  

where \(A_{n,z}\) is the number of codewords having output weight \(z\) generated by information words of weight \(w\).

From (14), and following the procedures illustrated in [26], CWEF, \(A^{C_{w1}}(W, z)\) of the outer constituent code \(C_{w1}\), can be calculated as:
\[
\begin{align*}
A^{C_{w1}}(W, 2) & = 7W^3 + 5W^4 + 3W^5 + W^6 \\
A^{C_{w1}}(W, 3) & = 0 \\
A^{C_{w1}}(W, 4) & = 6W^3 + 13W^4 + 16W^5 + 10W^6 + 14W^7 + 7W^8 \\
A^{C_{w1}}(W, 5) & = 0 \\
A^{C_{w1}}(W, 6) & = 3W^2 + 7W^3 + 3W^4 + 12W^5 + 12W^6 + W^7 + W^9 \\
A^{C_{w1}}(W, 7) & = 0 \\
A^{C_{w1}}(W, 8) & = 3W^4 + 2W^5(16)
\end{align*}
\]

From the above equation, we can calculate the parameters \(A^{C_{w1}}\), which is the number of codewords of the outer code having output weight \(z\) generated by information words of weight \(w\). For example, there are 7 codewords that have input weight \(w=3\) and output weight \(z=2\), 5 codewords have input weight \(w=4\) and output weight \(z=2\), and so on.

**B. Calculating CWEF of the inner code \(C_{w1}\)**

For a super-orthogonal convolutional code with \(k\)-stage shift register, the input-output weight enumerating function (IOWEF), \(A(Z, X, L)\), which enumerates the number of codewords with weight \(x\) generated from information word with weight \(z\) and whose length is \(l\) branches, is expressed as [28]
\[
A(Z, X, L) = \frac{Z^L}{1 - ZL(1 + L + \cdots + L^{k-2}) - XZL^L}
\]  

where \(Z = (X^2)^{k-3}\).

In this study, we investigated the performance of the SOPSCCC for an inner super-orthogonal convolutional code with memory size \(k = 4, 5, 6, 7, 8\) which results in an overall SOPSCCC rate \(R = 1/8, 1/16, 1/32, 1/64, 1/128\). For \(k = 4\), equation (17) can be written as
\[
A(Z, X, L) = \frac{X^{12} Z L^4}{1 - X^2 Z L - X^2 Z L^2 - X^4 Z L^4}
\]  

Using (18) and following the procedures described in [26] we can calculate the CWEF, \(A^{C_{w1}}(z, X)\) of the inner super-orthogonal convolutional code for \(k = 4\) which is get as:
\[
\begin{align*}
A^{C_{w1}}(1, X) & = 7X^{12} \\
A^{C_{w1}}(2, X) & = 6X^{14} + 9X^{16} + X^{24} \\
A^{C_{w1}}(3, X) & = 5X^{16} + 8X^{18} + 4X^{20} + 8X^{20} + 4X^{26} + 2X^{28} \\
A^{C_{w1}}(4, X) & = 4X^{18} + 15X^{20} + 12X^{22} + 6X^{24} \\
A^{C_{w1}}(5, X) & = 12X^{20} + 12X^{22} + 6X^{24} \\
A^{C_{w1}}(6, X) & = 2X^{22} + 10X^{24} \\
A^{C_{w1}}(7, X) & = X^{24}
\end{align*}
\]

CWEFs of \(C_{w1}\) for the remaining values of \(k\) can then be calculated. For each of the calculated CWEFs, substituting into (1) by the CWEF of the inner code, and by the parameters \(A^{C_{w1}}\), calculated from (16) for the outer code \(C_{w2}\), we can calculate the CWEF, \(A^{C_{w2}}(w, X)\) of the serially concatenated code \(C_{w2}\).

As the overall proposed coding scheme is a parallel concatenation of the two codes, \(C_{w1}\) and \(C_{w2}\), the CWEF of the SOPSCCC can be expressed using (2) as
\[
A^{SO}(w, X) = \frac{A^{C_{w1}}(w, X) \times A^{C_{w2}}(w, X)}{N_w}
\]  

Since the two outer codes \(C_{w1}\) and \(C_{w2}\) are identical and the two inner codes \(C_{w1}\) and \(C_{w2}\) are also identical, the two serially concatenated codes \(C_{w1}\) and \(C_{w2}\) are identical and then their CWEFs are the same, so, equation (20) can be rewritten as
\[
A^{SO}(w, X) = \frac{[A^{C_{w1}}(w, X)]^2}{N_w}
\]  

Substituting into the above equation by the calculated CWEFs of the code \(C_{w1}\) for each value of \(k\) we can calculate the CWEF of the SOPSCCC at rates \(R = 1/8, 1/16, 1/32, 1/64, \) and \(1/128\).

**VII. PERFORMANCE RESULTS OF THE SOPSCCC**

Bit error probability performance of the proposed low rate concatenated coding scheme is evaluated for rates \(R = 1/8, 1/16, 1/32, 1/64, \) and \(1/128\) at interleaver lengths \(N_i = N_{ij} = N = 200\) and results are illustrated in Fig.10. From performance figure we can see that increasing the memory size, \(k\) of the inner super-orthogonal convolutional code and hence, decreasing the rate of the SOPSCCC improves the performance of the code. At very low rates, the performance improvement is slightly increases for further decreasing in rate. On the other hand, increasing the memory size of the encoder results in increased decoding complexity. So, trade-off must be made between improving the performance and decreasing the complexity of the system.

In Fig. 11, the performance of the proposed code is compared with both conventional super-orthogonal turbo code (SOTC) [10], and repeat punctured super-orthogonal turbo code (RPSOTC) [14] at interleaver length \(N = 200\) and rate \(R = 1/15\). From results we can see that, the proposed coding scheme has superior performance than both codes. Also, performance of the concatenated coding scheme is studied in the presence of nonlinear high power amplifier, results for the bit error probability performance at OBO=5 and OBO=7 are illustrated in Fig. 12. From results we can see that, increasing the OBO value enhances the bit error probability performance of the system as it minimizes the nonlinear effects of the HPA. On the other hand, increasing the OBO values decreases the power efficiency of the HPA so, again, trade-off is taken between improving the system performance and increasing the power efficiency of the HPA.
VIII. CONCLUSIONS

A low rate concatenated convolutional coding scheme suitable for spread spectrum multiple access applications is investigated. The coding scheme is a parallel-serial concatenation of conventional convolutional codes and super-orthogonal convolutional codes. performance of the new coding scheme is studied in AWGN channel and results proved that, the proposed coding scheme has better performance than that of the existing super-orthogonal concatenated convolutional codes specially at high SNRs. Performance of the proposed concatenated coding scheme is also studied in the presence of nonlinear high power amplifier which is an essential component in satellite communications. The performance is studied for two OBO values 5 and 7. Smaller OBO values results in larger output power which means higher power efficiency of the amplifier but, on the other hand, this leads to more signal distortion resulting in more required signal-to-noise ratio for a given bit error rate.

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AUTHOR’S PROFILE

Prof. Hala M. A. Mansour
Electronics & Communications Deptt, Shoubra Faculty of Engineering, Benha University, Egypt.

As. Prof. Labib Francis Gergis
Mistr Academy of Engineering and Technology, Mansoura, Egypt.

Dr. Mostafa A. R. Eltokhy
Electronics Technology Deptt, Industrial education College, Helwan University, Egypt.

Eng. Hoda Z. Said
Ass. Lecturer, Electronics & Communications, Workers University, Egypt.