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Dynamic Traffic Management in Railway Traffic Networks

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***Abstract* — Current practice in the operational-level management of railway traffic networks is mostly based on predefined rules and on the ability of traffic controllers and train dispatchers to detect and avoid conflicting situations. However, train operations are affected by many varying factors, such as driver behavior, passenger volumes, weather conditions, etc. These factors are results in delays. Delays in railway traffic networks include initial, primary, consecutive, knock- on delays. These delays can be partly absorbed by a stable and robust timetable. Timetable stability is rapidly gaining attention due to the increasingly saturated European railway infrastructure, where a slightly delayed train may cause a domino effect of consecutive delays over the entire network. The essential structure of railway traffic operating under a periodic railway timetable can be modeled as a linear system in max-plus algebra. Max-plus-linear systems can be characterized as discrete event systems in which only synchronization and no concurrency or choice occurs. In this paper we present a controlled railway system model using the switching max-plus-linear system. If the model is affine in the controls, the optimization problem can be recast as a mixed-integer linear programming problem.**

***Keywords* — Delay, Linear system, Max- plus algebra, Optimization.**

I. INTRODUCTION

A railway traffic network consists of many interdependencies due to timetable constraints, passenger connections, infrastructure constraints, logistic constraints such as rolling stock circulations, etc. Such a system can be modeled effectively using a timed event graph, which can also be expressed as a linear system in max-plus algebra[1][2]. The graph structure of such a model allows fast calculations of delay propagation, making the model suitable for online use in decision support systems for dispatchers or in dynamic passenger information systems.

However, train operations are affected by many varying factors, such as driver behavior, passenger volumes, weather conditions, etc. The need for fast running time prediction algorithms on the one hand and the complexity of railway systems on the other hand leads to a trade-off between fast online train traffic prediction and a detailed calculation of the stochastic processes and interdependencies resulting from trains sharing the same railway infrastructure.

Note that in the railway context, synchronization[1] means that some trains should give pre-defined connections to other trains, and a fixed routing means that the order of departure is fixed. However, in the case of large delays, it is sometimes better — from a global performance viewpoint — to break a connection or to reschedule the order of trains, and to let a train depart anyway. In this way we prevent an accumulation of delays in the network. In this paper we will model a controlled railway system using the switching max-plus-linear system. In this description we use a number of MPL models, each model corresponds to a specific mode, describing the network by a different set of connection and order constraints. We control the system by switching between different modes, allowing us to break train connections and to change the order of trains. In this paper we define a control algorithm to optimize the performance of the network, and we show that the resulting optimization problem can be solved as a mixed integer problem or a mixed integer linear programming problem.

II. MAX-PLUS LINEAR SYSTEMS

A. Max- plus Algebra

In this section we give the basic definition of the max-plus algebra and we present some results on a class of max-plus functions.

Define $\varepsilon = -\infty$ and $R\varepsilon = R \cup \{\varepsilon\}$. The maxplus-algebraic addition (⊕) and multiplication (⊗) are defined as follows:

$$x \oplus y = \max(x, y)$$

$$x \otimes y = x + y$$

for numbers $x, y \oplus R_\varepsilon$, and

$$[A \oplus B]_{ij} = a_{ij} \oplus b_{ij} = \max(a_{ij}, b_{ij}) \quad \ldots\ldots\ldots\ldots\ldots(1)$$

$$[A \otimes C]_{ij} = \bigoplus_{k=1}^{n} a_{ik} \otimes c_{kj} = \max_{k=1,2,3\ldots n} (a_{ik} + c_{kj}) \quad \ldots\ldots\ldots\ldots(2)$$

for matrices $A,B \oplus R_{\in}^{mxn}$ and $C \oplus R_{\in}^{nxp}$.

B. Max-plus-linear systems

A discrete event system[3][6] in which there is synchronization but no concurrency can be described by a model of the form:

$$x(k) = A(k) \otimes x(k-1) \oplus B(k) \otimes u(k) \quad \ldots\ldots(3)$$

Systems that can be described by this model will be called max-plus-linear systems. The index k is called the event counter. For discrete event systems the state x(k) typically contains the time instants at which the internal events occur for the kth time, the input u(k) contains the time instants at which the input events occur for the kth time, and the output y(k) contains the time instants at which the output events occur for the kth time.

C. Switching max-plus-linear systems

A discrete event system that can switch between different modes of operation. In each different mode ℓ = 1, . . . , nm, the system is described by a max-plus-linear state equation:

$$x(k) = A^{(l)}(k) \otimes x(k-1) \oplus B^{(l)}(k) \otimes u(k) \quad ……(4)$$

in which the matrices A(ℓ), B(ℓ) are the system matrices for the ℓ-th mode. The switching allows us to change the structure of the system, to break synchronization and to change the order of events.

The moments of switching are determined by a switching mechanism. We define the switching variable z(k), which may depend on the previous state x(k − 1), previous mode ℓ(k − 1), the input variable u(k) and an additional control variable v(k):

$$z(k) = \phi(x(k-1), \ell(k-1), u(k), v(k)) \in R^{nz} \quad …..(5)$$

III. DELAYS IN RAILWAY TRAFFIC NETWORKS

Timetable stability[4] is rapidly gaining attention due to the increasingly saturated European railway infrastructure, where a slightly delayed train may cause a domino effect of consecutive delays over the entire network. There are two types of delays[5]: primary (or original) and secondary (or knock-on) delays. A primary delay is caused by a process that exceeds its scheduled process time. A secondary delay is caused by interaction with another train, such as a route conflict or a secured transfer. The delay of an event i scheduled in period k is denoted as:

$$Z_i(k) := x_i(k) - d_i(k) \quad ……………(6)$$

In the max-plus model delays are initiated from two sources:

Initial delays: Delays in the initial period,

$$ZI := \{(i, 0, z_i(0)) | z_i(0) = (x_0)_i - d_i(0) > 0\} ……(7)$$

Primary delays: Delays caused by exceeding scheduled process times,

$$Z_P := \{(i, k, z_i(k)) | z_i(k) = d_j(k - \mu_{ij}) + a_{ij}(k) - d_i(k) > 0, k \geq 1\} \quad ………(8)$$

Initially and primary delayed trains may cause secondary delays, which are computed from the max-plus system equations. Moreover, secondary delays may further generate more secondary delays. We distinguish between two types of secondary delays:

Consecutive delays: Existing, possibly partially recovered, train delays,

$$Z_C := \{(i, k, z_i(k)) | z_i(k) > 0, z_j(k - \mu_{ij}) > 0, a_{ij}(k) \text{ running or dwell time}, k \geq 1\} \quad …………(9)$$

Knock-on delays: Delays caused by interaction with other trains,

$$Z_K := \{(i, k, z_i(k)) | z_i(k) > 0, k \geq 1\} \setminus (Z_P \cup Z_C) \quad ….(10)$$

A delay may originate from different sources. For instance, a consecutive delay may increase by an additional primary delay. In this case, the delay is partially primary and partially consecutive. Also a primary delay or consecutive delay may be superseded by a larger knock-on delay from another train. The dynamic system behavior can be analyzed by simulating the max-plus linear systems.

IV. THE RAILWAY CONTROL PROBLEM

A. Model

Consider a periodic railway operations system that follows a schedule with period T. In nominal operation mode, we assume that all the trains follow a pre-scheduled route, with a fixed train order and predefined connections. If for any of the reasons mentioned before delays are introduced in the network, it might be advantageous to change the train order so as to minimize delays. In this case we will operate in a perturbed mode[7] with an associated new schedule.

Consider a network with n trains and define the vectors x(k) = [d1(k), . . . , dn(k), a1(k), . . . , an(k)] $\in R^{2n}$ and r(k) $\in R^{2n}$. By defining ε = −∞, R_ϵ = R U {ε}, and appropriate matrices Am $\in R^{2nX\,2n}$, m = m1, . . . ,m2 with m1 = mini,j(δij) and m2 = maxi,j(δij),

$$x_i(k) = \max\left(r_i(k), \max_{j,m}\left(x_j(k-m) + [A_m]_{ij}\right)\right) \quad …(11)$$

where [Am]ij is the (i, j)th entry of Am.

In max-plus notation (11) becomes

$$x_i(k) = r_i(k) \oplus \bigoplus_{j=1}^{2n} \bigoplus_{m=m_1}^{m_2} x_j(k-m) \otimes [A_m]_{ij} \quad ….(12)$$

and in matrix-notation we obtain

$$x(k) = \bigoplus_{m=m_1}^{m_2} A_m \otimes x(k-m) \oplus r(k) \quad ………(13)$$

A system operating in a perturbed mode ℓ(k) can be described as

$$x(k) = \bigoplus_{m=m_1}^{m_2} A_m(\ell(k)) \otimes x(k-m) \oplus r(k) \quad ……(14)$$

where mode ℓ(k) = 0 corresponds to the nominal timetable.

B. Timing Aspects

Discrete event systems are different from conventional time-driven systems in the sense that the event counter k is not directly related to a specific time. Let t be a given time instant and let k be such that x(k − m2), . . . , x(k − 1) are completely known, i.e., xi(k − j) ≤ t ∀ i, ∀ j ∈ {1, 2, . . . ,m2}. So at time instant t in cycle k, some of the components of x(k) may already be known while others may still lie in the future.

Due to the fact that time does not explicitly enter the max-plus recurrence in (14) it could happen that if an event is delayed by a precedence constraint and if due to a rescheduling action this hindering constraint in removed, then the delayed event could, in theory, be rescheduled in the past. To avoid this issue it should be considered that a departure event cannot be rescheduled before the current decision time t.

C. Control problem

Let Np be the prediction horizon and define the set U(k + j|t) ⊂ $\{0,1\}^{nu}$ for j = 0, 1, ...,Np with nu the dimension of the control space, as the set of possible future control actions for the k + jth cycle at time instant t.

To select the optimal set of possible future control actions, we define the following optimal control problem at time instant t:

$$\begin{cases} \min\limits_{u(k|t),u(k+1|t),\ldots,u(k+N_{\mathrm{p}}|t)} J(k|t) \\ \hat{x}(k+j|t) = \bigoplus\limits_{m=m_1}^{m_2} \hat{A}_m(\ell(k+j|t)) \otimes \\ \qquad\qquad \hat{x}(k+j-m|t) \oplus r(k+j) \\ \hat{x}(k-i) = x(k-i) \text{ for } i = 1,\ldots,m_2 \\ u(k+j|t) \in \mathcal{U}(k+j|t) \end{cases} \quad \text{.....(15)}$$

where the performance index J(k|t) is given by

$$J(k|t) = \sum_{j=0}^{N_{\mathrm{p}}} \left(\sum_{i=1}^{2n} \sigma_i \, \hat{e}_i(k+j|t) + \sum_{l=1}^{n_u} \rho_l \, u_l(k+j|t) \right). \quad \text{.....(16)}$$

The first term of (16) is related to the sum of all predicted delays, and the second term denotes the penalty for all switched train orders during cycle k + j. we now have all elements to solve the optimal control problem (15).

V. RECASTING OPTIMIZATION PROBLEM INTO MIXED-INTEGER LINEAR PROGRAMMING PROBLEM

By restricting the change in departure order of two successive trains, the railway network model can be written in affine form with respect to the controls, and the optimization problem in (15) can be recast into an MILP problem. Affinity with respect of the controls means that the system matrix can be written as:

$$\hat{A}_m(\ell(k)) = \hat{A}_{m,0} + \sum_{v=1}^{n_u} \hat{A}_{m,v}(k)\, u_v(k), \quad \text{.....(17)}$$

Equation (17) can be written as:

$$\hat{A}_m(\ell(k)) = \hat{A}_{m,0} + \sum_{\tau=1}^{n_t} \sum_{v=1}^{\sigma_\tau(k)} \hat{A}_{m,v}^{(\tau)}(k)\, u_v^{(\tau)}(k) \quad \text{.....(18)}$$

Now we show that the model predictive control problem (15) with Aˆm(ℓ(k)) given by (18) can be recast into an MILP problem. Assuming that in general m1 = 0 and m2 ≥ m1, we outline now the main ideas behind this transformation. For the sake of simplicity of notation we drop the notation $\hat{\cdot}$ and |t for a prediction from now on.

Define the vectors

$$\tilde{x}(k) = [x^{\mathrm{T}}(k), \ldots, x^{\mathrm{T}}(k+N_{\mathrm{p}})]^{\mathrm{T}},$$

$$\tilde{u}(k) = [u^{\mathrm{T}}(k), \ldots, u^{\mathrm{T}}(k+N_{\mathrm{p}})]^{\mathrm{T}},$$

$$\tilde{z}(k) = [x^{\mathrm{T}}(k-1), \ldots, x^{\mathrm{T}}(k-m_2)]^{\mathrm{T}},$$

$$\tilde{\ell}(k) = [\ell(k), \ldots, \ell(k+N_{\mathrm{p}})]^{\mathrm{T}},$$

$$\tilde{r}(k) = [r^{\mathrm{T}}(k), \ldots, r^{\mathrm{T}}(k+N_{\mathrm{p}})]^{\mathrm{T}}.$$

where ˜x(k) represents the partially known or completely unknown states and ˜z(k) represents the completely known states at cycle k. We can write

$$\tilde{x}(k) = \tilde{A}(\tilde{\ell}(k)) \otimes \tilde{x}(k) \oplus \tilde{B}(\tilde{\ell}(k)) \otimes \tilde{z}(k) \oplus \tilde{r}(k) \quad \text{......(19)}$$

Note that ˜ℓ(k) is a function of ˜u(k), which can be expressed as ˜ℓ(k) = ˜L(˜u(k)). The objective function J(k) is linear in ˜u(k) and ˜x(k), and can be written as:

$$J(k) = c_e^{\mathrm{T}}\, \tilde{x}(k) + c_u^{\mathrm{T}}\, \tilde{u}(k) \quad \text{...........(20)}$$

Equation (19) can be written as:

$$\tilde{x}_i(k) = \max\Big(\tilde{r}_i(k), \max_j(\tilde{x}_j(k) + [\tilde{A}(\tilde{\ell}(k))]_{ij}), \max_j(\tilde{z}_j(k) + [\tilde{B}(\tilde{\ell}(k))]_{ij})\Big), \quad \text{.......(21)}$$

which can be transformed into

$$\begin{cases} \tilde{x}_i(k) & \geq \tilde{r}_i(k), \\ \tilde{x}_i(k) & \geq \tilde{x}_j(k) + [\tilde{A}_0]_{ij} + \sum\limits_{v=1}^{n_u} [\tilde{A}_v]_{ij}\, \tilde{u}_v(k) \quad \forall j, \\ \tilde{x}_i(k) & \geq \tilde{z}_l(k) + [\tilde{B}_0]_{il} + \sum\limits_{v=1}^{n_u} [\tilde{B}_v]_{il}\, \tilde{u}_v(k) \quad \forall l. \end{cases} \quad \text{......(22)}$$

It is clear that all these constraints are linear in ˜x(k) and ˜u(k), and we end up with the linear inequality constraint:

$$A_c \begin{bmatrix} \tilde{x}(k) \\ \tilde{u}(k) \end{bmatrix} \leq b_c(k), \quad \text{......(23)}$$

So we have a linear objective function (20) that has to be minimized subject to the linear constraints (23) over real variables ˜x(k) and binary variables ˜u(k). Hence, we finally end up with an MILP.

VI. CONCLUSION

A railway traffic network consists of many interdependencies due to timetable constraints, passenger connections, infrastructure constraints, logistic constraints such as rolling stock circulations, etc. Such a system can be modeled effectively using a timed event graph, which can also be expressed as a linear system in max-plus algebra. In this paper we present a model predictive controller based on measurements of the actual train positions. The core of the model predictive control approach is the railway traffic model, for which a switching max-plus linear system is used. We also show how the control problem can be recast as a mixed-integer linear programming problem.

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