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Control of Rotary Flexible Joint by Two-surface Discrete Sliding Mode Control based on Approach Angle Reaching Law

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***Abstract* – In this paper a discrete time sliding mode controller with two surfaces is used to control rotary flexible joint and to reduce the vibration of the arm. Here, approach angle reaching law is used to take the trajectory to the surfaces. The advantage of using a two surface sliding mode control is that the system trajectory is brought to the first surface with maximum velocity thereby shortening the reaching phase. The scheme assures robustness, fast transients with reduced chattering when compared to single surface sliding mode.**

***Index Terms* – Approach angle, Discrete Sliding mode control, Rotary flexible joint, two surface sliding mode control.**

I. INTRODUCTION

Traditionally, robotic manipulators have been designed and built heavy and bulky for high structural stiffness; but it resulted in drawbacks such as high power consumption, low motion speed, high manufacturing cost etc. Modern industrial robots are designed with flexible joints. This has the advantage of increased payload capacity, cheaper construction and faster movement. However, the consequence of making mechanically flexible constructions resulted in new and complicated problems in modeling, identification and control. Moreover joint flexibility causes significant vibration at the end points. Research on the dynamic modeling and control of flexible robots has received increased attention in the last few decades. A first step towards designing an efficient control strategy for manipulators with flexible joints must be aimed at developing dynamic models that can characterize the flexibility of the joints accurately. The controller design that minimizes the effects of the flexible displacements in lightweight robots is highly demanded in many industrial and space applications that require accurate trajectory control. In control applications of robot, manipulators with flexible arms are targeted either to reach a target position or to follow a prescribed trajectory. In the first case to reach a target position, a short settling time is desired while a large robot arm displacement is planned in the second case to follow a prescribed trajectory. In both cases, strong control actions are applied to the robot arm, and, as a result, undesired behaviors could appear if vibrations induced in the robot arm are not considered. The control issue of the flexible joint is to design the controller so that link of robot can reach a desired position or track a prescribed trajectory precisely with minimum vibration to the link. In order to achieve these objectives, various methods using different technique have been proposed such as follow: linear quadratic regulation (LQR) control [1], adaptive output-feedback controller based on a backstepping design [2]–[4], nonlinear control based on integral manifold technique [5] control based on PD control [6], robust control [7], vibration control by PD fuzzy and neural network [8],[9] and so forth. In recent years, the sliding mode control methodology has been widely used for control design problem for a class of nonlinear systems. In 1990 GAO had presented the reaching law method, and extended the same to discrete time counter part [10],[11]. In the reaching law approach, switching function dynamics satisfying the reaching condition is chosen, and then the control input is derived from the inverse dynamics. There were some shortages for the Gao's reaching law namely the system states do not guarantee convergence to the origin and sometimes the limit loop could be formed in vicinity of origin. In order to avoid the above drawbacks, a new reaching law

$$s(k+1) = s(k) - \varepsilon T \| x(k) \| \operatorname{sgn} s(k) \quad \text{………..(1)}$$

was tried near the vicinity of the origin, which would allow the trajectory to converge to the origin [10],[13].This method also had some shortages regarding the determination of switching time and the input will be blown up when the two reaching laws are switched.

Thus new reaching laws were introduced [10], [13] some of which are

$$s(k+1) = (1-qT)s(k) - \varepsilon T arctg \| x(k) \|$$

$$s(k+1) = (1-qT)s(k) - Ts^2(k) \operatorname{sgn} s(k)$$

$$s(k+1) = (1-qT)s(k) - Te^{-|s(k)|} s^2(k) \operatorname{sgn} s(k)$$

Each one of the reaching laws had its own merits and demerits. In all the reaching law methods, either the distance to the surface or to the origin is considered. Later the approach angle reaching law was introduced, to make the design of control input more comprehensive [10],[12]. This method had reduced the switching problem between the reaching laws. In Approach angle methodology the trajectory is forced on to the chosen switching surface by the reaching control law for which the reaching angle is generally small. The direct implication of this constraint is that reaching phase is increased. This in turn affects the robustness of the control system.

A two surface sliding mode was proposed inorder to guarantee convergence to the origin with very less chattering and minimum control. In two surface sliding mode control two surfaces were defined, the first surface is a virtual surface that aids the fast motion of the trajectory on to its final destination. The motion from this to the second one is then effected by the approach angle reaching law with small approach angle to avoid chattering [14]. This sliding mode motion then guarantees that the system state is bounded under the existence of time varying disturbance and uncertainty.

In this paper, control of Rotary Flexible joint by two surface discrete sliding mode control using approach angle reaching law is presented. Firstly a linear system is

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described in order to facilitate the development of an efficient control system. Secondly the system is decoupled to form two subsystems. Thirdly a two surface sliding mode controller is developed so that arm is taken to desired position with minimum vibration.

The paper is organized as follows. In section II, the approach angle reaching law is explained. In section III the mathematical model of Rotary Flexible Joint is presented. In section IV simulation results are given to compare the performance of approach angle based on two surface sliding mode controllers with that of single surface sliding mode controllers. In section V, conclusion is given.

II. Approach Angle Reaching Law

Consider a single input linear discrete-time system

$$x(k+1) = \boldsymbol{A}x(k) + \boldsymbol{B}u(k)$$

where the state vector $x(k) \in \boldsymbol{R}^n$, scalar $u(k)$ is control input, $rank(\boldsymbol{B}) = 1$.The index k indicates the k^{th} sample We assume $(\boldsymbol{A},\boldsymbol{B})$ is a controllable pair.

Choosing the switching function

$$s(k) = \boldsymbol{C}x(k)$$

Approach angle θ is the angle between sliding surface and the trajectory Fig.1

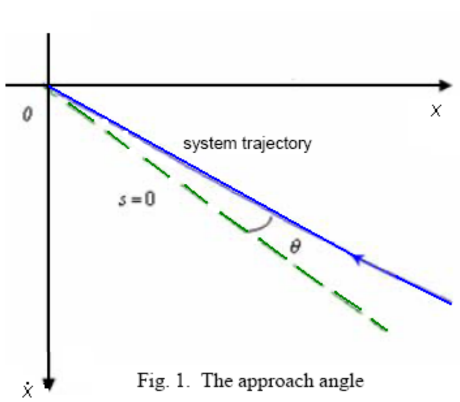

Fig. 1. The approach angle

Considering the approach angle [10] the reaching law is

$$s(k+1) = (1-qT)\,|\cos\theta\,|\,s(k) - \varepsilon Tarctg\,\|\,x\,\|\,\text{sgn}\,s(k)......(2a)$$

and

$$u(k) = (CB)^{-1}[-CAx(k) + (1-qT)\,|\cos\theta\,|\,s(k)$$
$$-\varepsilon Tarctg\,\|\,x\,\|\,\text{sgn}\,s(k)]......................................(2b)$$

In two surface sliding mode control a new surface is created so that the trajectory is brought to this surface by an appropriate approach angle for fast reaching. The trajectory is not allowed to slide on that surface but will be immediately taken at a suitable angle to the sliding surface [14]. The advantage of such a method is that minimum control effort. The strategy also guarantees faster transient response with considerable reduction in vibration of arm

III. Mathematical Modeling Of Rotary Flexible Joint (Quanser)

The Rotary flexible joint which is designed to emulate the flexible joint effects on a robot or space craft includes a SRV02 servomotor in the high gear ratio configuration, a body and a free beam attached to the body by two identical springs. The springs are mounted to an aluminum chassis which is driven by the servomotor. Rotating the base of the beam causes the entire beam to oscillate due to joint flexibility introduced by the springs. Different springs can be used to represent different flexibility effects. Also, a weight can be added to the beam. A Potentiometer on the servomotor generates an analog signal proportional to the angle of rotation. Depending on the make of RFJ, encoders may be provided to find the angular position of shaft and arm deflection.

A schematic representation of the system is shown in the Fig. 2. where θ denotes the motor angle and α denotes the angle of the flexible joint.

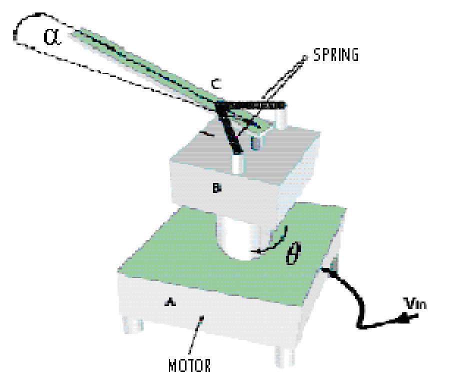

Fig.2 Schematic Diagram of Rotary Flexible Joint

Mathematical modeling is done using the Lagragian equation of motion, which makes use of the Kinetic energy and Potential energy of the system. It is assumed that inductance of the motor is negligible. Sensor dynamics is also neglected. Kinetic energy and potential energy of the system is given by

$$PE_{spring} = \frac{1}{2}K_{stiff}\alpha^2$$
$$KE_{hub} = \frac{1}{2}J_{hub}\dot{\theta}^2$$
$$KE_{load} = \frac{1}{2}J_{load}(\dot{\theta}+\dot{\alpha})^2$$
$$KE = KE_{hub} + KE_{load}$$
$$L = KE_{hub} + KE_{load} - PE_{spring}$$

Where PE_{spring} is the potential energy of the spring. KE_{hub} is the Kinetic energy of the motor ; KE_{load} is the Kinetic energy of the load. KE is the total Kinetic energy and L is the Lagrangian, which is the difference between the total Kinetic Energy and Total Potential energy. θ is the motor output angle and α is the measurement of the

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angular deflection of the arm. Lagrange equation of motion is given by

$$\frac{d}{dt}\frac{\partial L}{\partial \theta} - \frac{\partial}{\partial \dot{\theta}} = T$$

$$(J_{hub} + J_{load})\ \ddot{\theta} + J_{load}\ddot{\alpha} = T \text{ ....(3)}$$

$$\frac{d}{dt}\frac{\partial L}{\partial \alpha} - \frac{\partial}{\partial \dot{\alpha}} = 0$$

$$J_{load}(\ddot{\theta}) + J_{load}(\ddot{\alpha}) + K_{stiff}\alpha = 0 \text{ ....(4)}$$

For obtaining the state space representation of the Rotary Flexible Joint system substitute the equation of torque $T = \frac{K_g K_m V - K_g^2 K_m^2 \omega}{R_m}$ in equation 3 and define the states as $x_1 = \theta, x_2 = \dot{\theta}, x_3 = \alpha, x_4 = \dot{\alpha}$

The linear differential equation representing the states from equations 3 and 4 are as follows

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = \frac{K_m K_g V}{R_m J_{hub}} - \frac{K_m^2 K_g^2 x_2}{R_m J_{hub}} + \frac{K_{stiff} x_3}{J_{hub}}$$

$$\dot{x}_3 = x_4$$

$$\dot{x}_4 = \frac{K_m K_g V}{R_m J_{hub}} - \frac{K_m^2 K_g^2 x_2}{R_m J_{hub}} + \frac{K_{stiff}(J_{hub} + J_{load}) x_3}{J_{hub} J_{load}}$$

The state space representation is obtained as below

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -\frac{K_m^2 K_g^2}{R_m J_{hub}} & \frac{K_{stiff}}{J_{hub}} & 0 \\ 0 & 0 & 0 & 1 \\ 0 & \frac{K_m^2 K_g^2}{R_m J_{hub}} & \frac{K_{stiff}(J_{hub} + J_{load})}{J_{hub} J_{load}} & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{K_m K_g}{R_m J_{hub}} \\ 0 \\ -\frac{K_m K_g}{R_m J_{hub}} \end{bmatrix} V$$

Substituting the system parameters will give

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -53 & 766.6 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 53 & -1040 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 99 \\ 0 \\ -99 \end{bmatrix} V$$

Discretizing the system with zero-order hold and a sampling period of T=0.1sec gives

$$x(k+1) = \begin{bmatrix} 1 & 0.04237 & 0.5278 & 0.03304 \\ 0 & 0.5058 & -1.883 & 0.5278 \\ 0 & 0.03649 & -0.3404 & 0.03403 \\ 0 & -0.1302 & -7.42 & -0.3404 \end{bmatrix} x(k) + \begin{bmatrix} 0.1077 \\ 0.9231 \\ -0.06816 \\ 0.2431 \end{bmatrix} u(k)$$

The state space equation is decoupled into two different systems i) a system describing rotation ii) a system representing vibration. Hence here we will have two sliding surfaces, s_1 and s_2 as following

$$s_1(k) = \lambda_1 x_1(k) + x_2(k)$$

$$s_2(k) = \lambda_2 x_3(k) + x_4(k)$$

A and B matrix are split as follows

$$A\_new1 = \begin{bmatrix} 1 & 0.04237 \\ 0 & 0.5058 \end{bmatrix}$$

$$b\_new1 = \begin{bmatrix} 0.1077 \\ 0.9231 \end{bmatrix}$$

$$A\_new2 = \begin{bmatrix} -0.3404 & 0.03403 \\ -7.42 & -0.3404 \end{bmatrix}$$

$$b\_new2 = \begin{bmatrix} -0.06816 \\ 0.2431 \end{bmatrix}$$

Choose λ_1=10 and λ_2=12, obtain $u_1(k)$ and $u_2(k)$ by eqn.2b

IV. Simulation

Parameter	Description	Value	Unit
R_m	*Resistance of the armature*	*2.6*	*Ω*
J_{hub}	*Inertia of motor & gears*	*0.0021*	*Kgm^2*
J_{load}	*Load Inertia*	*0.0059*	*Kgm^2*
K_{stiff}	*Joint stiffness*	*1.61*	*Nm/rad*
K_m	*Mechanical Constant*	*0.00767*	*V/rad/s*
K_g	*gear ratio*	*70:1*	

Table.I. Numerical values for simulation

Both the simulations are carried out with the same initial state. i) A sliding mode controller was designed with single surface based on approach angle based reaching law, ii) a two surface sliding mode controller was designed with approach angle based reaching law. A comparison is done between the two methods. Virtual surface is not shown in figures. A step input of magnitude 1.04 (corresponding to 60º / 1.04 radians made by the motor angle) is given to show how better the motor angle is tracked.

The simulation results are as follows:

i) Single surface method

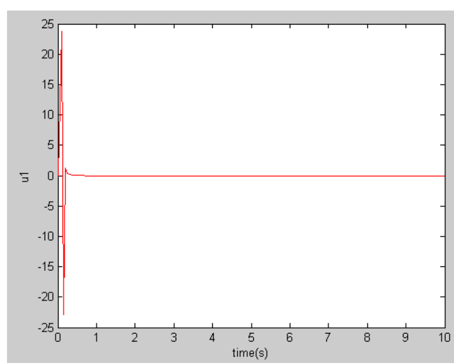

Fig.3. u1 vs time(s)

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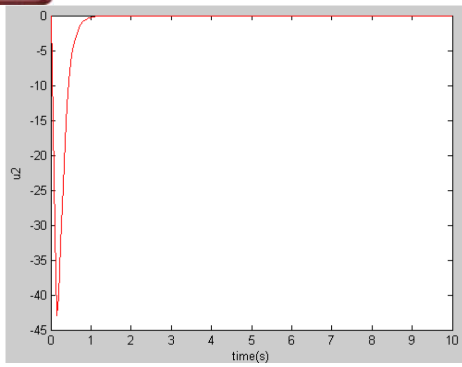

Fig.4.u2vs time(s)

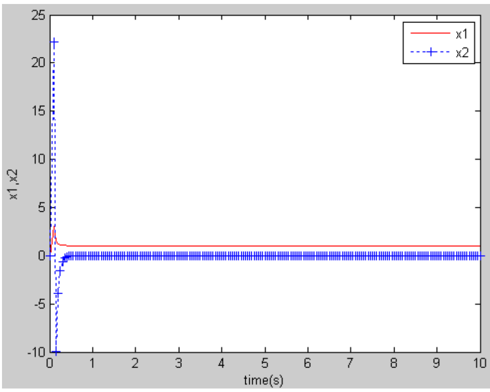

Fig.5. x1, x2 vs time(s)

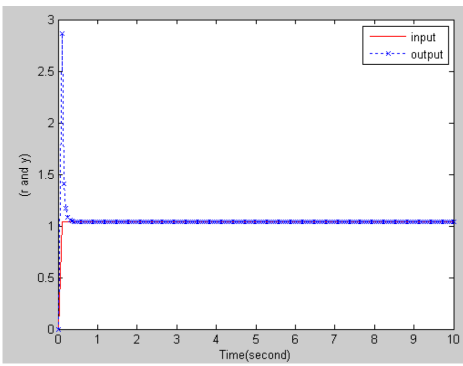

Fig.6. input(r), Output(y) vs time(s)

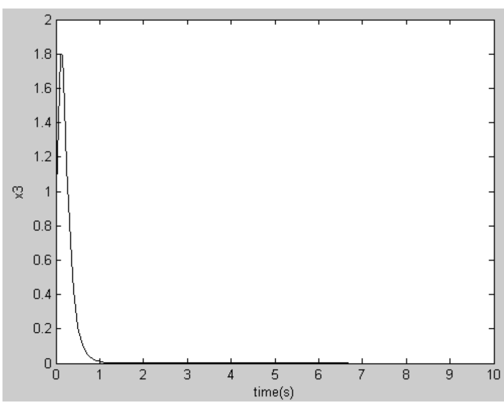

Fig.7. x3 vs time(s)

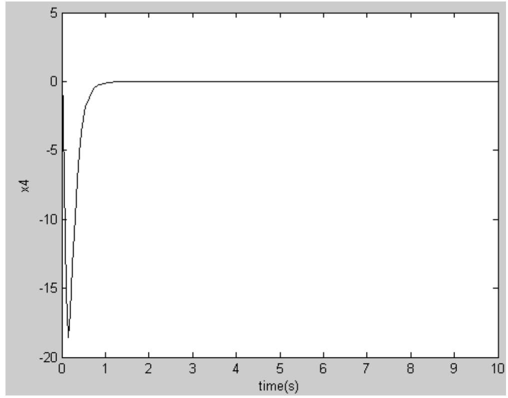

Fig.8. x4 vs time(s)

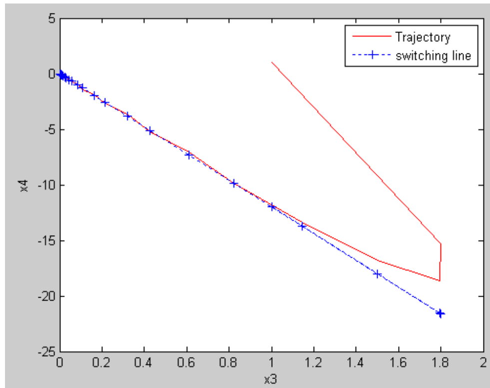

Fig.9. x3 vs x4

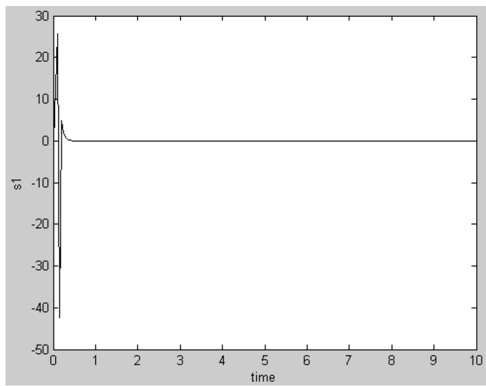

Fig.10. s1 vs time

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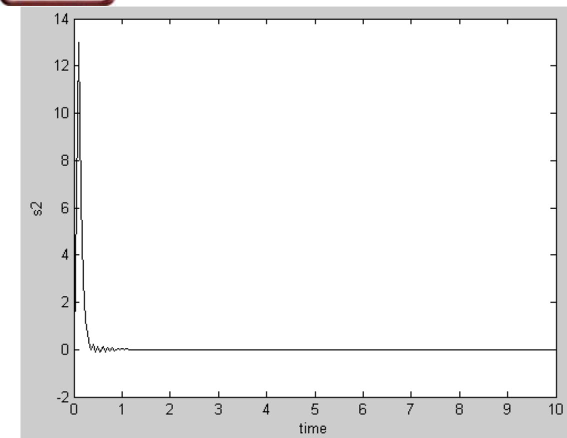

Fig.11. s2 vs time(s)

ii) Two surface method

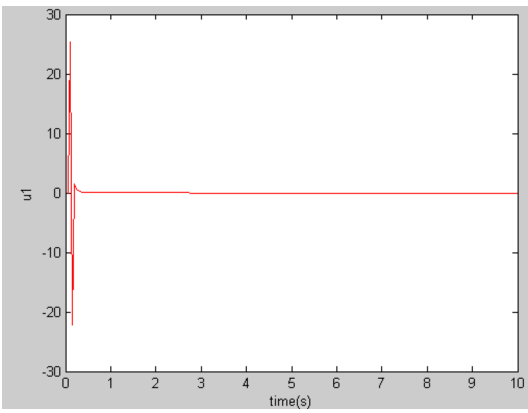

Fig.12. u1 vs time(s)

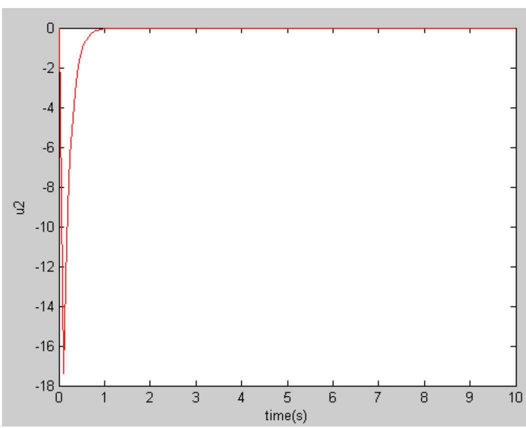

Fig.13. u2 vs time(s)

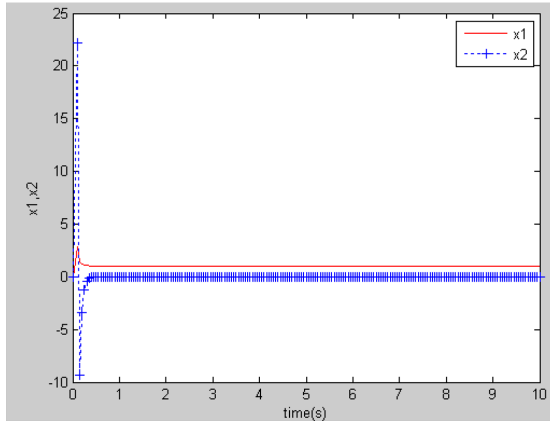

Fig14. x1, x2 vs time(s)

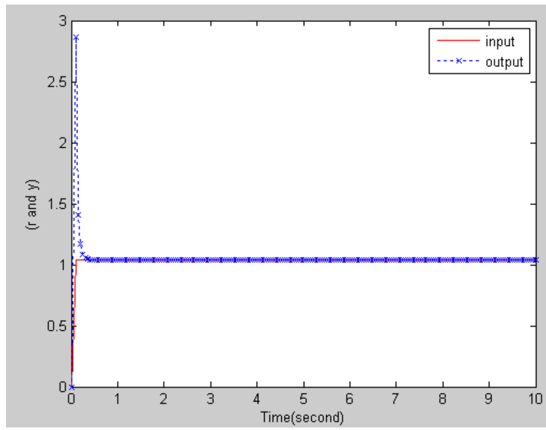

Fig.15. input(r), Output(y) vs time(s)

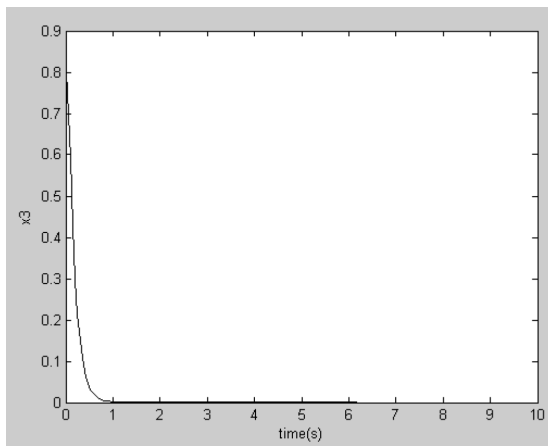

Fig.16. x3 vs time(s)

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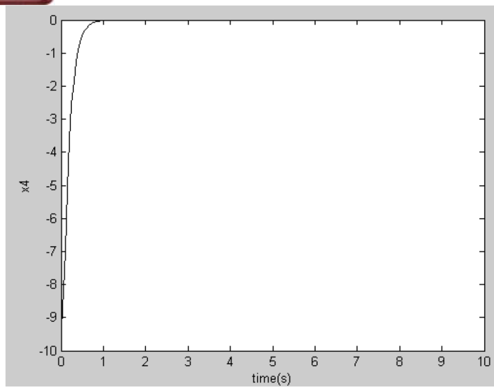

Fig.17. x4 vs time(s)

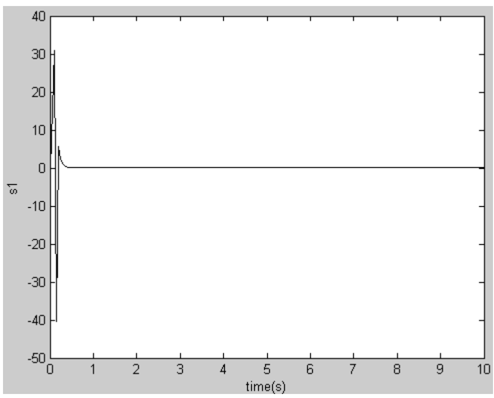

Fig.18. s1 vs time(s)

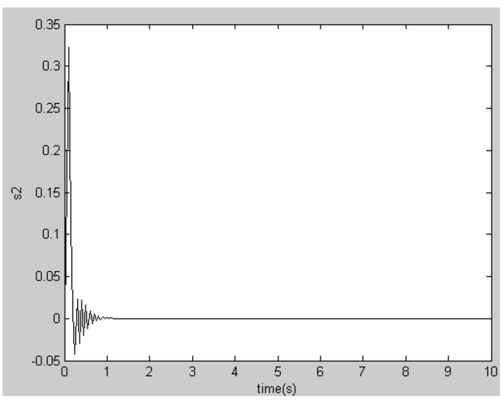

Fig.19. s2 vs time

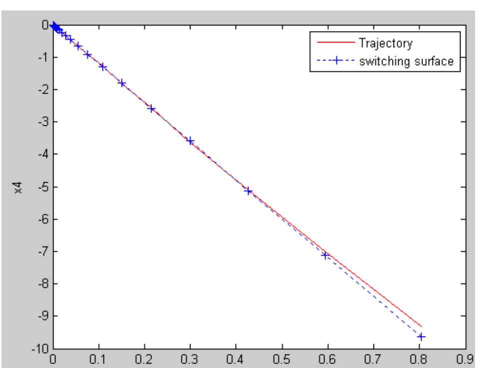

Fig.20. x3 vs x4

V. Conclusion

In this paper, a reaching law based two surface sliding mode controller was designed to control the rotary flexible joint and to reduce the vibration of the arm. Figures (1-11) shows the simulation results using approach angle reaching law with one surface. Figures (12-20) shows simulation results conducted assuming a virtual surface on which the trajectory is initially brought and then taken to the sliding surface by approach angle reaching law. On comparing the two simulations it is found that the two surface method guarantees faster transient response From fig. 4 and 13 it is understood that the two surface method assures minimum control effort and from fig. 9 and 20 it is concluded that the two surface method has comparatively less chattering .

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