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Output Feedback *H* Infinity Optimal Control of Macro-Micro-Flexible Link Manipulators

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Abstract **– In this paper, H_∞ optimal control of a 3-link macro-micro manipulator whose the first link is flexible is studied. It is supposed that the robot is subject to measurement noises and unknown coulomb and viscous frictions. The main contribution of the paper is to consider the flexible modes of the robot as sources of uncertainties while only the positions of the joints are measurable. Hence, by minimizetion of the effects of these sources of uncertainties on a regulated output, achieving a desired H_∞ performance is guaranteed for the robot. Simulation results for a 3-link macro-micro flexible link manipulator confirm the efficiency of the proposed approach.**

Keywords **– H_∞ Optimal Control, Flexible Link, Macro-Micro, Manipulator, Tracking.**

I. Introduction

Recent developments in various fields of science and technology have increased the applications of robotic systems in various categories such as tele-surgery, space mission, automation manufacturing systems, etc. Indeed, robotic systems are more reliable, safe, and robust compared with human operators, and especially are useful in doing tasks in hazardous environments where are dangerous for human operators [1]–[7].

In general, the robotic systems can be categorized in two classes, namely, mobile robots and manipulator robots. Mobile robots are a class of robots which are able to move in an environment in order to accomplish tasks. Unmanned ground vehicles [8], [9], unmanned underwater vehicles [10], [11], and unmanned air vehicles [12], [13] are various types of mobile robots. On the other hand, a manipulator is a set of jointed arms attached to a fixed surface with an end-effector at the bottom of the links [1], [2]. However, in many applications, a manipulator can be attached to a mobile platform which has the advantages of both mobile and manipulator robots. In other words, the manipulator can accomplish tasks while moving in an environment [14], [15].

In many practical applications, such as space missions, tele surgery, etc., in order to low mass and low dimension design, it is necessary to consider robotic structures with flexibility in the links [16]–[18]. Hence, research on control of manipulator robots with flexible links due to their complex models is worth to study. One of main problems in control of manipulator robots with flexible links is vibration of links which affects the tracking performance of the manipulator robots. There are a lot of techniques to damp the flexible modes of this class of robots such as output redefinition [19], [20], singular perturbation [21], [22], etc. One of popular techniques to cope with the mentioned problem is using robots with macro-micro structure. A macro-micro robot is consisting of a set of large and rigid links carrying out a small link with high performance. For instance, in [23]–[25], macro-micro structures were considered to cope with the problem of link flexibility in robots, and in [26], the output redefinition and singular perturbation techniques were employed for macro-micro manipulators, and a PD controller was employed to control the micro link.

In this paper, tracking control of a 3-link macro-micro manipulator whose the first link is flexible is studied. Hence, to attenuate the effects of measurement noises and coulomb and viscous frictions, the H_∞ controller is proposed. The main contribution of the paper compared with similar approaches in the literature can be listed as follows:

- It is supposed that only the information of the robot joints positions is available.
- The H_∞ optimal controller is formulated such that attenuates the effect of the flexible modes on the performance of the robot.

To achieve this goal, by considering measurement noises, frictions, and the flexible modes as sources of uncertainties in the robot model, and by designing a proper regulated output, the problem is stated via a standard H_∞ formulation. Then, by numerical solution of an optimi-zation problem, the parameters of a dynamic output fee-dback controller are designed to achieve a desired H_∞ per-formance.

We use the following notation in the paper. R is the set of real numbers. I_n is an $n \times n$ identity matrix. $0_{n\times m}$ is an $n \times m$ zeros matrix. $L_2[0,\infty)$ is the space of signals with bounded energy, and $\|.\|$ and $\|.\|_\infty$ denote L_2 and H_∞ norms, respectively.

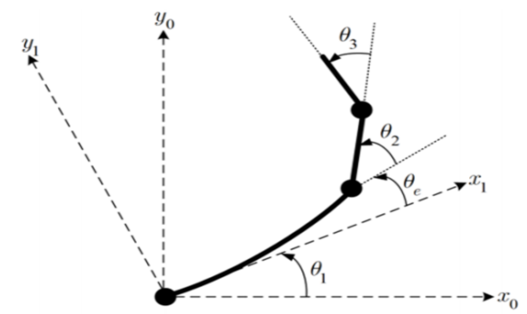

Fig.1. The macro-micro robot configuration.

The rest of the paper is organized as follows. In the next section, the dynamical model of the macro-micro robot is given, and its linearized model is obtained. The H_∞ op-timal

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control system will be designed in Section III. In Section IV, numerical solution of the optimization prob-lem is presented and the numerical simulations are pro-vided. Finally, the paper ends with conclusions in Section V.

II. Robot Model Definition

We consider a macro-micro manipulator depicted in Fig.1 whose dynamical equations are described as follows:

$$M_{ss}(\theta,q)\begin{bmatrix}\ddot{\theta}\\ \ddot{q}\end{bmatrix}+C_m(\theta,\dot{\theta},q,\dot{q})\begin{bmatrix}\dot{\theta}\\ \dot{q}\end{bmatrix}+ EI\begin{bmatrix}0_{3\times 3} & 0_{3\times 2}\\ 0_{2\times 3} & K_e\end{bmatrix}\begin{bmatrix}\theta\\ q\end{bmatrix}=\begin{bmatrix}\tau\\ 0\end{bmatrix}-\begin{bmatrix}K_\theta & 0\\ 0 & K_q\end{bmatrix}\begin{bmatrix}\dot{\theta}\\ \dot{q}\end{bmatrix}-\begin{bmatrix}F_\theta\\ F_q\end{bmatrix} \quad (1)$$

where $\theta \in R^3$ denotes the joints vector, $q \in R^2$ is the vector of flexible modes, $M_{ss}(\theta,q) \in R^{5\times 5}$ denotes the inertia matrix, $C_m(\theta,\dot{\theta},q,\dot{q}) \in R^{5\times 5}$ stands for the coriolis effects, *EI* is the Young's modulus, $K_e \in R^{2\times 2}$ presents the stiffness, $\tau \in R^3$is the input torque vector, $K_\theta \in R^{3\times 3}$ and $K_q \in R^{2\times 2}$ stand for the viscous frictions coefficients, and $F_\theta \in R^3$ and $F_q \in R^2$ are the coulomb frictions.

To design an H_∞ controller, it is necessary to linearize the robot model (1) about its equilibrium point. In this condition, without considering the friction terms, if we define

$$x_L = \begin{bmatrix}\theta^T & q^T & \dot{\theta}^T & \dot{q}^T\end{bmatrix},\quad y_L = \theta + n,\quad u_L = \tau$$

in which $n \in R^2$ is the measurement noises, the linearized form of the robot can be considered as follows:

$$\dot{x}_L(t) = A_L x_L(t) + B_L u_L(t),\quad y_L(t) = C_L x_L(t) + D_L u_L(t) \quad (2)$$

Where A_L , B_L , C_L , and D_L are the parameters of the linearized system with compatible dimensions. The robot closed loop system configuration is depicted in Fig. 2 in which θ_d is the desired value of θ, and A_c , B_c , C_c , and D_c are the parameters of a dynamic output feedback con-troller defined as follows:

$$\dot{x}_c(t) = A_c x_c(t) + B_c y(t),\quad u_L(t) = C_c x_c(t) + D_c y(t) \quad (3)$$

Where *y(t)* denotes the measurement output to be defined later, and x_c is the states of the dynamic output feedback controller system. Moreover, in Fig. 2, $\Delta_u$and Δ_c denote the effects of viscous and coulomb frictions, respectively, where

$$T_L = [0_{5\times 5}\ I_5]. \quad (4)$$

Therefore, the objective is to design a control strategy for the manipulator robot described in (2) in order to attenuate the effects of uncertainties such as noise, frictions, and flexible modes on the performance of the robot to track a desired trajectory which is studied in the following section.

III. H_∞ Control System Design

One of popular strategies to cope with uncertainties in dynamical systems is H_∞ control whose standard configuration is depicted in Fig. 3. The main objective in H_∞ control is to design the control input *u* from a stat-ic/dynamic controller *K* based on the measurement *y* in order to attenuate the effects of the uncertainties vector *w* (exogenous signals) on a desired performance which will be defined via the regulated output *z* [27], [28]. Therefore, the objective is to design the controller such that the effect of the L_2 norm of *w* on the L_2 norm of *z* is minimized. In other words, the H_∞ norm of the transfer function from *w* to *z* should be minimized as follows:

$$\min_{K \text{ stabilizing}} \sup_{w} \frac{\|z\|_2}{\|w\|_2} = \min_{K \text{ stabilizing}} \|T_{zw}\|_\infty .$$

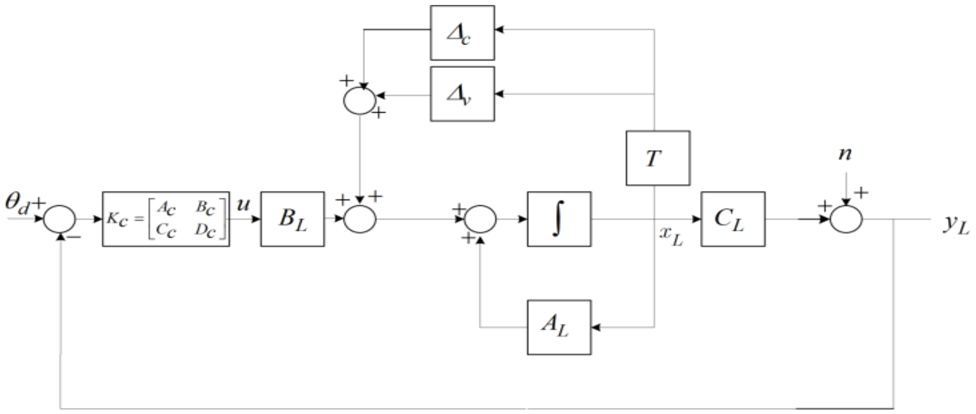

Fig. 2. The closed loop system configuration.

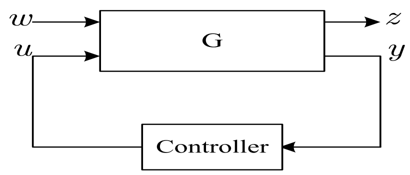

Fig. 3. The configuration of H_∞ control.

To design the H_∞ controller, at first a state space representation of the system should be obtained as follows:

$$\dot{x} = Ax + B_1 w + B_2 u,$$
$$z = C_1 x + D_{11} w + D_{12} u,$$
$$y = C_1 x + D_{21} w + D_{22} u. \qquad (5)$$

To obtain these parameters, it is necessary to modify the system configuration depicted in Fig. 2 by considering the effects of flexible modes as source of uncertainties instead of the states of the system. At first, let us state A_L, B_L and C_L by their block entries as follows:

$$A_L = \begin{bmatrix} A^{11}_{11(3\times3)} & A^{12}_{11(3\times2)} & A^{11}_{12(3\times3)} & A^{12}_{12(3\times2)} \\ A^{21}_{11(2\times3)} & A^{22}_{11(2\times2)} & A^{21}_{12(2\times3)} & A^{22}_{12(2\times2)} \\ A^{11}_{21(3\times3)} & A^{12}_{21(3\times2)} & A^{11}_{22(3\times3)} & A^{12}_{22(3\times2)} \\ A^{21}_{21(2\times3)} & A^{22}_{21(2\times2)} & A^{21}_{22(2\times3)} & A^{22}_{22(2\times2)} \end{bmatrix},$$

$$B_L = \begin{bmatrix} B^1_{\theta(3\times3)} \\ B^1_{q(2\times3)} \\ B^2_{\theta(3\times3)} \\ B^2_{q(2\times3)} \end{bmatrix},$$

$$C_L = \begin{bmatrix} C^1_{\theta(3\times3)} & C^1_{q(3\times2)} & C^2_{\theta(3\times3)} & C^2_{q(3\times2)} \end{bmatrix}.$$

Now, if we define

$$A_{11} = \begin{bmatrix} A^{11}_{11(3\times3)} & A^{11}_{12(3\times3)} \\ A^{11}_{21(3\times3)} & A^{11}_{22(3\times3)} \end{bmatrix},$$

$$A_{12} = \begin{bmatrix} A^{12}_{11(3\times2)} & A^{12}_{12(3\times2)} \\ A^{12}_{21(3\times2)} & A^{12}_{22(3\times2)} \end{bmatrix},$$

$$A_{21} = \begin{bmatrix} A^{21}_{11(2\times3)} & A^{21}_{12(2\times3)} \\ A^{21}_{21(2\times3)} & A^{21}_{22(2\times3)} \end{bmatrix},$$

$$B_\theta = \begin{bmatrix} B^1_{\theta(3\times3)} \\ B^2_{\theta(3\times3)} \end{bmatrix}, B_q = \begin{bmatrix} B^1_{q(2\times3)} \\ B^2_{q(2\times3)} \end{bmatrix},$$

$$C_\theta = \begin{bmatrix} C^1_{\theta(3\times3)} & C^2_{\theta(3\times3)} \end{bmatrix}, C_q = \begin{bmatrix} C^1_{q(3\times2)} & C^2_{q(3\times2)} \end{bmatrix}, \qquad (6)$$

the system configuration can be represented as depicted in Fig. 4. Indeed, in the mentioned matrices, the joints positions vector is decomposed from the vector of flexible modes. Therefore, A_{11} is corresponding to the effect of $[\theta\ \dot{\theta}]$ on $[\dot{\theta}\ \ddot{\theta}]$, and A_{12} and A_{21} imply the effect of $[q\ \dot{q}]$ on $[\dot{\theta}\ \ddot{\theta}]$ and the effect of $[\theta\ \dot{\theta}]$ on $[\dot{q}\ \ddot{q}]$, respectively. B_q and B_θ denote the effect of u on $[\dot{q}\ \ddot{q}]$ and $[\dot{\theta}\ \ddot{\theta}]$, respectively, and C_q and C_θ are output matrices corresponding to $[q\ \dot{q}]$ and $[\theta\ \dot{\theta}]$, respectively. According to Fig. 4, at first the exogenous signals are formulated and a desired regulated output is designed. Then, by formulation of the measurement output, all the matrix parameters in (5) are obtained.

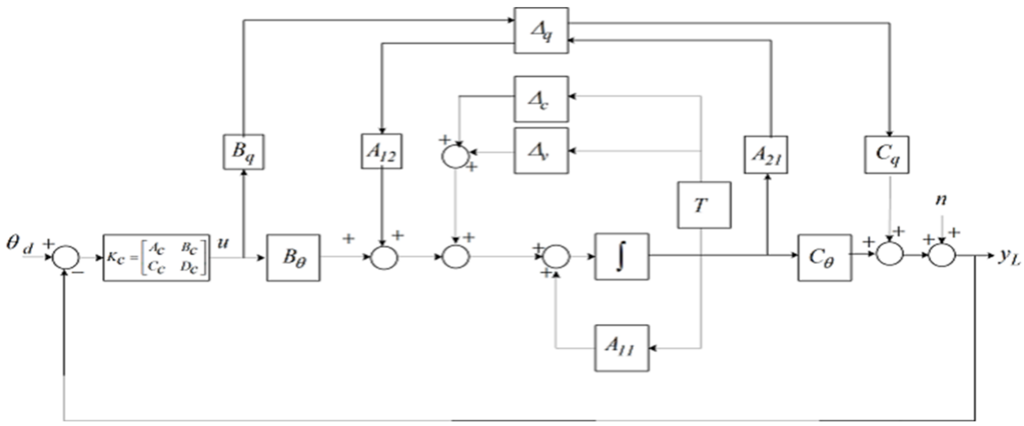

Fig. 4. The closed loop system configuration while considering the flexible modes as sources of uncertainties.

A. Exogenous Signals Formulations

According to the sources of uncertainties, an exogenous signals vector should be designed. There are four sources of uncertainties in the system listed below:

- The effect of the viscous frictions on the robot: w_d^v.
- The effect of the coulomb frictions on the robot: w_d^c.
- The effect of the flexible modes on the robot: w_d^q.
- The measurement noises vector: n.

On the other hand, the reference signal q_d also should be considered as an exogenous signal. Therefore, considering all the above-mentioned issues, the exogenous signals vector can be defined as follows:

$$w_{(16\times1)} = \begin{bmatrix} w^{vT}_{d(3\times1)} & w^{cT}_{d(3\times1)} & w^{qT}_{d(4\times1)} & n^{T}_{(3\times1)} & \theta^{T}_{d(3\times1)} \end{bmatrix}^T$$

Accordingly, considering Fig. 4, one can observe that

$$\dot{x} = A_{11}x + \begin{bmatrix} \dfrac{I_3}{0_3} & \dfrac{I_3}{0_3} & A_{12(6\times4)} & 0_{6\times3} & 0_{6\times3} \end{bmatrix} w + B_\theta u, \qquad (7)$$

and therefore, the matrix parameters A, B_1, and B_2 can be stated as follows:

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$$A = A_{11},\quad B_1 = \begin{bmatrix} \frac{I_3}{0_3} & \frac{I_3}{0_3} & A_{12(6\times4)} & 0_{6\times3} & 0_{6\times3} \end{bmatrix},\quad B_2 = \begin{bmatrix} B_\theta \end{bmatrix}. \tag{8}$$

B. Regulated Output Formulation

The next step is to design a regulated output such that minimizes effects of the source of uncertainties in the system while guaranteeing a desired tracking performance. Moreover, it should guarantee tracking of the reference signal with reasonable control efforts. Therefore, we can design a regulated output as follows:

$$z_{(16\times1)} = \begin{bmatrix} y^{vT}_{d(3\times1)} & y^{cT}_{d(3\times1)} & y^{qT}_{d(4\times1)} & e_{(3\times1)}^{\ T} & u_{(3\times1)}^{\ T} \end{bmatrix}^T$$

where y_d^v, y_d^c, and y_d^q are the sources of the viscous friction, the coulomb friction, and the flexible modes, respectively, and e is the tracking error which by considering Fig. 4, it can be stated as follows:

$$e = \theta_d - C_\theta x - C_q w_d^q. \tag{9}$$

Therefore, by considering y_d^v, y_d^c, and y_d^q in Fig. 4, the regulated output can be restated as follows:

$$z = \begin{bmatrix} y_d^v \\ y_d^c \\ y_d^q \\ e \\ u \end{bmatrix} = \begin{bmatrix} \delta_v T x \\ \delta_c T x \\ A_{21}x + B_q u \\ \theta_d - C_\theta x - C_q w_d^q \\ u \end{bmatrix} = \begin{bmatrix} \delta_v T \\ \delta_c T \\ A_{21} \\ -C_\theta \\ 0_{3\times6} \end{bmatrix} x + \left[\begin{array}{c|ccc} \multicolumn{4}{c}{0_{10\times16}} \\ \hline 0_{6\times12} & -C_q & 0_3 & I_3 \\ & 0_{3\times4} & 0_3 & 0_3 \end{array}\right] w + \begin{bmatrix} 0_3 \\ 0_3 \\ B_q \\ 0_3 \\ I_3 \end{bmatrix} u$$

in which

$$T = \begin{bmatrix} 0_{3\times3} & I_3 \end{bmatrix}. \tag{10}$$

which implies that the matrix parameters C_1, D_{11} and D_{12} are as follows:

$$C_1 = \begin{bmatrix} \delta_v^T T & \delta_c^T T & A_{21} & -C_\theta & 0_{3\times6} \end{bmatrix}^T,\quad D_{11} = \left[\begin{array}{c|ccc} \multicolumn{4}{c}{0_{10\times16}} \\ \hline 0_6 & -C_q & 0_3 & I_3 \\ & 0_{3\times4} & 0_3 & 0_3 \end{array}\right],\quad D_{12} = \begin{bmatrix} 0_3 \\ 0_3 \\ B_q \\ 0_3 \\ I_3 \end{bmatrix}. \tag{11}$$

C. State Space Representation of the H_∞ Control Problem

The measurement output of the robot will be studied.

According to Fig. 4, the measurement output is a function of θ and uncertainties as follows:

$$y = \theta_d - C_\theta x - C_q w_d^q - n = -C_\theta x + \begin{bmatrix} 0_3 & 0_3 & -C_{q(3\times4)} & -I_3 & I_3 \end{bmatrix} w + 0_3 u \tag{12}$$

and therefore, the matrix parameters C_2, D_{21}, and D_{22} are as follows:

$$C_2 = \begin{bmatrix} -C_\theta \end{bmatrix},\quad D_{21} = \begin{bmatrix} 0_3 & 0_3 & -C_{q(3\times4)} & -I_3 & I_3 \end{bmatrix},\quad D_{22} = \begin{bmatrix} 0_3 \end{bmatrix}. \tag{13}$$

Therefore, considering all the mentioned issues in (8), (11), and (13), all the matrix parameters in (5) were obtained, and the system G can be formulated in the following standard form:

$$G = \begin{matrix} \dot{x} \\ z \\ y \end{matrix} \left[\begin{array}{c|cc} A & B_1 & B_2 \\ \hline C_1 & D_{11} & D_{12} \\ C_2 & D_{21} & D_{22} \end{array}\right] \begin{matrix} x \\ w \\ u \end{matrix} \tag{14}$$

In this condition, it is sufficient to numerically solve an optimization problem to find the controller parameters A_c, B_c, C_c, and D_c such that the H_∞ norm of the transfer function from w to z is minimized.

Remark 1: It is worth mentioning that although the problem was formulated for a macro-micro three link manipulator, the proposed strategy can be extended for every manipulator robot with any number of flexible links.

IV. Simulation Results

The performance of the proposed H_∞ optimal controller is studied in this section. By considering the obtained standard H_∞ formulation in (14), we have employed the robust H_∞ optimization toolbox in MATLAB to obtain the parameters of a dynamic output feedback controller defined in (3) while satisfying a desired H_∞ performance.

We consider a scenario where the desired trajectories of the joints are considered as follows:

$$\begin{aligned} \theta_{1d} &= 0.2\sin(t/10)\ \text{rad} \\ \theta_{2d} &= 0.1\sin(t/3)\ \text{rad} \\ \theta_{3d} &= 0.3\cos(t/5)\ \text{rad} \end{aligned}$$

and the initial states of the robots are supposed to be as follows:

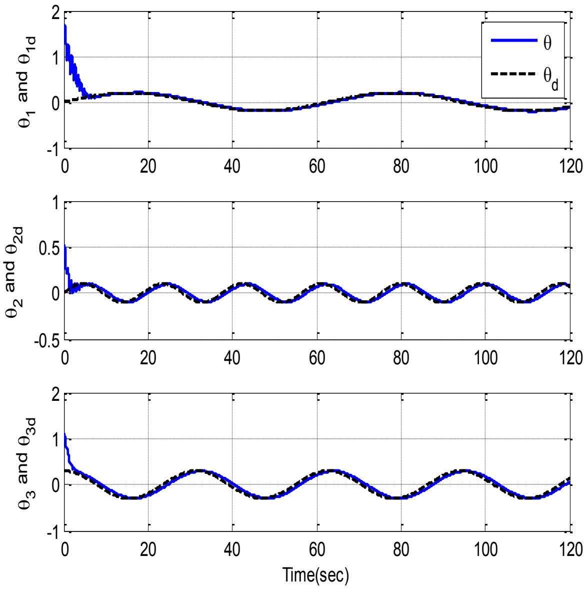

Fig.5. The trajectories of the robot joints..

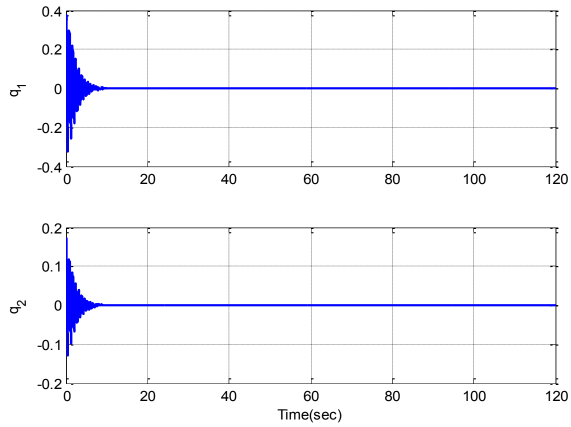

Fig.6. The trajectories of the flexible modes.

$$\begin{bmatrix}\theta_{01} & \theta_{02} & \theta_{03} & q_{01} & q_{02}\end{bmatrix} = [1.6\ 0.5\ 1.2\ 0.4\ 0.2]\,\text{rad}$$
$$\begin{bmatrix}\dot\theta_{01} & \dot\theta_{02} & \dot\theta_{03} & \dot q_{01} & \dot q_{02}\end{bmatrix} = [0.02\ 0.04\ 0\ 0.02\ -0.004]\,\text{rad s}^{-1}$$

For simulation, we have considered a measurement noise with noise power of 10^{-5} and sample time of 0.1. Moreover, the robot parameters are supposed to be as giv-en in Table I.

Table I. The Robot Parameters

Parameter	Value
The first link length	0.63m
The second and third links length	0.018m
The Young modulus	2.051Nm^2
The first link density	0.47Kg/m
The second and third links density	0.003Kg/m
The first motor moment of inertia	0.67Kgm^2
The second and third links moment of inertia	0.003Kgm^2
The first link mass	5Kg
The second and third links mass	0.326Kg

In this condition, the robot joints trajectories are depicted in Fig. 5. According to the figure, the robot tracks the desired trajectory with negligible errors. Moreover, as depicted in Fig. 6, the flexible modes are damped to zero. Furthermore, the input torques to the robot joints are shown in Fig. 7 confirming that the desired performance is achieved by reasonable control efforts.

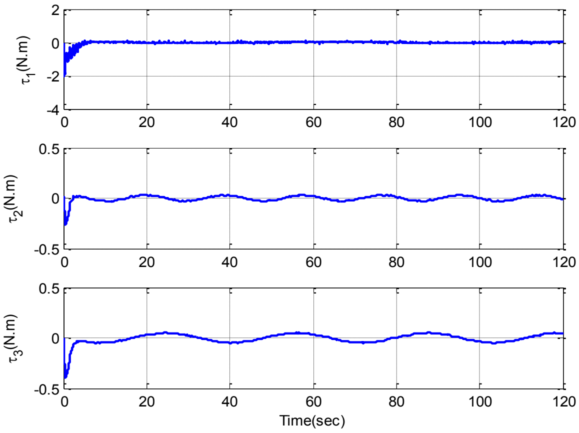

Fig.7. The input torques to the robot joints.

V. Conclusion and Future Work

H_∞ optimal control of 3-link macro-micro manipulators was studied in this paper. It was supposed that the first link of the robot is flexible, and the robot is subject to measurement noises and unknown frictions. Then, by considering the flexible modes, measurement noises, and the frictions as sources of uncertainties, and by designing a proper regulated output, a standard H_∞ optimal control problem was formulated. Hence, by using only the information of the robot joints positions, a dynamic output-feedback controller guaranteeing a desired H_∞ performance was obtained. H_∞ optimal impedance control of the robot in the presence of aforementioned uncertainties is another challenging issue to be studied as future work.

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