A Combinatorial Algorithm with Transportation Map Regularization for Image Restoration from Salt and Pepper Noise

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Abstract – In this paper we proposed combinatorial Algorithm which is applicable to image processing. Basically combinatorics are relating to mathematical tools these comes under set theory which is also called Hypergraph Theory. Combinatorics refers to arranging the elements into sub sets. We proposed a application in particular to Denoising of Image using combinatorial analysis. First the image can be converted into Hypergraph model by using Image Adaptive Neighbourhood Hypergraph (IANH) model then this model can be used for different applications of image processing. Impulse noise is detected by analysing unit cardinality pixels in IANH model, that can be removed by different traditional methods and comparing PSNR and MAE of these methods. We present the algorithm and conduct some experimental results proving its efficiency.

Keywords – Combinatorial, Hypergraph Theory, Denoising, IANH Model.

I. INTRODUCTION

In many areas of research such as, Biomedical, Criminology, Artificial intelligence, etc., the relation between objects expressed by binary relations. The area which is model these relations is combination of graph theory and image processing. In a graph the vertices models to objects and edges corresponds to the interrelations of these objects. A digital image can also be considered as a graph when the topography (connectivity) of the support grid is taken into detail. But it is difficult to model the direct graph into image, so we go for hyper graphs which expressed by binary relations.

Combinatorial (Hypergraph) theory initially proposed by C.Berge [10] it is generalization of graph theory by grouping sets into edges, then combining these edges into family of Hypergraph. This mathematical concept can be represented to networks, communication systems, process scheduling, data structures and a variety of other systems where connectivity between the objects in the system play a dominant role. We can consider it as a model of image applications.

Salt and pepper (SP) noise is very common problem in digital images, several filtering techniques have been proposed over years to eliminate this noise but these techniques have limitation that most of them are related to statistical approaches rather than lexical considerations such as threshold fixing for distance and intensity levels.

In order to overcome these difficulties we use the combinatorial technique. In this first image can be converted into Hypergraph model by using IANH model, then applying our unit cardinality algorithm to detect SP noise and this noise is removed by different filters such as contra harmonic mean filter, median filter and transposition and map regularization techniques. Performance was compared in terms of PSNR and Mean Absolute Error (MAE). Proposed method is able to clean images efficiently without loss of image details.

II. METHODOLOGY

The methodology relating to graphs and hypergraphs is similar to [10, 13]. A graph is denoted by G= (V; E) where V is vertices set and E is Edge set and \( \Gamma(x) \) is neighbourhood of a vertex x.

\[
\Gamma(x) = \{ y \in V, \{ x, y \} \in E \}
\]

Example of a graph is given in Fig. 1, which is associated with neighbourhood Hypergraph associated with this graph is

\[
S = \{ x_1, x_2, x_3, x_4 \} \quad E_{s_1} = \{ x_1, x_2 \} \quad E_{s_2} = \{ x_1 , x_2 , x_3 \}, \\
E_{s_3} = \{ x_1 , x_2 , x_4 \} , \quad E_{s_4} = \{ x_3 , x_4 \}
\]

The cardinality of star H(x) denoted by

\[
dx = \text{Card}(H(x)).
\]
As shown in Fig. 2. We have the set of vertices 
\[ S = \{x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9\}, \]
and the set of hyperedges 
\[ E_1 = \{x_1, x_2, x_3, x_9\}, \]
\[ E_2 = \{x_2, x_3, x_4\}, \]
\[ E_3 = \{x_2, x_7\}, \]
\[ E_4 = \{x_5, x_6, x_7, x_8, x_9\} \]
the star centred on 
\[ x_2 \text{ is } H(x_2) = \{E_1, E_2, E_3\}, \]
with degree \( dx_2 = 3 \)
and the star centred on 
\[ x_4 \text{ is } H(x_4) = \{E_2\}, \]
with degree \( dx_4 = 1 \).

### III. IMAGE ADAPTIVE NEIGHBORHOOD HYPERGRAPH (IANH) MODEL

In this paper the Digital image can be represented into Hypergraph using the IANH Model for different applications. The digital image is a two dimensional discrete form this is digitized in both spatial and magnitude feature value. The digital image can be represented by

\[ I : X \subseteq Z^2 \rightarrow C \subseteq Z^n \]

Where \( C \) denotes the feature intensity level and \( X \) denotes a set of image points. The set \( \{x, I(x)\} \) is called a pixel which is an acronym for picture element. Assume \( d \) be a distance on \( C \), then the neighbourhood relation on an image is given by

\[ \forall x \in X, \Gamma_{\alpha, \beta}(x) = \{x' \in X, x' \neq x \mid d(I(x), I(x')) < \alpha\} \]

And \( d'(x, x') \leq \beta \) Where \( \Gamma_{\beta}(x) \) is neighbourhood of \( x \) on the grid of \( \beta \). If \( \beta = 4 \) it is the 4 neighbourhood. If \( \beta = 8 \) then it is 8 neighbourhood. So each image we can apply a hyper graph called Image Adaptive Neighbourhood Hypergraph (IANH) which is denoted by \( H_{\alpha, \beta} \) which is defined by

\[ H_{\alpha, \beta} = \{x \{x\} \Gamma_{\alpha, \beta}(x)\} \subseteq X \]

The attribute \( \alpha \) can be standard deviation of centre pixel and its neighbourhood pixels i.e., \( \{x\} \cup \Gamma_{\beta}(x) \).

### Algorithm for IANH model

The IANH model converts digital image into Hypergraph model. Take a digital image of size \( M \times N \) an neighbourhood order \( \beta \).

\( X = \phi \)

**For each pixel x of I, do;**

\[ \alpha = \text{Standard deviation of the pixels } \{x\} \cup \Gamma_{\beta}(x); \]

**For each pixel y of } \Gamma_{\beta}(x), do;**

if \( d(I(x), I(x')) < \alpha \) then

\[ \Gamma_{\alpha, \beta}(x) = \Gamma_{\alpha, \beta}(x) \cup \{y\}; \]

end if

end for

\[ X = X \cup \{x\}; \]

\[ E_{\alpha, \beta}(x) = [\Gamma_{\alpha, \beta}(x) \cup \{x\}]; \]

end for

\[ H_{\alpha, \beta}(x) = (X, \{E_{\alpha, \beta}(x)\}_{x \in X}); \]

End

### IV. SALT AND PEPPER NOISE REMOVAL USING COMBINATORIAL ANALYSIS

We now develop some low level image processing application based on geometrical properties of the IANH model. Specifically, Salt and Pepper Noise removal.

- Salt and Pepper noise detection
- The reduction of noise appearing data is main task in computer vision. The accuracy of this task can be determined by the overall system performance. The two main things in this case are how much the noise granularity has been removed and how good the edges are preserved. Many filtering techniques have been developed to suppress the noise. In this paper we proposed an algorithm for transforming an image so that it satisfies the property[11] by using IANH algorithm.

If is a noise hyperedge then it satisfies any one of the following conditions:

- It will detect the noise and noise will be cancelled by traditional methods such as
  - The cardinality of is equal to one, and is not contained in the intersection of any number of hyperedges.
  - When \( E(x) \) is an isolated hyperedge (of perhaps cardinality larger than one), there exists \( y \) belonging to open neighbourhood of \( E(x) \) on the grid such that \( E(y) \) is isolated.

Take 5x5 image grid as shown in Figure 3, it contain grey-level of value every vertex \( x_i \) is an integer in the interval \([0, 255]\). Intensity 0 represents black and 255 represents white.

The hyper edges are \( F_1 \) through \( F_{14} \) are given bellow.

\[ F_1 = \{x_1, x_2, x_3, x_8\} \]
\[ F_2 = \{x_2, x_3, x_4, x_9\} \]
\[ F_3 = \{x_4, x_5\} \]
\[ F_4 = \{x_5, x_6, x_7, x_8\} \]
\[ F_5 = \{x_7, x_8, x_9\} \]
\[ F_6 = \{x_9\} \]

(Blurred Noise)

(Blurred Noise)

(Blurred Noise)

(Blurred Noise)

(Blurred Noise)

(Blurred Noise)
A. Noise removal

The Root Mean Square (RMS)[2] approximation of restored image is given by the expression

$$f(x, y) = \left( \frac{1}{mn} \sum_{s,t}(g(s, t))^2 \right)^{1/2}, \quad (s, t) \in S_{xy}$$

Where $g(s,t)$ points the gray level of the image pixel at $(s, t)$. 

Hypergraph based Root Mean square approximation (HGRMS) has advantages of following

1) To make INHG more adaptive.
2) INHG is flexible enough to accommodate RMS approximation for ensuring better clearance of the noise.
3) Image details are preserved accurately by detecting the noisy pixels using INHG parameters & denoising only specified affected pixels.

B. The Yaroslavsky filter

In this paper we give some new insight on neighbour filters using only pixel wise information to compute image similarity which removes SP noise effectively. The Yaroslavsky filter[6] works on the principle of Transportation map response(TMR) defined as the difference between the original and the corrected image, which removes the SP noise as well as brighten the image. The mathematical formulation is

$$Y^F_Y(x) = \frac{\sum_x \frac{(Y(x') - Y(x))/g}{L(|x'/x|/g).L(|x'/x|/h).Y(x')}}{K(|Y(x') - Y(x)|/g).L(|x'/x|/h)}$$

Where $x'$ runs in image pixels, $K$, $L$ are kernel functions, $g>0$ and $h>0$ are size parameters. For simplicity we used filter with both the spatial kernel $L$ and photo metric kernel $K$ being box kernels. The advantage of TMR is which removes the SP noise effectively compared to RMS filter and increases the image brightness.

C. Performance Measure

The Image quality is evaluated by following metrics

- Peak signal to noise ratio (PSNR)
- Mean absolute error (MAE)

The peak signal to noise ratio is most commonly used method for measure of quality of restored images.

$$PSNR = 10 \log_{10} \frac{255^2}{\frac{1}{mn} \sum (r_y - x_y)^2}$$

The Mean absolute error is one of the number of ways of comparing forecasts with their eventual outcomes. The MAE measures the average magnitude of the errors in a set of forecasts, without considering the direction. It measures accuracy for continuous variables. The MAE is the average over the verification sample of the absolute values of the differences between forecast and the corresponding observation. The MAE is a linear score which means that individual differences are weighted equally in average.

$$MAE = \frac{1}{mn} \sum \left| r_y - x_y \right|$$
Where \( r_y \) and \( x_y \) denote the pixel intensity values of the restored and original image respectively.

V. RESULTS AND PERFORMANCE ANALYSIS

Fig. 5.1 is the original image of size , we applied Hypergraph algorithm to this image. Fig. 5.2 shows the image subjected to 20% salt & pepper noise. The hyperedges are calculated from the noisy image from two hypergraph parameters \( \alpha \) and \( \beta \). We found that optimal value of \( \alpha \) and \( \beta \) empirically. Then isolated stars have been identified by using Helly property. Noisy hyperedges have been found. The calculated noisy pixels are denoised by using Root mean square(RMS) approximation. Various values for the hypergraph values are used for experimentation. Finally we got the optimized values \( \alpha=20 \) and \( \beta=2 \).

Fig. 5.3 shows the denoised image using HGRMS (Hypergraph based Root Mean Square) method.

Performance analysis of image is subjected to salt and pepper noise with various noise ratio for PSNR and MAE. The HGTMR is compared with HGRMS and HGCHM. HGTMR is outperforms compared to remaining filters.
Table I: Results in PSNR for the Lena image subjected to various noise levels for Different filters

<table>
<thead>
<tr>
<th>Noise ratio (%)</th>
<th>Hypergraph based Root Mean Square (HGRMS)</th>
<th>Hypergraph based Contra harmonic filter (HGCHM)</th>
<th>Hypergraph based Transportation Map Regularize (HGTMR)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>55.23</td>
<td>59.39</td>
<td>65.25</td>
</tr>
<tr>
<td>15</td>
<td>55.25</td>
<td>58.32</td>
<td>62.23</td>
</tr>
<tr>
<td>20</td>
<td>52.37</td>
<td>57.41</td>
<td>61.72</td>
</tr>
</tbody>
</table>

Table II: Results in MAE for Lena image subjected to various noise levels for different filters

<table>
<thead>
<tr>
<th>Noise Level (%)</th>
<th>Hypergraph based Root Mean Square (HGRMS)</th>
<th>Hypergraph based Contra harmonic filter (HGCHM)</th>
<th>Hypergraph based Transportation Map Regularize (HGTMR)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>0.07</td>
<td>0.12</td>
<td>0.06</td>
</tr>
<tr>
<td>15</td>
<td>0.10</td>
<td>0.15</td>
<td>0.08</td>
</tr>
<tr>
<td>20</td>
<td>0.13</td>
<td>0.20</td>
<td>0.13</td>
</tr>
</tbody>
</table>

VI. CONCLUSION

In traditional image representation, the Contra Harmonic Filter (CHM) is effectual in removing Salt and Pepper (SP) noise, but it has a drawback: it tends to blur the image. On the other hand, in hypergraph (HG) image representation, the parameters α and β track the image by means of hyperedges, which results in a better distinction of SP noise. Subsequently, Bretto[9] has been verified hat Root Mean Square(RMS) filter worked best in combinatorial representation in the removal of Impulse noise and Gaussian noise, but HGRMS also limitation that it leaves few stars unprocessed. The ability of the proposed HTMR filter to deal with these artifacts while restoring the fine details of images has been demonstrated on various examples. Several extensions of this work are foreseen. First, notice that the computation time of the TMR operator is similar to those of the bilateral filter or nonlocal means.

REFERENCES