

Performance Analysis of Adaptive M -PSK Communications Using Equal Gain Combining

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Abstract – Channel estimation at the receiver is essential for Variable Rate (adaptive) modulation schemes and this prohibits systems that are less complex from using (adaptive) variable rate and/or variable power transmissions. Providing a solution to this problem, we introduce a variable-rate (VR) M -PSK modulation scheme, for communications over fading channels, without channel gain estimation at the receiver. The choice of the constellation size is based on the signal-plus-noise ($S+N$) value rather than on the signal-to-noise ratio (S/N). It is analytically shown that $S+N$ can serve as an excellent simpler criterion, alternative to S/N , for determining the modulation order in adaptive systems. In this way, low complexity transceivers can use VR transmissions to increase their spectral efficiency under an error performance constraint. As an application, we utilize the proposed adaptive modulation scheme in equal gain combining (EGC) diversity receivers.

Keywords – Equal Gain Combining, Variable-Rate (VR), Adaptive M -PSK Communications.

I. INTRODUCTION

Adaptive transmission is the transfer of data or media onto a communication channel carrier where the characteristics of the transmission can be adapted to better match the characteristics of the transmission channel that have changed. A common technique for coping with fading in wireless communications is transmission or reception diversity, which improves the performance using an extra hardware or increase in bandwidth. Alternatively, if a feedback link is available, the fading can be reduced by allowing the receiver to monitor the channel conditions and request compensatory changes in certain parameters of the transmitted signal. This technique is called adaptive transmission and its basic concept is the real-time balancing of the link budget through adaptive variation of the transmitted power level, symbol transmission rate, constellation size, coding rate/scheme, or any combination of these parameters. Adaptive transmission was first proposed by Hayes [1], who considered a Rayleigh fading channel where the amplitude of the transmitted signal is under control of the receiver through a feedback channel.

II. PREVIOUS WORKS AND CONTRIBUTION

In [14], the performance of a blind adaptive modulation scheme is investigated, which necessitates channel knowledge only at the receiver. In this scheme, the order

of an M -ary quadrature amplitude modulation (M -QAM) sequentially decreases or increases according to positive/negative acknowledgments (ACK/NACK) sent to the transmitter. In this paper, we propose a rate adaptation technique, which is based on the signal-plus-noise ($S+N$) samples, instead of the instantaneous S/N . Owing to this particularity, no channel gain estimation is necessary for determining the transmission rate. In [15] the $S+N$ technique has been applied to selection diversity communication systems, where its main advantage is it takes into account the instantaneous noise power, in contrast to S/N . From [17] we utilize $S+N$ can serve as an excellent alternative criterion to S/N , in adaptive systems; the same spectral efficiency can be achieved, when S/N or $S+N$ is used for determining the constellation size. The main contribution of this paper can be summarized as follows.

We introduce an $S+N$ based adaptive M -ary phase shift keying (M -PSK) modulation scheme that does not require any channel gain estimation at the receiver. we utilize the proposed adaptive modulation scheme in equal gain combining (EGC) diversity receivers. Also, M PSK modulation schemes have some important advantages over QAM schemes, e.g. improved performance when used with high power amplifiers. Another advantage of using M -PSK instead of M -QAM for Orthogonal Frequency-Division Multiplexing (OFDM) systems is the availability of techniques to reduce the signal's Peak-to-Average-power Ratio (PAR), also known as PAPR or PMEPR (Peak-to-Mean Envelope Power Ratio) [16].

III. ADAPTIVE SYSTEMS WITH $S+N$ CRITERION

We consider a discrete-time channel, assuming that the fading amplitude $a[i]$ follows a Rayleigh distribution with probability density function (pdf)

$$f_a(a) = \frac{2a}{\Omega} e^{-\frac{a^2}{\Omega}} \quad \text{----- (1)}$$

Where $\Omega = \{a^2\}$. The instantaneous received signal

$$r[i] = a[i]e^{j\theta[i]}s[i] + n[i] \quad \text{-----(2)}$$

where $s[i]$ is the complex transmitted symbol, $\theta[i]$ is the phase introduced by the fading channel and $n[i]$ is a zero mean circularly symmetric complex Gaussian noise with $E\{n[i]*n[i]\} = N_0 = 2\sigma^2$.

The symbol $s[i]$ is selected from a signal constellation with M symbols (e.g. M -QAM or M -PSK) each with energy E_{s_k} and total average symbol energy, $E_s = 1$. Then,

the received S/N will be

$$\gamma[i] = E_s \frac{\alpha^2[i]}{BN_0} \quad \text{----- (3)}$$

Where B is the bandwidth of the received signal. The average channel gain is denoted as γ , which is assumed to be $\gamma=1$, without loss of the generality.

In [17], an alternative criterion for determining the modulation order in adaptive systems, which does not require the estimation of the fading amplitude and is only based on a scaled $S+N$ sample.

$$\xi = \frac{\Re\{r[i]e^{-j\theta[i]}\}^2}{N_o} = \frac{(\alpha\sqrt{E_{S_k} + n_l})^2}{N_o} \quad \text{--- (4)}$$

Where $\Re\{z\}$ denotes the real part of z and n_l is the in-phase noise component with variance $N_o/2$.

In the proposed $S+N$ scheme, the modulation order is chosen similarly to the conventional one, with the difference that γ is replaced by ξ , i.e. the selected modulation order, will be

$$M=M_j, \text{ if } \begin{cases} \gamma_j \leq \xi < \gamma_{j+1} \\ \gamma_j \leq \frac{(\alpha\sqrt{E_{S_k} + n_l})^2}{N_o} < \gamma_{j+1} \end{cases} \quad \text{---- (5)}$$

The effectiveness of the proposed scheme will be compared against the conventional one by considering the S/N as the reference criterion for determining the maximum modulation order.

IV. ADAPTIVE M-PSK

In this section, we introduce a constant power, adaptive M -PSK modulation scheme, in which the decision on the modulation order is not based on the S/N samples, but rather on the $S+N$ ones. The mode of operation is illustrated in Fig.1. An ideal coherent phase detection is assumed to be available at the receiver at time instant, i , while no channel gain estimation is necessary. The receiver estimates the $S+N$ samples at time, i , and decides on the constellation size $M_0=0, M_j=2^j, j=1, \dots, N$, that will be used at the next time interval, $i+1$, according to this sample. The decision on the modulation order is sent back to the transmitter, transmitting N bits through a feedback channel, that does not introduce any errors. Usually, the elimination of the estimation errors can be assured by increasing its delay time and using an ARQ transmission protocol.

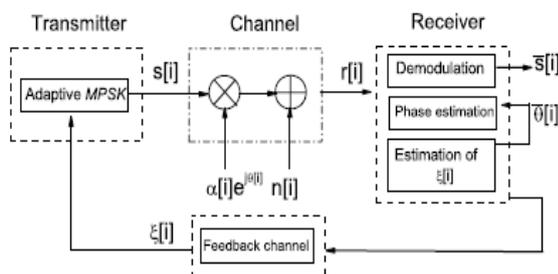


Fig.1. The Adaptive M-PSK modulation scheme.

Similarly to the S/N based adaptive modulation schemes, the estimation of the modulation order for the $S+N$ based schemes, would require a closed-form formula that relates, the $S+N$ with the error probability. Because of the fact that such formulas are not available, S/N related formulas will be used instead. Moreover, formulas relating $S+N$ with the SER would be useless for the adaptation scheme, since the instantaneous noise components between two time instances are different. Therefore, for the case of the M -PSK we can use the approximation for the SER, i.e., [18, Ch. 7]

$$P_M \approx \text{erfc}(\sqrt{\gamma} \sin \frac{\pi}{M}) \quad \text{----- (6)}$$

Solving above equation with respect to M , the maximum constellation size for a given SER is given by

$$M = \frac{\pi}{\text{arc sin}(\frac{1}{\sqrt{\gamma}} \text{erfc}^{-1} P_M)} \quad \text{---- (7)}$$

Therefore, the selected order, M , will be

$$M=M_j, \text{ if } \begin{cases} M_j \leq M < M_{j+1} \\ \gamma_j \leq \frac{(\alpha\sqrt{E_s + n_l})^2}{N_o} < \gamma_{j+1} \end{cases} \quad \text{----- (8)}$$

where

$$\gamma_j = \frac{\text{erfc}^{-1} P_M}{\sin \frac{\pi}{M_j}} \quad \text{----- (9)}$$

The proposed scheme gives the ability to communications systems with no channel gain estimation capabilities to adapt their transmission rate, so that their spectral efficiency can be increased. We should also note that in some applications, constant envelope modulations, such as M -PSK, are more desirable than other ones, because of specific advantages like their improved performance when used with high power amplifiers.

Spectral Efficiency:

The normalized spectral efficiency of the $S+N$ based VR M -PSK scheme is obtained as

$$S_{S+N} = \frac{R}{B} = \sum_{j=1}^N [\log_2(M_j) P_r \{ \gamma_j \leq \xi < \gamma_{j+1} \}] \quad \text{----- (10)}$$

The probability, $\Pr \{ \gamma_j \leq \xi < \gamma_{j+1} \}$ can be rewritten as

$$P_r \{ \gamma_j \leq \xi < \gamma_{j+1} \} = P_r \left\{ \sqrt{\gamma_j N_o} \leq \alpha\sqrt{E_s + n_l} < \sqrt{\gamma_{j+1} N_o} \right\} = \int_{\gamma_j}^{\gamma_{j+1}} f_z(z) dz \quad \text{----- (11)}$$

Where $f_z(z)$ is the pdf of the random variable $z = \alpha\sqrt{E_s + n_l}$, which is the sum of a Rayleigh and a Gaussian random variable, and is given by [15]

$$f_z(z) = \frac{1}{\sqrt{2\pi}\sigma^2} \frac{2\sigma^2}{E_s\Omega + 2\sigma^2} e^{-\frac{z^2}{2\sigma^2}} + \frac{z\sqrt{E_s\Omega}}{E_s\Omega + 2\sigma^2} \frac{1}{\sqrt{E_s\Omega + 2\sigma^2}} X e^{-\frac{z^2}{E_s\Omega + 2\sigma^2}} \left(1 - \text{erfc} \left(-\frac{\sqrt{E_s\Omega} z}{\sqrt{2\sigma^2} \sqrt{E_s\Omega + 2\sigma^2}} \right) \right)$$

Where $\text{erf}(\cdot)$ is the error function.

A closed-form solution to the indefinite integral

$$J(z) = \int f_z(z) dz \quad \text{----- (12)}$$

can be found using [20] and Finally, the spectral efficiency of the $S+N$ based VR M -PSK scheme is obtained as

$$S_{S+N} = \sum_{j=1}^N \left[\log_2(M_j) \int_{\gamma_j}^{\gamma_{j+1}} f_z(z) dz \right] \quad \text{-----(13)}$$

or

$$S_{S+N} = \sum_{j=1}^N \left[\log_2(M_j) \{ \tau_{(\gamma_{z+1})} - \tau_{(\gamma_z)} \} \right] \quad \text{----- (14)}$$

On the other hand, the normalized capacity of a VR M-PSK scheme, with the modulation order determined by the received S/N, is given by

$$S_{S/N} = \sum_{j=1}^N \left[\log_2(M_j) P_r \{ \gamma_j \leq \gamma < \gamma_{j+1} \} \right] \quad \text{----- (15)}$$

For the case that the fading amplitude, a , follows the Rayleigh distribution (1), γ will be exponentially distributed as

$$f(\gamma) = \frac{1}{\gamma} e^{-\frac{\gamma}{\gamma}} \quad \text{----- (16)}$$

Where $\gamma = \Omega E_s / N_0$ is the average SNR.

Symbol Error Probability:

Regarding the SER of a S/N based VR M-PSK scheme, it can be obtained by averaging over the SER for M-PSK in AWGN [5], i.e.,

$$P_s = \sum_j \int_{\gamma_j}^{\gamma_{j+1}} P_{AWGN}(M_j, \gamma) f_{\gamma}(\gamma) d\gamma \quad \text{----- (17)}$$

The SER of M-PSK over AWGN is given by [22, Ch.8.1]

$$P_{AWGN}(M_j, \gamma) = Q(\sqrt{2\gamma}) + \frac{2}{\sqrt{\pi}} \int_0^{\infty} \exp\{- (u - \sqrt{\gamma})^2\} Q(\sqrt{2\gamma} \tan \frac{\pi}{M} du) \quad \text{----- (18)}$$

The above approach results in some cases to closed-form solutions for the error probability.

For the case of the S+N, however, a similar approach cannot be followed, since a formula for the SER as a function of ξ is required. Moreover, when physically realizing S+N schemes, though, by sampling the output of a matched filter, the noise is a random variable [23]. Thus, it is inexact to specify the performance of S+N systems using a constant noise analysis. Because of the above reasons, the SER for the proposed scheme is calculated via computer simulations.

V. APPLICATION TO EQUAL-GAIN COMBINING RECEIVERS

The system model of the EGC receiver employing VR M-PSK is shown in Fig. 2.

Consider a multichannel diversity reception system with L branches operating in a discrete-time channel, in which the receiver employs symbol-by-symbol detection. The signal received over the k th diversity branch, at the time instant i can be expressed as

$$r_k[i] = \Re\{ (a_k[i] e^{j\theta_k[i]} s[i] + n_k[i]) \} = \Re\{ R_k[i] \} \quad \text{----- (19)}$$

$$k = 1 \dots L$$

where $[i]$ is the random magnitude, $\theta_k[i]$ is the random phase of the k th diversity branch gain and $n_k[i]$, represents

the additive noise with

$$\{ [i]^n k [i] \} = Nk = 2\sigma^2 = N_0 \quad \text{----- (20)}$$

Assuming that the random phase $[i]$ are known at the receiver, the received signals are co-phased and transferred to baseband so that the signal at the k th branch will be

$$u_k[i] = \Re \{ a_k [i] s(t) + n_k [i] e^{j\theta_k [i]} \} \quad \text{----- (21)}$$

At the combination stage the signals $[i]$ are weighted and summed to produce the decision variable

$$u[i] = \sum_{k=1}^L w_k [i] u_k [i] \quad \text{----- (22)}$$

where $[i]$ is the weight of the k th branch. Then, applying Maximum Likelihood Detection (MLD), the combiner's output is compared with all the known possible transmitted symbols in order to extract the decision metric. The coherent equal-gain-combining (EGC) receiver [22, Ch. 9.3] cophases and equally weights each branch before combining and therefore does not require estimation of the channel (path) fading amplitudes, but only knowledge of the channel phase, in order the demodulator to undo the random phase shifts introduced on the diversity channels. As a result, for $[i]=1$, the output combined signal will be

$$u[i] = \sum_{k=1}^L u_k [i] \quad \text{----- (23)}$$

which is actually the summation of the S+N at each diversity branch. As a result, the VR M-PSK modulation scheme can be directly applied to EGC receivers with the decision on the modulation order to be based on

$$\psi[i] = \frac{u[i]^2}{N_0} = \frac{[\sum_{k=1}^L \Re \{ a_k [i] s(t) + n_k [i] e^{j\theta_k [i]} \}]^2}{N_0} \quad \text{----- (24)}$$

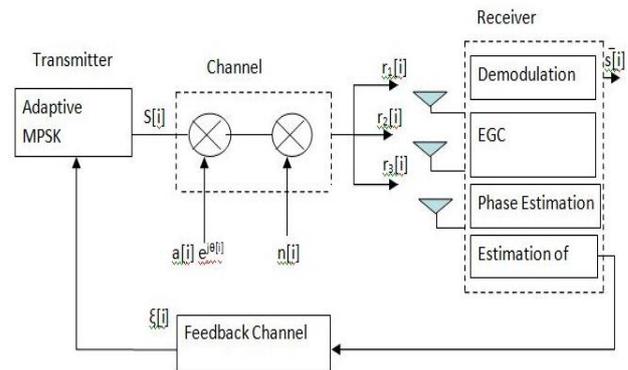


Fig.2.The Variable Rate M-PSK modulation scheme with EGC diversity technique.

VI. SIMULATION RESULTS

The difference between the S/N and S+N based adaptive M-PSK schemes in terms of spectral efficiency are shown in Fig.3 It is seen that both schemes achieve the same spectral efficiency, while the difference in their SER is not significant as indicated in Fig. 4. In general, it can be observed that the S+N can serve as an attractive criterion for determining the channel quality, when channel gain estimation is not available. Moreover, as expected, the efficiency of the S+N improves as the average S/N

increases, since the instantaneous noise components become small, so that the divergence between $(aE_S+n)^2$ and $a^2 E_S$ decreases.

The explanation for the efficiency of the $S+N$ criterion is as follows: in the conventional adaptive M -PSK, the decision on the modulation order is based on the instantaneous S/N , i.e. on $a^2 E_S/N_0$, which means that the instantaneous noise power is ignored, since only the average noise power is taken into account. On the contrary, the $S+N$ samples, $(aE_S+n)^2$ include the instantaneous noise component and therefore an indirect estimation of the instantaneous noise power is obtained. Similar results can be obtained also for the EGC employing adaptive M -PSK. As shown in Fig.6, the spectral efficiency is increased compared to a fixed rate M -PSK scheme, while the SER is maintained under the required levels, whenever this is possible.

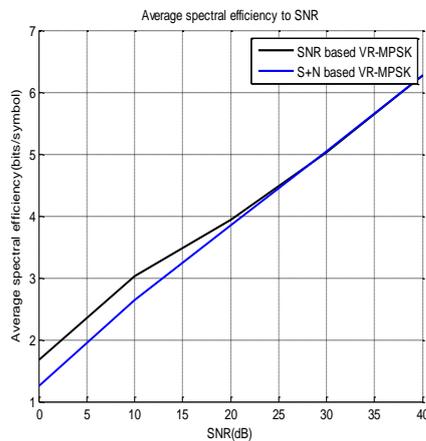


Fig.3. The spectral efficiency of VR M -PSK schemes for SNR and $S+N$ based systems.

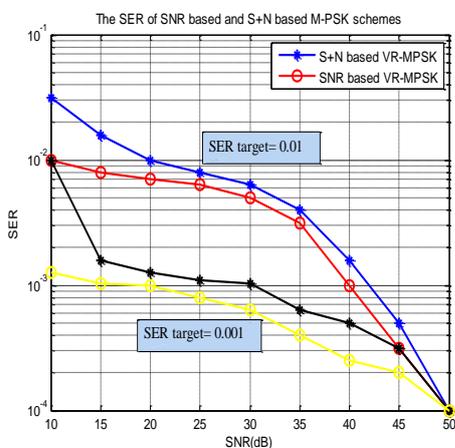


Fig.4. The SER of VR M -PSK schemes, for different SER targets.

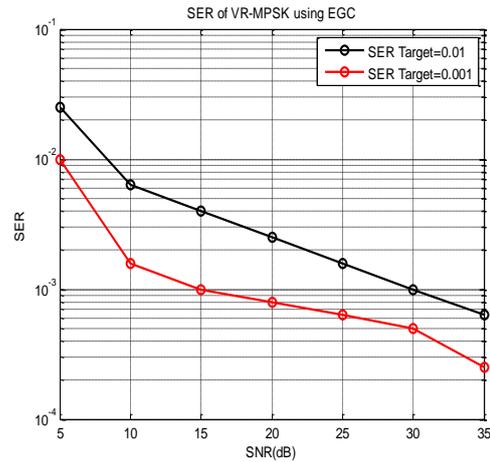


Fig.5. The SER of EGC with VR M -PSK schemes, for different SER targets.

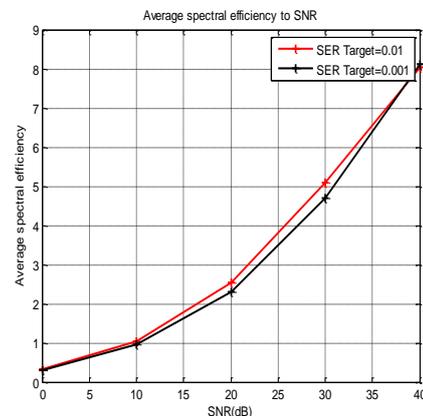


Fig.6. The spectral efficiency of EGC with VR M -PSK schemes, for different SER targets.

VII. CONCLUSIONS

A adaptive M -PSK modulation scheme was introduced for wireless communication systems without the requirement of channel gain estimation. Adaptive M -PSK also increases the spectral efficiency of M -PSK systems under the instantaneous error rate requirement. The choice of the modulation order is not based on the S/N samples, but rather on the $S+N$ ones. Also $S+N$ criterion is an attractive alternative to S/N for choosing the appropriate modulation order in adaptive communication systems, the same spectral efficiency can be achieved, when either S/N or $S+N$ is used. Moreover, the proposed adaptive modulation scheme was applied to EGC receivers, enabling low-complexity diversity systems to increase their spectral efficiency.

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